673. Parametric synthesis of rod spatial vibroisolation system under arbitrarily directed external disturbance

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Abstract. A mathematical model of flexible-rod spatial vibroisolating suspension, describing the motion of protected object, is considered. An algorithm is proposed for solving the problem of spatial displacement of the object under the action of static forces applied in an arbitrary direction. The coefficients of stiffness matrix of the suspension are determined depending on the position of static equilibrium. It is demonstrated that, depending on the requirements by varying geometrical parameters of the rods, different dynamic properties of the suspension may be obtained.

Keywords: vibroisolation, spatial suspension, flexible rods, numerical simulation.

1. Introduction

For vibration isolation of devices and equipment mounted on transport facilities different constructions of elastic suspensions are used [1]. In cases when external influences are oriented arbitrarily relative to the object of vibration isolation, three-dimensional systems with natural frequencies in different directions close to each other are used. Devices of navigation, connection, orientation have special importance of decreasing acting vibration loads [2].

In addition to vibration the object of vibration isolation may be under the action of constant accelerations oriented in any direction. These accelerations lead to change of position of static equilibrium of the object relative to which its oscillations occur. In a nonlinear system such change of position of equilibrium leads to changing of suspension stiffness [3].

In the present paper peculiarities of calculation of elastic characteristics are considered and results of determination of dynamic properties of three-dimensional suspension are presented. The suspension design is carried out on the basis of the system of planar curvilinear rods (Fig. 1) [4]. Here for definiteness it is accepted that the protected object has the form of a sphere, which is connected with external rigid case established directly on the vibrating foundation with the help of six elastic rods of constant curvature.

2. Suspension shape and design model

Position of structural elements in space and their motion is considered in moving coordinate system $OX_1X_2X_3$ with unit vectors $\mathbf{i}_1$, $\mathbf{i}_2$, $\mathbf{i}_3$. The system is rigidly bound to the case. The beginning of the chosen coordinate system coincides with the position of equilibrium of sphere center of mass in the undeformed state of the system.

Points of rods fastening on the case lie in the plane OX_2X_3 at equal distance from the point O. Three rods are above the plane X_2OX_3, three other rods lie under the plane X_2OX_3. The angle between the straight lines passing through the point O and the points of rod fastening on the case for each pair of adjacent rods lying in the same half-spaces is equal to 120^0. Such position of rods and the corresponding choice of their cross-section form allows to obtain close stiffness characteristics of the suspension in different directions.

We shall note that in the general case upper and lower groups of three rods can be rotated relative to each other at the arbitrary angle ψ in the projection on the plane OX_2X_3.

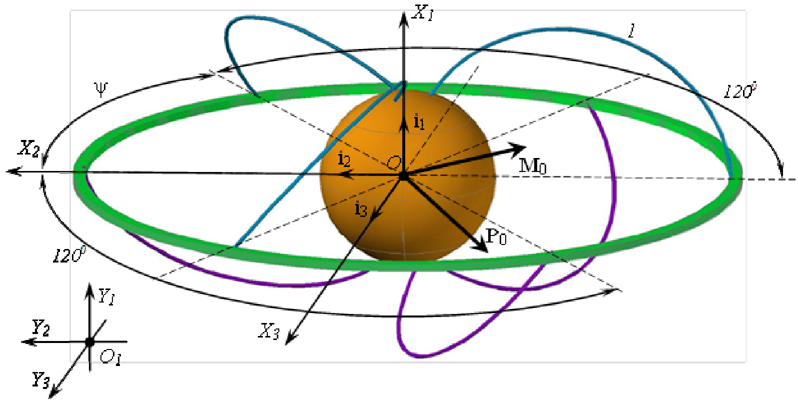

Fig. 1. Design model of elastic suspension

For the description of shape and position in space of each elastic element it is sufficient to consider one arbitrarily chosen element, e.g. element *1*. The elastic element (Fig. 1) in the initial state is a planar curvilinear rod of constant circular cross-section, which axial line is the arc of the circle of radius R (Fig. 2).

Both ends of the rod are rigidly bound to the case (point A, Fig. 2) and to the object (point B), and the point of fastening to the object lies in the plane OX_1X_2 (plane α). The position of point B is established by the angle φ_0 between straight lines OA and OB, and by the sphere radius R_0. In the general case the plane β, in which the rod axis lie, crosses the plane α in the line AB and makes with this plane an angle φ_1.The center of rod curvature (point O_c) lies in the plane β, and in the point A the tangent to the rod axis must be perpendicular to the line OA.

Elastic elements are made of homogeneous isotropic material with constant modulus of elasticity E and Poisson's ratio μ. The mass of the rods is negligibly small in comparison with the sphere mass and is therefore neglected.

Kinematic influence on the case is described by the case position vector $\mathbf{Y_0}$, i.e. the case motion is described by the coordinates of point O and angles of rotation of moving coordinate system $OX_1X_2X_3$ relative to the steady coordinate system $O_1Y_1Y_2Y_3$.

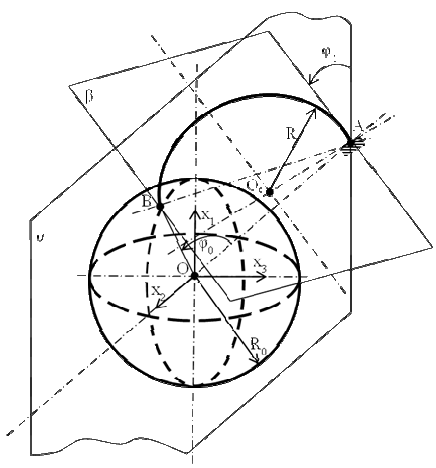

Fig. 2. Position of separate rod in space

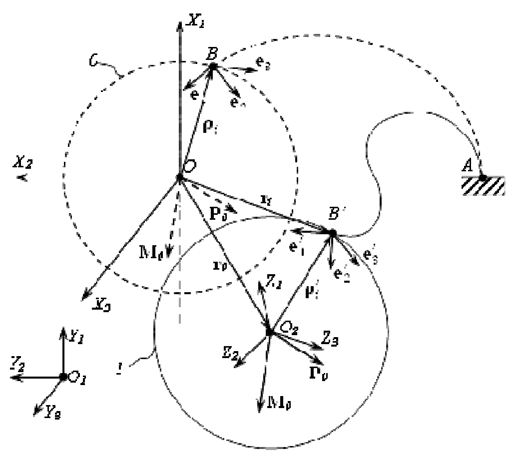

Fig. 3. To the derivation of kinematic relations for sphere displacements

The equation of the forced oscillations of the sphere in the moving system $OX_1X_2X_3$ is [5]:

$$\mathbf{G}\ddot{\mathbf{X}}(t)+\mathbf{C}\mathbf{X}(t)=\mathbf{Q}_0(t),$$

where **G** is the matrix of the system inertia parameters, **X** is the vector of the sphere position in the moving coordinate system $OX_1X_2X_3$, **C** is the suspension stiffness matrix, $\mathbf{Q}_0=-\mathbf{G}\ddot{\mathbf{Y}}_0$ is the vector of external action (vector of forces of moving space). Points indicate differentiation with respect to time.

External influence is specified as the vector of loads $\mathbf{Q}_0(t)=\mathbf{Q}_0{}^{c}+\mathbf{Q}_0{}^{v}(t)=\begin{bmatrix}\mathbf{P}_0\\ \mathbf{M}_0\end{bmatrix}$, where $\mathbf{Q}_0{}^{c}$ and $\mathbf{Q}_0{}^{v}(t)$ are respectively vectors of constant and vibrational components of forces of moving space, $\mathbf{P}_0$ and $\mathbf{M}_0$ are, respectively, vectors of forces and moments.

For description of sphere motion we shall enter coordinate system $O_2Z_1Z_2Z_3$ (Fig. 3) with unit vectors $\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3$. The system is roughly bounded to the sphere and its beginning (point O_2) coincides with its mass center. Then the sphere position in arbitrary point in space is described by the radius-vector of its mass center $\mathbf{r}_0=\overline{OO_2}$ and the vector of rotation angles $\boldsymbol{\vartheta}_0$ of coordinate system $O_2Z_1Z_2Z_3$ relative to the coordinate system $OX_1X_2X_3$, i.e. $\mathbf{X}(\mathbf{t})=\begin{bmatrix}\mathbf{r}_0\\ \boldsymbol{\vartheta}_0\end{bmatrix}$.

Position of the *i*-th rod bounding point on the sphere is described by the radius-vector $\boldsymbol{\rho}_i$ rigidly bound with the sphere, with the beginning in its mass center, and by the radius-vector $\mathbf{r}_i$, defining the position of the same point *B* relative to the point *O*, and the orientation of the rod section in this point is defined by rigidly bound with the sphere vectors $\mathbf{e}_{1,i}$, $\mathbf{e}_{2,i}$, $\mathbf{e}_{3,i}$ of the natural basis of the *i*-th rod.

The rotation of the sphere relative to the undeformed state of the suspension can be described with the help of matrix of rotation:

$$\mathbf{L}=\mathbf{L}_3\mathbf{L}_1\mathbf{L}_2, \tag{1}$$

$$\mathbf{L}_1=\begin{bmatrix}1 & 0 & 0\\ 0 & cos(\vartheta_1) & sin(\vartheta_1)\\ 0 & -sin(\vartheta_1) & cos(\vartheta_1)\end{bmatrix},\ \mathbf{L}_2=\begin{bmatrix}cos(\vartheta_2) & 0 & -sin(\vartheta_2)\\ 0 & 1 & 0\\ sin(\vartheta_2) & 0 & cos(\vartheta_2)\end{bmatrix},$$
$$\mathbf{L}_3=\begin{bmatrix}cos(\vartheta_3) & sin(\vartheta_3) & 0\\ -sin(\vartheta_3) & cos(\vartheta_3) & 0\\ 0 & 0 & 1\end{bmatrix},$$

where ϑ_1, ϑ_2, ϑ_3 are components of vector $\boldsymbol{\vartheta}_0$. Then the *i*-th rod bounding point coordinates in deformed state:

$$\mathbf{r}_i=\mathbf{r}_0+\boldsymbol{\rho}_i',\ \text{where}\ \boldsymbol{\rho}_i'=\mathbf{L}^{-1}\boldsymbol{\rho}_i, \tag{2}$$

and the vectors of natural basis of *i*-th rod in this state:

$$\mathbf{e}_{j,i}'==\mathbf{L}^{-1}\mathbf{e}_{j,i},\quad (j=1,2,3). \tag{3}$$

Thus for investigation of vibro-protecting properties of the suspension it is necessary to establish its stiffness matrix coefficients and the sphere displacements vector. The stiffness matrix coefficients are established from the solution of curvilinear rods static problem depending on the sphere position of static equilibrium in space.

In the present work the problem of probable sphere positions under the influence of set static load determination and, therefore, the problem of stiffness matrix coefficients and natural frequencies corresponding to these sphere positions in space determination is solved.

3. Mode of deformation of arbitrary rod.

Mode of deformation of i-th rod is described by the equations of its element static equilibrium, the equations of displacements of its axis points and the equations connecting the moments and curvature of the rod [6, 7]. The position of rod section is explicitly defined by the basis unit vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$, connected with the axial line and cross-section of the rod (here and further for this basis the indices corresponding to the rod number are omitted). The unit vector $\mathbf{e}_1$ directed along the tangent to the rod axial line, and the unit vectors $\mathbf{e}_2, \mathbf{e}_3$ are directed along the centroidal principal axes of the section (Fig. 3).

Equilibrium equations have the standard form:

$$\frac{d\mathbf{P}}{ds} = 0, \quad \frac{d\mathbf{M}}{ds} + \mathbf{e}_1 \times \mathbf{P} = 0, \tag{4}$$

where s is the section angular position, **P** and **M** – force and moment acting in the cross section. Unlike [6], in the present work the position of rod element is described with the help of radius-vector **r** and vectors of natural basis, expressed in terms of the rod curvature vector in deformed state $\boldsymbol{\chi} = \chi_1\mathbf{e}_1 + \chi_2\mathbf{e}_2 + \chi_3\mathbf{e}_3$, where χ_1, χ_2, χ_3 are torsion and curvatures of the deformed rod, i.e.:

$$\frac{d\mathbf{r}}{ds} = \mathbf{e}_1\,; \frac{d\mathbf{e}_1}{ds} = \boldsymbol{\chi} \times \mathbf{e}_1\,; \frac{d\mathbf{e}_2}{ds} = \boldsymbol{\chi} \times \mathbf{e}_2\,; \frac{d\mathbf{e}_3}{ds} = \boldsymbol{\chi} \times \mathbf{e}_3. \tag{5}$$

In projections on the axes $OX_1X_2X_3$ the vector:

$$\boldsymbol{\chi} = k_1\mathbf{i}_1 + k_2\mathbf{i}_2 + k_3\mathbf{i}_3, \text{ where } k_j = \sum_{i=1}^{3} e_{ij}\chi_i, \ (e_{ij} = \mathbf{e}_i \cdot \mathbf{i}_j). \tag{6}$$

The dependence of bending and torque moments in the section on the increment of curvature in the case of small rod deformations (displacements and rotations can be finite) is linear:

$$\mathbf{M} \cdot \mathbf{e}_1 = GJ_1(\chi_1 - \chi_{10}); \mathbf{M} \cdot \mathbf{e}_2 = EJ_2(\chi_2 - \chi_{20}); \mathbf{M} \cdot \mathbf{e}_3 = EJ_3(\chi_3 - \chi_{30}), \tag{7}$$

where EJ_2, EJ_3 are principal bending stiffnesses of the section, GJ_1 – torsional stiffness of the section, $\chi_{10}, \chi_{20}, \chi_{30}$ – components of rod curvature vector in the undeformed state.

The system of equations (4)-(7) contains 18 unknown variables: forces and moments vectors (**P** and **M**) components, radius-vector **r** and unit vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ components. The rod curvatures χ_{20}, χ_{30} and torsion χ_{10} components in undeformed state are considered stated.

Boundary conditions at $s=0$ (point A of bounding the rod to the case):

$\mathbf{r}(0) = \mathbf{r}_A; \quad \mathbf{e}_1(0) = \mathbf{e}_{1,A}; \quad \mathbf{e}_2(0) = \mathbf{e}_{2,A}; \quad \mathbf{e}_3(0) = \mathbf{e}_{3,A}.$

At $s=l$ (point B of bounding the rod to the sphere): $(l) = \mathbf{P}_B; \quad \mathbf{M}(l) = \mathbf{M}_B$.

4. Determination of static equilibrium position of the sphere under the influence of constant external forces

For determination of stiffness matrix coefficients corresponding to probable sphere equilibrium positions we shall consider the suspension loaded in the general case by the constant force $\mathbf{P}_0$, applied to the sphere mass center, and constant moment $\mathbf{M}_0$, acting in arbitrary directions.

For the solution of statically indeterminate problem of sphere displacement we developed an algorithm, which essence is to state arbitrary sphere displacements $\tilde{\mathbf{X}}_0$ instead of the load $\mathbf{Q}_0$ and to establish corresponding loads $\mathbf{Q}_0(\tilde{\mathbf{X}}_0)$. The algorithm is oriented at iterative search of such displacements for which $\mathbf{Q}_0(\tilde{\mathbf{X}}_0) \to \mathbf{Q}_0$. According to the proposed algorithm the vector components of the sphere position are considered as variable

$\tilde{\mathbf{X}}_0 = [C_1 \quad C_2 \quad C_3 \quad C_4 \quad C_5 \quad C_6]^T.$

When stating C_i arbitrarily the displacements of rod bounding points on the sphere are defined by the formulas (1)-(3). Reactions $\mathbf{P}_i$ and $\mathbf{M}_i$ in these points are found from the solution of elastic rod static problem when displacements are given [6, 8, 9], and the components $\mathbf{Q}_0(\tilde{\mathbf{X}}_0)$ of loads vector corresponding to this sphere position of equilibrium will be defined from the equilibrium equations:

$$\mathbf{P}_0 = \sum_{i=1}^{6} \mathbf{P}_i, \quad \mathbf{M}_0 = \sum_{i=1}^{6} (\mathbf{M}_i + \boldsymbol{\rho}_i \times \mathbf{P}_i). \tag{8}$$

For $\tilde{\mathbf{X}}_0$ to be the searched solution $\mathbf{X}_0$ the following condition must be met:

$$\mathbf{Q}_0(\tilde{\mathbf{X}}_0) - \mathbf{Q}_0(\mathbf{X}_0) = \mathbf{F}(C_1, C_2, C_3, C_4, C_5, C_6) = 0. \tag{9}$$

As a result, determination of sphere displacements comes to solution of algebraic equations of the nonlinear system (9), which is carried out using Newton method together with the continuation of the parameter of loading [6, 8, 9].

5. Suspension stiffness matrix

For determination of suspension stiffness matrix **C** in the considered equilibrium state, which is defined by the position vector $\mathbf{X}_0$ and loads vector $\mathbf{Q}_0$, it is necessary to define the resulting vector of reactions of all the elastic elements brought to the sphere mass center. The vector appears when moving the sphere in the specified direction from this equilibrium state. For example, we shall specify a small displacement to the sphere from the current equilibrium position in the direction OX_1. Then the new position is defined by the vector:
$\mathbf{X}_0' = \mathbf{X}_0 + \Delta\mathbf{X} = [x_{10} + \Delta x_1 \quad x_{20} \quad x_{30} \quad x_{40} \quad x_{50} \quad x_{60}]^T$,
and the coordinates of rods bounding points on the sphere are defined by the formulas (1)-(3). Reactions in these points will be defined from the solution of elastic rod static problem when the displacements of one of its end are specified, and the components of loads vector $\mathbf{Q}_0'$ corresponding to this sphere equilibrium position will be defined by the formulas (8). Then the components of the first column of stiffness matrix will be specified by the formula:
$c_{i1} = \frac{\Delta Q_i}{\Delta x_1}, i = 1 \dots 6$, where $\Delta Q_i = Q_{0i}' - Q_{0i}$.
Similarly, specifying small displacements or angles of sphere rotation in other directions, other stiffness matrix components can be determined.

6. Choice of rational positioning parameters for suspension elastic elements

Natural frequencies p of the suspension determined from the equation $\det(-\mathbf{G}\mathbf{p}^2 + \mathbf{C}) = 0$ [5] with specified dimensions of the case and sphere diameter depend on the parameters φ_0 and φ_1 (Fig. 2). Rational values of parameters φ_0 and φ_1 must satisfy the following criteria:

- Minimum of maximum natural frequency value, i. e.

$$\Phi_1(\varphi_0, \varphi_1) = \min \max(\mathrm{p}(\varphi_0, \varphi_1)); \tag{10}$$

- The difference in the values of the system natural frequencies doesn't exceed a priory specified value λ, which provides invariance of the suspension dynamic properties relative to the excitation vector, i. e.

$$\Phi_2(\varphi_0, \varphi_1) = \frac{\max(\mathrm{p}(\varphi_0, \varphi_1))}{\min(\mathrm{p}(\varphi_0, \varphi_1))} \leq \lambda. \tag{11}$$

We shall note that in this statement the material and cross-section diameter of the rod do not matter.

The choice of rational parameters φ_0 and φ_1 is carried out for the suspension equilibrium position corresponding to its initial undeformed state. Naturally, the obtained rational values of the parameters φ_0 and φ_1 can change in other positions of equilibrium of the system. However, as the comparison results showed, the dispersion of these parameters values turned out to be practically non-sensitive to the state of static equilibrium.

7. Results of calculation

With the purpose of choosing rational rods positioning in suspension structure we carried out calculation of its natural frequencies with stated mass and dimensional parameters of the structure and variable parameters $\varphi_0 = [0^0, 180^0]$ and $\varphi_1 = [0^0, 90^0]$. For definiteness, the value of the criterion (11) takes λ = 1, 2. The calculation was carried out numerically in MatLab. As a result of calculation we obtained the graph of suspension maximum natural frequency variation (Fig. 4) and the graph of variation of the ratio of maximum frequency to minimal one (Fig. 5) depending on the angles φ_0 and φ_1, which characterize length and spatial positioning of the rods.

From the graph in Fig. 4 it is obvious that excluding a relatively small area near zero values of parameters φ_0 and φ_1, monotonous decreasing of the values of suspension natural frequencies can be observed at increasing parameter φ_0. It is explained by increasing of the rod axial line length at the expense of increase in distance between its bounding points on the case and on the sphere one from another, which leads to decreasing rod stiffness and therefore decreasing stiffness of the whole suspension. The influence of parameter φ_1 on the value of maximum natural frequency is non-monotonous, which substantially depends on the value of the parameter φ_0. Minimal values of suspension maximum natural frequencies are achieved at $\varphi_0 = 180^0$ and $\varphi_1 \approx 40^0$.

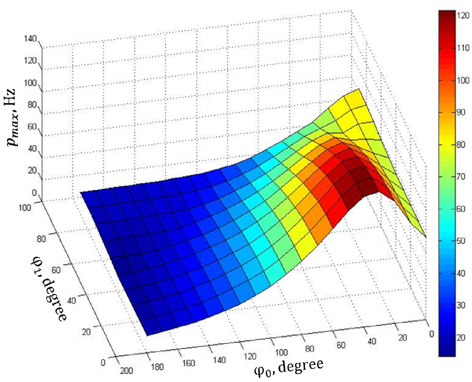

Fig. 4. Dependency of the suspension maximum natural frequencies on the angles φ_0 and φ_1

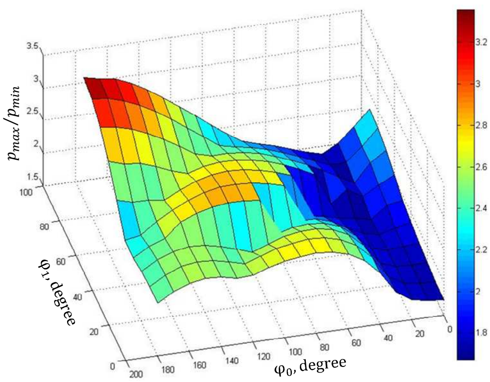

Fig. 5. Dependency of the ratio of suspension maximum natural frequency to minimal frequency on the angles φ_0 and φ_1

From the analysis of Fig. 5 it follows that the minimum value of the ratio of maximum natural frequency to the minimal one is achieved at zero values of parameters φ_0 and φ_1 and is 1,2. When increasing these parameters one can observe a non-monotonous (with expressed local extremes) changing of frequencies ratio with a tendency to its increasing.

So in the case of specified mass and dimensional parameters of the suspension structure it is impossible to satisfy simultaneously the criteria (10) and (11). So for the choice of rational parameter values it is necessary to establish the priorities of the criteria on the basis of the demands for the suspension functional characteristics.

For effective vibro-protection in low-frequency area the most important is criterion (10) of maximum natural frequency minimum, according to which rational values $\varphi_0 = 180^0$ and $\varphi_1 \approx 40^0$. For the chosen values of φ_0 and φ_1 the values of natural frequencies are represented in the Table 1, and the ratio of suspension maximum natural frequency to minimum one is 2,2.

From the Table 1 it follows that except for the lowest natural frequency all the other frequencies differ from one another at maximum by 20%.

Table 1. Suspension natural frequencies in undeformed state at $\varphi_0 = 180^0$ and $\varphi_1 \approx 40^0$.

Frequency No.	1	2	3	4	5	6
Frequency, Hz	6,49	12,06	12,06	13,67	13,67	14,28

Fig. 6 and 7 present the diagrams of changing of maximum and minimum suspension frequencies at chosen rational values of parameters φ_0 and φ_1 depending on the value and direction of the static force applied to the sphere mass center (the force acts in the plane OX_1X_2, and its direction is determined by the angle γ, which is counted from the negative direction of the vertical axis OX_1). From Fig. 6 it is obvious that when increasing the external force maximum, the value of natural frequencies initially decreases and, having reached its minimum, increases. For a given force, increasing of the angle γ from zero to 90^0 leads to an increase in maximum natural frequency. We shall note that changes of maximum natural frequency at certain values of force and its direction do not exceed 2,8%. When increasing static force and angle γ the value of minimum natural frequency increases (Fig. 7).

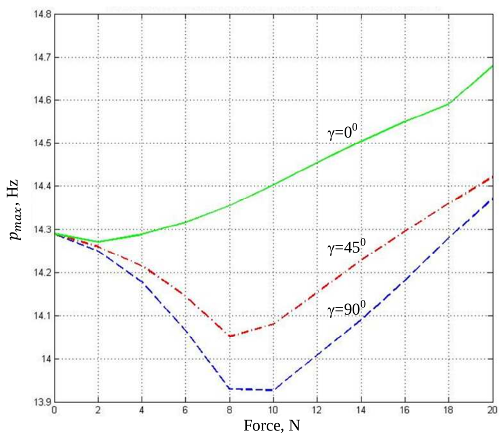

Fig. 6. Maximum natural frequency as a function of static force

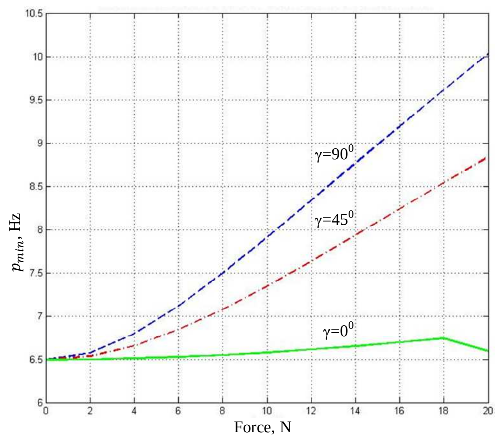

Fig. 7. Minimum natural frequency as a function of static force

8. Conclusions

The present paper proposed the mathematical model describing the motion of vibro-isolated object on the elastic suspension, which is a spatial system of six massless curvilinear rods.

The algorithm of solving the problem of object spatial motion under the influence of static forces applied in arbitrary directions is developed. Suspension stiffness matrix coefficients are determined depending on static equilibrium position.

The problem of the choice of rational values of geometrical parameters of rods determining their length and plane of location is considered.

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The values of suspension natural frequencies depending on the modulus and direction of external force are established for the given mass and dimensional parameters of suspension and chosen values of rods geometrical parameters.

It is demonstrated that, depending on the requirements, at the expense of varying the rods geometrical parameters one can obtain different dynamical properties of the suspension: minimum value of the highest natural frequency or minimum difference of natural frequencies.

References

[1] Vibrations in Engineering: Hangbook. In 6 Volumes. V. 6. Protection from Vibrations and Impacts. Moscow: Mechanical Engineering, 1981. – 456 p.
[2] **Gouskov A. M., Korovaytseva E. A., Panovko G. Ya., Shokhin A. E.** Investigation of external vibratory field influence on quartz oscillator dynamics. Mechanical Industry and Engineering Education, 2011, No. 3. p. 22-28.
[3] Vibrations in Engineering: Handbook. In 6 Volumes. V. 2. Oscillation of Nonlinear Mechanical Systems. Moscow: Mechanical Engineering, 1979. – 351 p.
[4] **Panovko G. Ya., Shokhin A. E.** Synthesis of rod spatial suspension. Scientific articles «Control Vibration Technologies and Machines» in 2 Volumes. V. 2. Kursk: Kursk State Technical University, 2010. – p. 80-88.
[5] **Biderman V. L.** Theory of Mechanical Oscillations. Moscow: Vysshaya Shkola, 1980. – 408 p.
[6] **Svetlitsky V. A.** Mechanics of Rods: P. I. Statics (in Russian), Moscow: Vysshaya Shkola, 1987. – 320 p.
[7] **Svetlitsky V. A., Naraykin O. S.** Elastic Elements of Machines. Moscow: Mechanical Engineering, 1989. – 264 p.
[8] **Usyukin V. I.** Structural Mechanics of Structures of Space Technology. Moscow: Mechanical Engineering, 1988. – 392 p.
[9] **Ilyin V. G., Karpov V. V., Maslennikov A. M.** Numerical Methods for Solving Problems of Structural Mechanics. Minsk: Vysshaya Shkola, 1990. – 349 p.