669. A model of a thermal feedback in a biological object taking into account the processes of thermal self-regulation and their dynamics

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Abstract. The paper proposes a model for temperature distribution in an organism taking into account the internal heat sources and the processes of thermal self-regulation. The model describes temperature distribution and the processes of its self-regulation in hypothermal zones, where abnormal phenomena in a biological object are accompanied by a decrease of temperature. The presented model can be applied for diagnostics of inflammatory processes and tumors as well as for control of their dynamics by means of thermography.

Keywords: biological object, thermography, model of temperature distribution, processes of self-regulation and their dynamics.

Introduction

The obtained distribution of internal temperature provides an opportunity for the assessment of biomedical applications of the method while using thermographs [1, 2] as well as for a correct interpretation of the thermal image on examining abnormal thermal zones on surface of the skin integument of biological objects upon applying the method of infrared (IR) thermography. This is very important for the control of the functional status of the organism *in vivo* [3-5]. In the paper [6], the author proposed a model of the thermal field in a pathologic zone by applying the method of solving electrostatic problems based on an analogy between the electrostatic field and the thermal one. However, in the presented work, the processes of self-regulation of temperature in a living organism were not taken into account. Exothermal biochemical processes in cells and tissues of all internals of biological object result in generation of heat that is redistributed in the organism. The said continuous process takes place within the total period of vital activity of biological object upon essential interaction with the environment. The thermal disorders carry diagnostic information on the functional status of the organism and may be used for interpretation of thermographic images as indicators (markers) of various pathological conditions. Therefore for perception of the mechanism of the thermal disorders it is important to take into account the processes of self-regulation while analyzing the algorithm of formation of the surface temperature of biological object.

Theoretical background and temperature distribution in a living organism

As it is known, the processes of temperature redistribution are described by the thermal conductivity equation that is based on the condition of "continuity" of thermal energy flow [7]:

$$\frac{\partial q}{\partial t} + div\vec{j} = \dot{Q}, \qquad (1)$$

where $q = c\rho(T - T_0)$ – density of thermal energy; c, ρ – thermal capacity and density of the substance, respectively; T – ambient temperature with respect to a certain fixed value T_0; $\vec{j}$ –

vector of density of the thermal flow; $\dot{Q}$ – power of the sources of heat exposed or absorbed by the environment.

Density of thermal flow is supposed to be proportional to the thermal gradient:

$$\vec{j} = -\chi \nabla T, \tag{2}$$

where χ – coefficient of proportionality (coefficient of thermal conductivity); $\nabla = e_x \frac{\partial}{\partial x} + e_y \frac{\partial}{\partial y} + e_z \frac{\partial}{\partial z}$ – nabla operator; $\vec{e}_i$ – unit vectors of the system of coordinates.

After insertion of (2) in (1), the thermal conductivity equation will be acquire the form:

$$c\rho \frac{\partial T}{\partial t} - \nabla(\chi \nabla T) = \dot{Q}, \tag{3}$$

In terms of mathematics, the equation (3) is the one of a parabolic type. In a general case, its coefficients vary from point to point. For a homogenous environment with constant coefficients, the equation (3) is simplified considerably and transformed into an equation of the following type:

$$c\rho \frac{\partial T}{\partial t} - \chi \Delta T = \dot{Q}(\vec{r}, t), \tag{4}$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ – Laplace operator; $\vec{r} = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z$ – radius-vector.

The boundary conditions should be introduced for unambiguous determination of the thermal field:

$$T_i = T_j$$

$$\chi_i \frac{\partial T_i}{\partial n} = \chi_j \frac{\partial T_j}{\partial n} \bigg| \vec{r} \in S_{ij} \tag{5}$$

where S_{ij} – the conventional symbol of the interface between the *i*-th and *j*-th sector of the environment; $\frac{\partial}{\partial \vec{n}}$ − partial derivative with respect to the normal to the interface.

If the object is in thermal balance with the environment, its temperature is stable and $\frac{\partial T_j}{\partial t} = 0$. In such a case, the thermal conductivity equation (4) is simplified considerably:

$$-\chi_i \Delta T_i = \dot{Q}_i(\vec{r}) \tag{6}$$

In equation (6) it was supposed that power of heat sources does not depend on time. Now the equation (6) coincides with Poisson equation. This theory is well developed in the field of electrostatic problems [8]. This similarity was used in the work [6], where a tumor is considered a specific source $\dot{Q}$ and the thermal properties of the body are described by a certain coefficient of heat conductivity χ. The task includes determination of the depth and shape of the formations according to their thermal projection on the surface of skin integument. It shows a principal possibility of diagnostics of inflammatory processes by means of thermography. If temperature of biological objects falls below T_0~37°C (the internal temperature of the body), the oxidation processes in muscular tissues are activated and cause heat generation. In other words, when the temperature of the muscular tissue is below T_0, the tissue emits heat. When the temperature increases up to $T \geq T_0$, heat emission within the said sectors cancels and thermal balance is possible due to the heat exchange with environment through the skin surface. The sweating processes accelerate cooling of the body [9]. Therefore the mechanisms of tissue heating and cooling are quite different. It may be concluded from the above described that in case of benign tumors when the temperature of muscular tissue of the body falls, the internal heat sources are activated and they equalize the temperature $T \to T_0$. The said processes cause a specific temperature distribution in the organism of biological object. We will demonstrate the specific peculiarities of temperature distribution caused by the self-regulation processes in a simple example. Taking into account that temperature in internal tissues of the body strives to T_0, and outside is a thermostat with the temperature $T_e < T_0$, the temperature of the surface layer will lie in the range $T_e < T < T_0$. The temperature of muscular tissues close to the surface of the body is $T < T_0$, therefore they emit heat. The process of heat emission in muscular tissues can be accounted upon analyzing the power of internal heat sources $\dot{Q}$. Their intensity depends on the difference $T - T_0$. The dependence $\dot{Q}(T - T_0)$ may vary. But if the difference of temperatures is small $\frac{T - T_0}{\Delta T} \leq 1$, where ΔT – the temperature range (from 42°C to 35°C) tolerable by a living organism for a long time, the power of internal heat sources can be expressed as power expansion of $T - T_0$:

$$\dot{Q}(T - T_0) = \gamma(T_0 - T) + ..., \tag{7}$$

where γ – heat generation coefficient which value depends on the location of the analyzed sector and the physical condition of the body. Taking into account the internal heat source (7) the thermal conductivity equation (6) can be written in the following form:

$$\begin{aligned} &\delta^2 \Delta T = (T - T_0); \quad T - T_0 < 0 \\ &\Delta T = 0; \qquad\qquad T - T_0 > 0, \end{aligned} \tag{8}$$

where $\delta = \sqrt{\frac{\chi}{\gamma}}$ – the length of relaxation of inhomogeneous temperature.

We will clarify the physical meaning of the value δ by solving a thermal problem for the structure shown in Fig. 1.

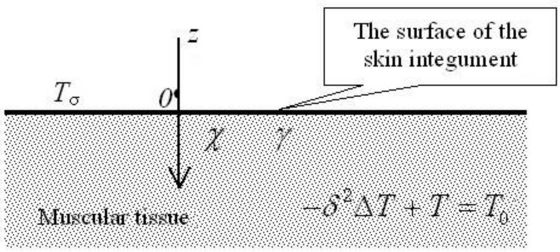

Fig. 1. Schematic view of the structure: T – temperature of the body; χ – coefficient of heat conductivity of tissues of the body; γ –heat generation coefficient

In this case, we'll confine ourselves to temperature distribution in internal tissues of the body. It is evident that upon a homogenous status of an organism, its temperature in the plane *x0y* depends on the coordinate *z* only. The temperature T_σ on the surface of the body is provided as a boundary condition. If the environmental temperature is lower than T_0, it is evident that $T_\sigma < T_0$. In this a case, we do not need to analyze the details of the process of generation of the thermal balance between a biological object and the environment. It may be based on various processes (diffusion, convection, thermal radiation) when the body releases the heat. If a thermograph is available, a reliably measured value is namely distribution of skin integument temperature T_σ. For determination of temperature distribution in biological object the following boundary value problem should be solved:

$$-\delta^2 \frac{d^2T}{dz^2} + T = T_0 \cdot \quad (9)$$

Upon the following boundary conditions:

$$\begin{cases} T = T_\delta; & z = 0 \\ T \to T_0; & z \to \infty \end{cases} \quad (10)$$

Solution of the boundary value problem (9), (10) provides the following result:

$$T(z) = T_\delta + (T_0 - T_\delta)(1 - e^{-z/\delta}) \quad (11)$$

The diagram of temperature distribution is presented in Fig. 2.

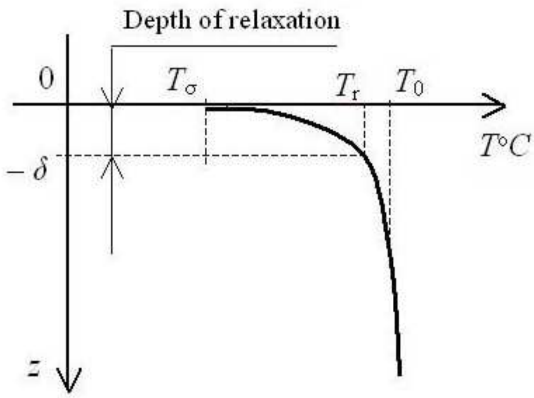

Fig. 2. Temperature distribution in the frame of solving the thermal problem (see Fig. 1): T_r − temperature of relaxation

In the depth δ, temperature T of the body slightly differs from the nuclear temperature T_0. Therefore we consider δ to be the depth of relaxation of inhomogeneous temperature of the subsurface layer where the difference between the nuclear and skin integument temperature falls e times. It equals to:

$$\frac{T_0 - T_r}{T_0 - T_\delta} = \frac{1}{e} \approx \frac{1}{2.71} \quad (12)$$

If the processes of the self-regulation of temperature are taken into account, it is found that the temperature of the body differs from the nuclear temperature T_0 only in the subsurface layer with the thickness δ. On going deep into the body, inhomogeneity of the temperature decreases according to the exponential law, while the approach proposed in work [6] provides the dependence according to the power-law.

Analysis on a local bloodstream disorder in muscular tissues

Let us suppose that a disorder of bloodstream results in reduction of the intensity of the above-mentioned internal heat sources. If a zone of a disorder of bloodstream is of a small volume $V \le \delta^3$, it may be considered as a spot object. The location of such a zone is preset by Dirac δ-function and the equation (6) is expressed as follows:

$$-\delta^2\Delta T + T = T_0 - \frac{\Delta\gamma}{\gamma}V(T_0 - T)\delta(r - r_0), \tag{13}$$

where $\Delta\gamma$ – deflection of the coefficient of heat sources intensity in a zone of pathology from the normal value γ; V – volume of the zone of pathology; $r_0 = (x_0, y_0, z_0)$ – coordinates of the abnormal area; $\delta(r - r_0) = \delta(x - x_0)\delta(y - y_0)\delta(z - z_0), \delta(x - x_0)$ – Dirac δ-function.

It may be concluded from equation (1) that the point heat source is "activated" when the temperature of the abnormal zone falls under the nuclear temperature T_0.

Solution of the equation (13) may consist of two parts:

$$T = T_1 + T_2 \tag{14}$$

The first part T_1 is defined in equation (11) and it describes temperature distribution in tissues without the pathologic changes

$$T_1 = T_\delta + (T_0 - T_\delta)(1 - e^{-z/\delta}) \tag{15}$$

The second part T_2 describes the impact of the spot abnormality of internal heat sources. The equation describing the abnormalities of the thermal field is expressed as follows:

$$-\delta^2\Delta T_2 + T_2 = -\frac{\Delta\gamma}{\gamma}V(T_0 - T_1(z_0))\delta(x - x_0)\delta(y - y_0)\delta(z - z_0) \tag{16}$$

For unambiguous determination of temperature distribution, the boundary conditions should be preset. It is evident that far from the abnormal zone, the temperature of the surface of the body equals T_δ. In addition, heat streams on the body surface should satisfy a certain condition:

$$\chi_e\frac{\partial T_2^e}{\partial z} = \chi_i\frac{\partial T_2^i}{\partial z}, \text{ if } z = 0 \tag{17}$$

The condition (17) results from inhomogeneity of heat stream on the interface of media. The index "*i*" identifies physical values inside the body, while the index "*e*" identifies parameters of the environment.

If $\chi_i \geq \chi_e$, for example, for outdoors situation the boundary conditions may be replaced by simpler ones:

$$\frac{\partial T_2^i}{\partial z} = 0 \text{ if } z = 0. \tag{18}$$

Such a simple boundary condition (18) will allow excluding the heat exchange processes in environment on solving the boundary value problem. The boundary problem for determination of temperature inhomogeneity caused by a presence of an abnormal zone in muscular tissues can be expressed as follows:

$$-\delta^2\Delta T_2^i + T_2^i = -\frac{\Delta\gamma}{\gamma}V(T_0 - T_1(z_0))\delta(x)\delta(y)\delta(z - z_0)$$

$$\frac{\partial T_2^i}{\partial z} = 0, \text{ if } z = 0; \ T_2^i \to T_\delta, \quad z = 0, \ \rho \to \infty \tag{19}$$

where $\vec{\rho} = x\vec{e}_x + y\vec{e}_y$.

In this case it is supposed that $x_0 = y_0 = 0$.

Therefore the solution of the boundary problem (19) will be expressed as follows:

$$T_2^i = -\frac{\Delta\gamma}{\gamma}\frac{V}{4\pi\delta^2}(T_0 - T_1(z_0))\left(\frac{e^{-\sqrt{\rho^2+(z-z_0)^{2/\delta}}}}{\sqrt{\rho^2+(z-z_0)^2}} + \frac{e^{-\sqrt{\rho^2+(z+z_0)^{2/\delta}}}}{\sqrt{\rho^2+(z+z_0)^2}}\right) \quad (20)$$

The ratio (20) enables determining the temperature distribution in the volume of a biological organism. However, non-invasive methods provide the values or the thermal field on the surface of biological object only, i.e. when $z = 0$:

$$T^i(r)_{z=0} = T_\sigma - \Theta\cdot\frac{\exp\left((-z_0-\sqrt{\rho^2+z_0^2})/\delta\right)}{\sqrt{\rho^2+z_0^{2/\delta}}} \quad (21)$$

where $\Theta = \frac{\Delta\gamma}{\gamma}\frac{V}{2\pi\delta^3}(T_0 - T_\sigma)$ – representative temperature of a zone with a thermal abnormality.

A schematic distribution of thermal field on the body surface close to a zone with a thermal abnormality is provided in Fig. 3.

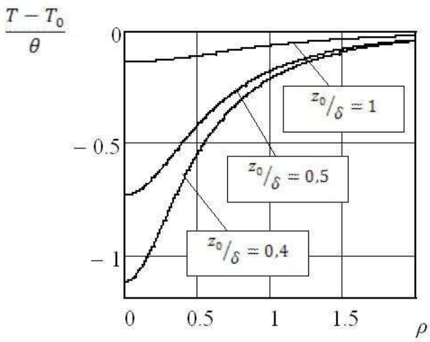

Fig. 3. Distribution of thermal field on the surface of skin integument when *z*=0

It may be noticed from (9) that observation of spot zones with thermal abnormality is possible on the depth δ. If the depth exceeds the said value, thermal inhomogeneity on the surface of biological object will be slightly expressed, because spatial relaxation of temperature takes place in accordance with the exponential law. Finally, we will mention an important particular case when information about the depth of a spot thermal abnormality of biological object may be obtained. Then the depth of thermal inhomogeneity is not large, i.e. $\delta \geq z_0$, the expression (9) for the zone close to its projection to the surface $\rho \geq \delta$ may be transformed as follows:

$$\frac{T_\sigma - T}{\theta} = \frac{1}{\sqrt{(\rho/\delta)^2 + (z_0/\delta)^2}} \quad (22)$$

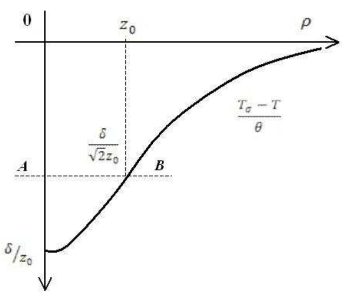

Fig. 4. Distribution of thermal field on the skin integument surface, when $z_0 \leq \delta$

The diagram of the dependence of $\frac{T_\sigma - T}{\theta}$ on ρ is provided in Fig. 4. It may be concluded from the diagram that crossing point of the curve $\frac{T_\sigma - T}{\theta}$ by the straight AB lying on the height equal to $\frac{1}{\sqrt{2}}$ of the maximum value is situated in the distance z_0 from the point 0 that is the projection of the thermal abnormality onto the skin surface.

The methodology of establishing the heat generation coefficient

For establishing the parameter δ, a stationary thermographic image resulted by the impact of the spot cooling source upon skin integument was used. Zone of the right forearm was brought into contact with the tip of a container filled with a water/ice mixture upon temperature $T = 0^0 C$. Because of a small cross-section of the tip, its impact upon skin integument can be considered spot. The duration of the contact was 3–4 minutes. It was sufficient for setting a steady temperature distribution in the zone of the contact point. The thermograph ThermaCAM E300 was used for measurement of the temperature. Data processing was performed using software Microsoft Excel 2003 and MathCAD 13. The thermal image of the skin surface that was in contact with the spot cooling source upon $T = 0^\circ C$ and the relevant graph of shearing temperature are presented in Fig. 5.

It is evident that the theoretical dependence (21) is not applicable to a zone of the point of contact because of divergence $T \xrightarrow{\rho \to 0} -\infty$ (see Fig. 6). Therefore calculation of δ was carried out on a peripheral sector of the graph of shearing temperature.

The input data for the problem solution include the predetermined temperature distribution on skin surface $T(\rho)$ and the average value of temperature of skin integument before the contact.

Let's discuss upon two points of the graph $T_1(\rho_1)$ and $T_2(\rho_2)$ and write down a system of equations for them:

$$T_1 = T_\sigma - \theta \frac{e^{-\frac{\rho_1}{\delta}}}{\rho_1 / \delta},$$

$$T_2 = T_\sigma - \theta \frac{e^{-\frac{\rho_2}{\delta}}}{\rho_2 / \delta}. \qquad (23)$$

Here, the parameters δ and θ are unknown. Upon excluding θ, we find the equation for δ:

$$\delta = \frac{\rho_2 - \rho_1}{\ln\left(\frac{\rho_1}{\rho_2} \cdot \frac{T_\sigma - T_1}{T_\sigma - T_2}\right)}. \qquad (24)$$

The value $a^2 = \frac{\chi}{c \cdot \rho}$, (where c and ρ – heat capacity and the density of the body, respectively) was found in [10]. But $\delta = \sqrt{\frac{\chi}{\gamma}}$, so:

$$\gamma = \frac{a^2}{\delta^2} c\rho. \qquad (25)$$

The depth of relaxation of temperature inhomogeneity $\delta \approx 1{,}259\,cm$ is determined by using the formula (24) for various values of ρ and T. Upon using the normal value a^2 for a pathology-free sector of the body $a^2 = 7{,}4\times10^{-4}\dfrac{cm^2}{s}$ [10] and taking into account the average values of the heat capacity and the body density ($c = 1{,}51\times10^7 \dfrac{sm^2}{s^2\cdot{}^0C}$ and $\rho = 1{,}036\dfrac{g}{sm^3}$), we obtain the value of the heat generation coefficient: $\gamma \approx 7{,}3\times10^3 \dfrac{erg}{s^0C}$.

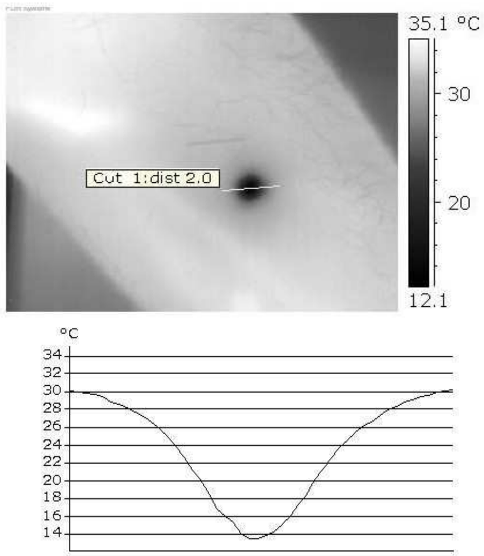

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Fig. 5. Thermal image of the forearm and the relevant graph of shearing temperature

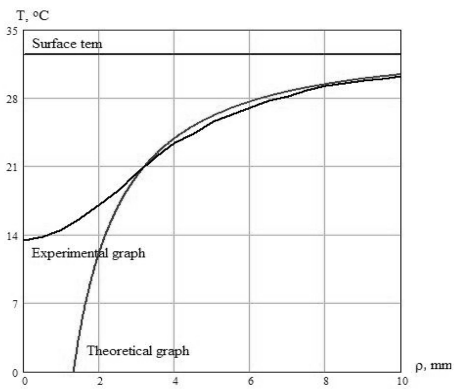

Fig. 6. Experimental and theoretical data of temperature distribution

Conclusions

1. The proposed model describes temperature distribution and the processes of its self-regulation in hypothermal zones when abnormal phenomena in biological object are accompanied by a temperature drop.

2. Calculation results reveal certain limitations of temperature visualization in the depth equal to about δ.
3. The area of thermal abnormality on skin integument surface in the zone of projection of the internal organ exceeds its real size because of heat dissipation in tissues of biological object.
4. Thermography can be effectively applied for non-invasive research of abnormal thermal zones inside a living organism and surface thermal fields that is a subject for further studies.

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