668. Improved smoothing procedure for determination of stresses in the coating of a vibrating beam

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Abstract. Stresses in the coating of a straight beam performing transverse vibrations are analyzed. The numerical procedure is based on the technique of conjugate approximation with smoothing. In the paper the smoothing procedure for one dimensional Lagrange quadratic elements is proposed. This procedure has advantages over the conventional smoothing procedure. Comparisons of the results obtained by using both smoothing procedures are presented.

Keywords: beam, vibrations, eigenmode, coating, stresses, conjugate approximation, conjugate smoothing

Introduction

The numerical calculation and analysis of stresses in photo-elastic coatings is important in hybrid experimental – numerical procedures [1, 2]. Transverse vibrations of a straight beam are analyzed. It is assumed that the coating is thin and has no effect to the vibrations of the beam. The model for the analysis of beam bending described in [3] is used. The numerical procedure is based on the technique of conjugate approximation [4, 5] with smoothing [6].

The calculation of the stress field in hybrid experimental – numerical procedures was analyzed in the papers [7, 8]. In this paper the improved smoothing procedure for one dimensional Lagrange quadratic elements is proposed, which has advantages over the conventional smoothing procedure.

The conventional smoothing procedure for calculation of stresses in the coating of a vibrating beam was analyzed in [8, 9, 10]. This paper can be considered as a continuation of investigations presented in the above mentioned papers.

Model of the beam and the proposed smoothing procedure

Further x, y and z denote the axes of the system of coordinates. The beam bending element has two nodal degrees of freedom: the displacement w in the direction of the z axis and the rotation Θ_y about the y axis. The displacement u in the direction of the x axis is expressed as $u=z\Theta_y$.

The mass matrix has the form:

$$[M] = \int [N]^T \begin{bmatrix} \rho h & 0 \\ 0 & \rho \dfrac{h^3}{12} \end{bmatrix} [N] dx, \tag{1}$$

where ρ is the density of the material of the beam, h is the thickness of the beam and:

$$[N]=\begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix}, \tag{2}$$

where N_1, N_2, N_3 are the shape functions of the one dimensional Lagrange quadratic finite element.

The stiffness matrix has the form:

$$[K]=\int\left([B]^T\left[\frac{E}{1-\nu^2}\frac{h^3}{12}\right][B]+\left[\tilde{B}\right]^T\left[\frac{E}{2(1+\nu)1.2}h\right]\left[\tilde{B}\right]\right)dx, \tag{3}$$

where E is the modulus of elasticity of the beam, v is the Poisson's ratio of the beam and:

$$[B]=\begin{bmatrix} 0 & \frac{dN_1}{dx} & 0 & \frac{dN_2}{dx} & 0 & \frac{dN_3}{dx} \end{bmatrix}, \tag{4}$$

$$\left[\tilde{B}\right]=\begin{bmatrix} \frac{dN_1}{dx} & N_1 & \frac{dN_2}{dx} & N_2 & \frac{dN_3}{dx} & N_3 \end{bmatrix}. \tag{5}$$

The nodal values of the stress in the coating are determined from:

$$\left[\hat{K}\right]\{\delta_c\}=\left\{\hat{F}\right\}, \tag{6}$$

where $\{\delta_c\}$ is the vector of nodal values of the stress in the coating and:

$$\left[\hat{K}\right]=\int\left(\left[\hat{N}\right]^T\left[\hat{N}\right]+\left[\hat{B}\right]^T\lambda\left[\hat{B}\right]\right)dx, \tag{7}$$

$$\left\{\hat{F}\right\}=\int\left[\hat{N}\right]^T\sigma_c dx, \tag{8}$$

where λ is the smoothing parameter, σ_c is the stress in the coating and:

$$\left[\hat{N}\right]=\begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix}, \tag{9}$$

$$\left[\hat{B}\right]=\begin{bmatrix} N_1-\overline{N}_1 & N_2 & N_3-\overline{N}_2 \end{bmatrix}, \tag{10}$$

$$\sigma_c=\left[\frac{E_c}{1-\nu_c^2}\right]\frac{h}{2}[B]\{\delta\}, \tag{11}$$

where $\overline{N}_1$, $\overline{N}_2$ are the shape functions of the one dimensional Lagrange linear finite element, E_c is the modulus of elasticity of the coating, v_c is the Poisson's ratio of the coating, $\{\delta\}$ is the vector of generalized displacements of the analyzed eigenmode.

For the conventional smoothing procedure:

$$\left[\hat{B}\right]=\begin{bmatrix} \frac{dN_1}{dx} & \frac{dN_2}{dx} & \frac{dN_3}{dx} \end{bmatrix}. \tag{12}$$

Results of calculation of stresses in the coating of a vibrating beam

At both ends of the beam both generalized displacements are assumed equal to zero. It is assumed that the modulus of elasticity of the beam $E = 8$ Pa, Poisson's ratio of the beam $\nu = 0.3$, thickness of the beam $h = 0.1$ m, density of the material of the beam $\rho = 0.8$ kg/m^3, modulus of elasticity of the coating $E_c = 2$ Pa, Poisson's ratio of the coating $\nu_c = 0.3$, length of the beam 64 m.

The stress field for the fifth eigenmode obtained by using the procedure of conjugate approximation and the stress field obtained by using the proposed procedure of conjugate smoothing with $\lambda = 0.6$ are presented in Fig. 1. The corresponding results for the tenth eigenmode are presented in Fig. 2.

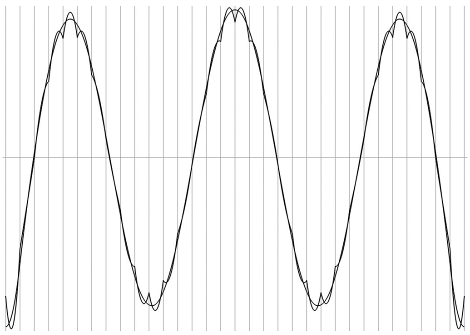

Fig. 1. The fifth eigenmode: stress field obtained by using the procedure of conjugate approximation and stress field obtained by using the proposed procedure of conjugate smoothing with $\lambda = 0.6$

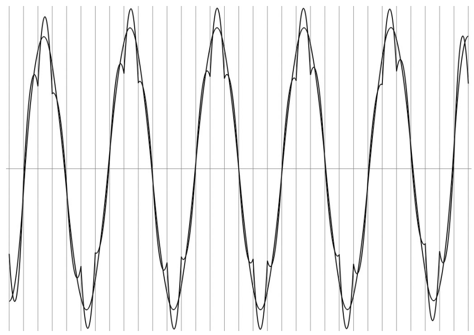

Fig. 2. The tenth eigenmode: stress field obtained by using the procedure of conjugate approximation and stress field obtained by using the proposed procedure of conjugate smoothing with $\lambda = 0.6$

The stress field for the fifth eigenmode obtained by using the procedure of conjugate approximation and the stress field obtained by using the proposed procedure of conjugate smoothing with $\lambda = 10000$ are presented in Fig. 3. The corresponding results for the tenth eigenmode are presented in Fig. 4.

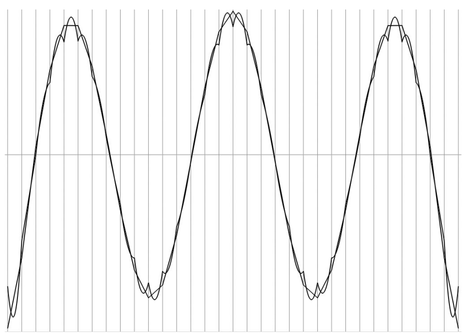

Fig. 3. The fifth eigenmode: stress field obtained by using the procedure of conjugate approximation and stress field obtained by using the proposed procedure of conjugate smoothing with $\lambda = 10000$

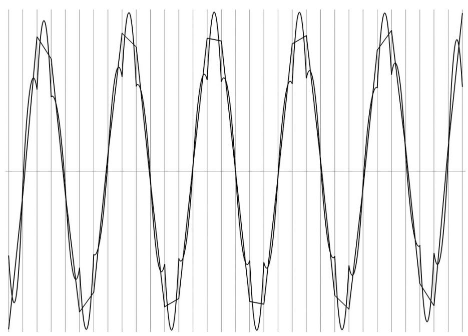

Fig. 4. The tenth eigenmode: stress field obtained by using the procedure of conjugate approximation and stress field obtained by using the proposed procedure of conjugate smoothing with $\lambda = 10000$

From the obtained results it is seen that for large values of the smoothing parameter the results approach piecewise linear approximation.

The stress field for the fifth eigenmode obtained by using the procedure of conjugate approximation and the stress field obtained by using the conventional procedure of conjugate smoothing with $\lambda = 0.6$ are presented in Fig. 5. The corresponding results for the tenth eigenmode are presented in Fig. 6.

From the obtained results it is seen that for the conventional smoothing procedure the results for the fifth eigenmode are acceptable, while for the tenth eigenmode they are over-smoothed (too much flattened).

The stress field for the fifth eigenmode obtained by using the procedure of conjugate approximation and the stress field obtained by using the conventional procedure of conjugate smoothing with $\lambda = 10000$ are presented in Fig. 7. The corresponding results for the tenth eigenmode are presented in Fig. 8.

From the obtained results it is seen that for large values of the smoothing parameter the results of the conventional smoothing procedure approach a constant value and thus are totally unacceptable. From the presented results the precision of the proposed procedure of conjugate smoothing is evident.

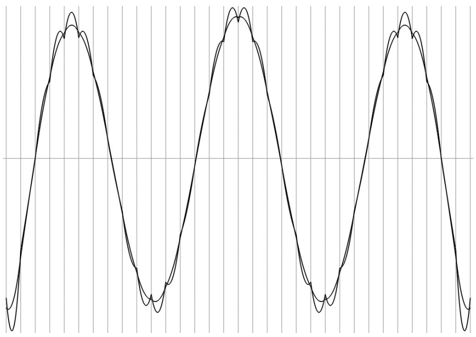

Fig. 5. The fifth eigenmode: stress field obtained by using the procedure of conjugate approximation and stress field obtained by using the conventional procedure of conjugate smoothing with $\lambda = 0.6$

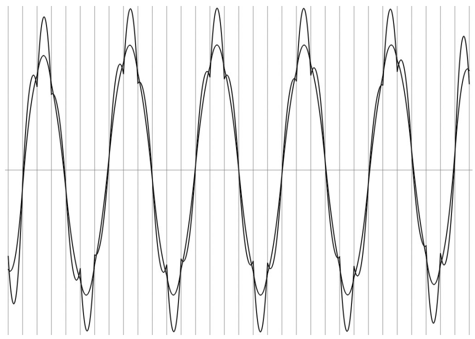

Fig. 6. The tenth eigenmode: stress field obtained by using the procedure of conjugate approximation and stress field obtained by using the conventional procedure of conjugate smoothing with $\lambda = 0.6$

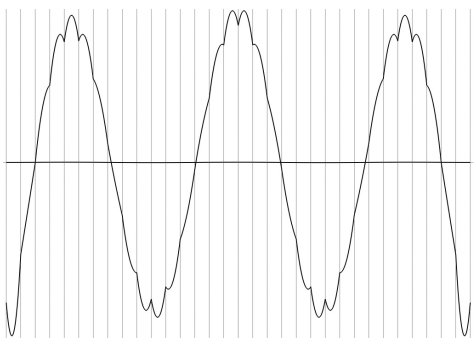

Fig. 7. The fifth eigenmode: stress field obtained by using the procedure of conjugate approximation and stress field obtained by using the conventional procedure of conjugate smoothing with $\lambda = 10000$

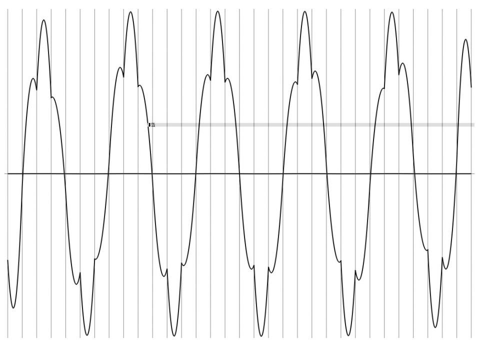

Fig. 8. The tenth eigenmode: stress field obtained by using the procedure of conjugate approximation and stress field obtained by using the conventional procedure of conjugate smoothing with $\lambda = 10000$

Conclusions

The improved smoothing procedure for the analysis of stresses in the coating of a straight beam performing transverse vibrations is presented. The developed numerical procedure is based on the technique of conjugate approximation with smoothing using one dimensional Lagrange quadratic elements.

From the presented graphical results the precision of the proposed procedure of conjugate smoothing is evident. Comparisons of the results obtained by using both smoothing procedures for various values of the smoothing parameter are presented. This procedure has no flattening effect of the stress field, which may be observed when applying conventional smoothing procedures.

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