660. Vibrations of elements of packages with supplementary stiffening bio-degradable strips

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Abstract. This paper analyzes the eigenmodes of bending vibrations of a paperboard, which is treated as a plate, taking into account the additional effect of bending vibrations of a supplementary bio-degradable strip included in the paperboard. Results of experimental investigations by means of method of time-averaged projection moiré are presented in the paper and comparisons between the experimental and numerical results are performed. The study revealed that the paperboard with supplementary stiffness elements of bio-degradable strip is more ecological: the introduction of additional stiffness elements to the composition of the paperboard in the packaging material has an effect of additional rigidity of the structure and thus reduces the amplitudes of resonance vibrations. The obtained results are applied in the process of design of elements of packages.

Keywords: element of package, finite elements, paperboard, plate bending, supplementary stiffness, bio-degradable strip, vibrations, eigenfrequencies, eigenmodes, experimental investigation, time averaging, projection moiré, experimental setup, glued strip, bio-degradable material.

Introduction

In recent years a lot of efforts are devoted to solving ecological problems of packaging technologies in order to minimize the mass of materials used for production of packages. For this purpose narrow strips of bio-degradable materials glued into the walls of the package may be applied. They increase the strength of the package for compression as well as for tension (Fig. 1). In this case the materials from which the strips are produced can be only such that their strength against loading is much higher than the strength of the paperboard.

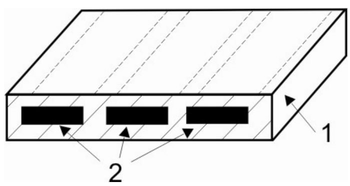


Fig. 1. View of the paperboard wall of the package with glued strips of bio-degradable material: 1 – paperboard sheet; 2 – strips of bio-degradable material

This paper analyzes the eigenmodes of bending vibrations of paperboard, which is treated as a plate, taking into account the additional effect of bending vibrations of a supplementary bio-degradable strip included in the paperboard. The model for the analysis of vibrations of paperboard is proposed on the basis of descriptions provided in [1, 2].

The materials used for packaging may be investigated not only with conventional methods, but also applying the techniques of nondestructive testing, such as geometrical, shadow, projection moiré techniques using stroboscopic or time-averaging approaches. Extensive experimental research work applying these methods has been performed [3 - 21]. Basic results of the investigations of the analyzed problem are presented in the paper, followed by comparison of experimental and numerical results, which leads to the conclusions.

In the process of transportation packages experience vibrations. When those vibrations are of substantial amplitude, they may lead to destruction of the package. The performed investigations demonstrate that the supplementary stiffness elements may be used for reduction of amplitudes of those resonance vibrations, thus enhancing the durability of the packages.

The obtained results are used in the process of design of elements of packages.

Model for the analysis of vibrations of paperboard with strips of bio-degradable material

Further x, y and z denote the axes of the system of coordinates. The plate bending element has three nodal degrees of freedom: the transverse displacement of the paperboard w and the rotations Θ_x and Θ_y about the axes of coordinates x and y.

The strip of bio-degradable material is represented by one-dimensional elements which have the same nodal degrees of freedom. The values of w, Θ_x, Θ_y in the element are represented as:

$$\begin{Bmatrix} w \\ \theta_x \\ \theta_y \end{Bmatrix} = [N]\{\delta\}, \tag{1}$$

where $\{\delta\}$ is the vector of generalized nodal displacements and:

$$[N] = \begin{bmatrix} N_1 & 0 & 0 & \dots \\ 0 & N_1 & 0 & \dots \\ 0 & 0 & N_1 & \dots \end{bmatrix}, \tag{2}$$

where N_i are the shape functions of the one-dimensional finite element.

Further the rotations about the longitudinal axis of the strip of bio-degradable material s and about the axis t perpendicular to s and located in the plane $x0y$ are denoted as Θ_s and Θ_t. Thus:

$$\begin{Bmatrix} w \\ \theta_s \\ \theta_t \end{Bmatrix} = [T]\begin{Bmatrix} w \\ \theta_x \\ \theta_y \end{Bmatrix}, \tag{3}$$

where:

$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \dfrac{dx}{d\xi} \Big/ \dfrac{ds}{d\xi} & \dfrac{dy}{d\xi} \Big/ \dfrac{ds}{d\xi} \\ 0 & -\dfrac{dy}{d\xi} \Big/ \dfrac{ds}{d\xi} & \dfrac{dx}{d\xi} \Big/ \dfrac{ds}{d\xi} \end{bmatrix}, \tag{4}$$

where ξ is the local coordinate of the finite element and:

$$\frac{ds}{d\xi}=\sqrt{\left(\frac{dx}{d\xi}\right)^2+\left(\frac{dy}{d\xi}\right)^2}. \tag{5}$$

The following notation is introduced:

$$\left[\bar{N}\right]=\begin{bmatrix}\left[\bar{N}_1\right]\\ \left[\bar{N}_2\right]\\ \left[\bar{N}_3\right]\end{bmatrix}=[T][N]. \tag{6}$$

The derivatives of w, Θ_x, Θ_y are represented as:

$$\begin{Bmatrix}\frac{dw}{ds}\\ \frac{d\theta_x}{ds}\\ \frac{d\theta_y}{ds}\end{Bmatrix}=[N']\{\delta\}, \tag{7}$$

where:

$$[N']=\begin{bmatrix}\frac{dN_1}{ds} & 0 & 0 & \dots\\ 0 & \frac{dN_1}{ds} & 0 & \dots\\ 0 & 0 & \frac{dN_1}{ds} & \dots\end{bmatrix}. \tag{8}$$

It is assumed that:

$$\begin{Bmatrix}\frac{dw}{ds}\\ \frac{d\theta_s}{ds}\\ \frac{d\theta_t}{ds}\end{Bmatrix}=[T]\begin{Bmatrix}\frac{dw}{ds}\\ \frac{d\theta_x}{ds}\\ \frac{d\theta_y}{ds}\end{Bmatrix}. \tag{9}$$

The following notation is introduced:

$$\left[\bar{N}'\right]=\begin{bmatrix}\left[\bar{N}'_1\right]\\ \left[\bar{N}'_2\right]\\ \left[\bar{N}'_3\right]\end{bmatrix}=[T][N']. \tag{10}$$

The mass matrix of the strip of bio-degradable material has the form:

$$[M] = \int [\bar{N}]^T \begin{bmatrix} \rho h & 0 & 0 \\ 0 & \frac{\rho h^3}{12} & 0 \\ 0 & 0 & \frac{\rho h^3}{12} \end{bmatrix} [\bar{N}] h_w ds, \tag{11}$$

where ρ is the density of the material of the strip, h is the thickness of the strip and h_w is the width of the strip.

The stiffness matrix of the strip of bio-degradable material has the form:

$$[K] = \int \left([B]^T \frac{h^3}{12} [D][B] + [\bar{B}]^T \frac{E_s E_t h}{(E_s + E_t + E_s \nu_{ts} + E_t \nu_{st})1.2} [\bar{B}] \right) h_w ds, \tag{12}$$

where E_s and E_t are the modulus of elasticity of the strip, ν_{st} and ν_{ts} are the Poisson's ratios of the strip and:

$$[B] = \begin{bmatrix} [\bar{N}'_3] \\ [0] \\ -[\bar{N}'_2] \end{bmatrix}, \tag{13}$$

$$[\bar{B}] = \left[[\bar{N}'_1] + [\bar{N}_3] \right], \tag{14}$$

$$[D] = \begin{bmatrix} \frac{E_s}{1-\nu_{st}\nu_{ts}} & \frac{E_t \nu_{st}}{1-\nu_{st}\nu_{ts}} & 0 \\ \frac{E_s \nu_{ts}}{1-\nu_{st}\nu_{ts}} & \frac{E_t}{1-\nu_{st}\nu_{ts}} & 0 \\ 0 & 0 & \frac{E_s E_t}{E_s + E_t + E_s \nu_{ts} + E_t \nu_{st}} \end{bmatrix}, \tag{15}$$

where:

$$E_s = E_t \frac{\nu_{st}}{\nu_{ts}}. \tag{16}$$

Results of numerical analysis of vibrations of paperboard with a strip of bio-degradable material

The square piece of paperboard is analyzed. On the lower and upper boundaries all the generalized displacements are assumed equal to zero. The strip of bio-degradable material is located in the center of the paperboard. The following parameters of the paperboard are assumed: modulus of elasticity E_y=0.34·10^9 Pa, Poisson's ratio ν_{xy}=0.4, Poisson's ratio ν_{yx}=0.14, thickness h_p=0.0001 m, density of the material ρ_p=785 kg/m^3. The following parameters of the strip of bio-degradable material are assumed: modulus of elasticity E_t=0.68·10^9 Pa, Poisson's ratio ν_{st}=0.4, Poisson's ratio ν_{ts}=0.14, thickness h=0.0002 m, density of the material ρ=1570 kg/m^3, width h_w=0.002 m. The first eigenmodes of paperboard are presented in Fig. 2.

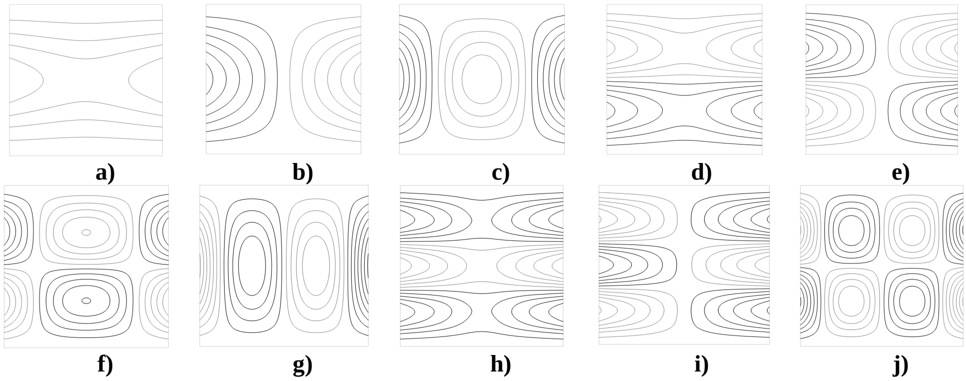


Fig. 2. The first eigenmodes of the paperboard

From the presented results it may be observed that the eigenmodes of the analyzed ecological paperboard are influenced by the presence of the stiffening strip of bio-degradable material. From Fig. 2a, Fig. 2d, Fig. 2h it is clearly evident that the supplementary stiffening strip has the effect of reduction of the amplitude of vibrations.

The method and the results of experimental investigations of paperboard with a strip of bio-degradable material

The specially developed experimental setup for the analysis of vibrations using the method of time-veraged projection moiré was used. For the experimental investigations the paperboard MC Mirabell was chosen. Technical characteristics of this paperboard are as follows: surface density 400 g/m^2, thickness 565 μm. The square piece of the paperboard is analyzed: paperboard sheet was 0.2 m width and 0.2 m length. Some of the obtained results of experimental investigations are presented further.

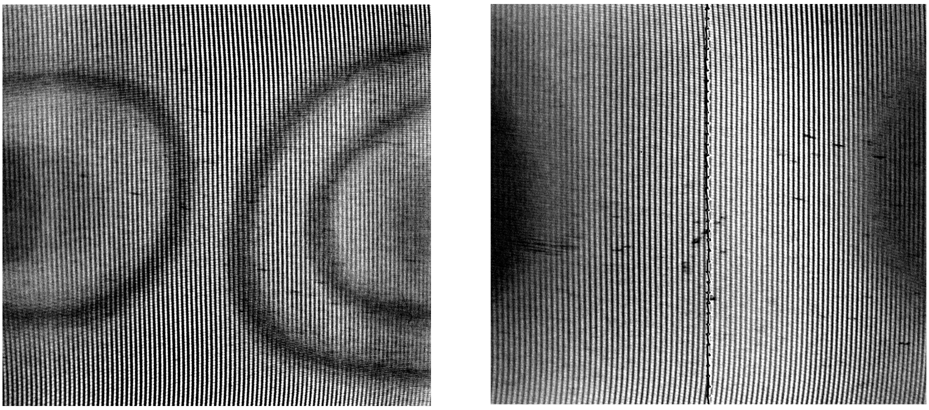
The first eigenfrequencies for the paperboard without supplementary stiffening strip of bio-degradable material and with supplementary stiffening strip are presented in Table 1.

Table 1. The first eigenfrequencies of paperboard MC Mirabell 400 g/m^2 without supplementary stiffening strip of bio-degradable material and with supplementary stiffening strip (amplitude 3×10^{-4} m, loading force 25.5 N)

	Eigenfrequency 1	Eigenfrequency 2	Eigenfrequency 3
Without supplementary stiffening strip	50 Hz	70 Hz	92 Hz
With supplementary stiffening strip	58 Hz	95 Hz	102 Hz

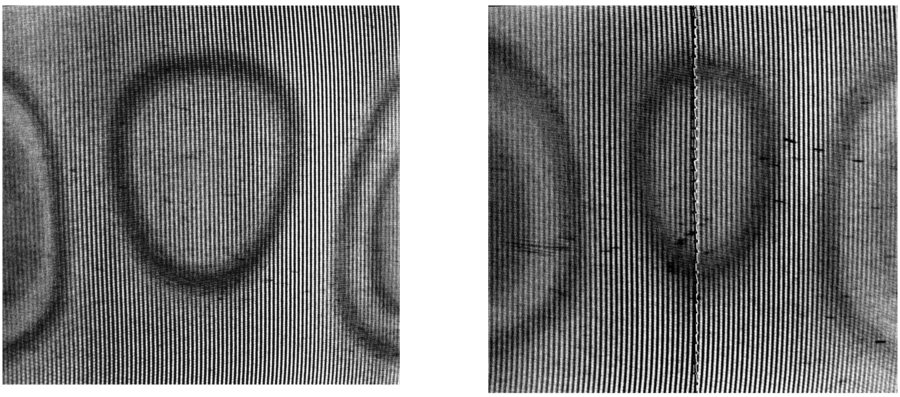
From the experimental results presented in Table 1 it is observed that the first, second and third eigenfrequencies of bending vibrations of the paperboard with supplementary stiffness elements of strip type have higher values if compared with less ecological paperboard without supplementary stiffness elements. This indicates the stiffening effect of the supplementary stiffness elements for the lower eigenmodes.

For the paperboard without supplementary stiffening strip MC Mirabell the image of the second eigenmode is presented in Fig. 3a, while the corresponding image of the paperboard with stiffening strip is presented in Fig. 3b. For the paperboard without supplementary stiffening strip MC Mirabell the image of the third eigenmode is presented in Fig. 4a, while the corresponding image of the paperboard with stiffening strip is presented in Fig. 4b.



a) b)

Fig. 3. The second eigenmode of paperboard MC Mirabell 400 g/m^2: a) without supplementary stiffening strip and b) the corresponding eigenmode of paperboard with supplementary stiffening strip (amplitude 3×10^{-4} m, loading force 25.5 N)



a) b)

Fig. 4. The third eigenmode of paperboard MC Mirabell 400 g/m^2: a) without supplementary stiffening strip and b) the corresponding eigenmode of paperboard with supplementary stiffening strip (amplitude 3×10^{-4}m, loading force 25.5 N)

The experimental results presented in Figs. 3-4 indicate that the paperboard with supplementary stiffness elements of strip type is more ecological. The presented results also reveal that the introduction of additional stiffness elements to the composition of the paperboard in the packaging material has an effect of additional rigidity of the structure and thus reduces the amplitudes of resonance vibrations.

By analyzing the results of experimental and numerical investigations one can note that the sheet of paperboard in which there are strips of bio-degradable material glued into it undergoes vibrations of lower amplitude, while the sheet without the strips of bio-degradable material vibrates with higher amplitude. This allows to state that the packages that are produced from paperboard with glued strips from bio-degradable materials are more stable in terms of vibrational behavior and thus during transportation may experience lower level of defects.

Conclusions

The model for the analysis of vibrations of paperboard with supplementary stiffness elements of bio-degradable strip type is proposed. The eigenmodes of bending vibrations of a plate are calculated taking into account the bending vibrations of a strip located in the paperboard. Presented results indicate that the eigenmodes of the analyzed paperboard without supplementary stiffening elements and paperboard with supplementary stiffening elements may have substantial differences.

The performed experimental and numerical investigations demonstrated that the first eigenfrequencies of the paperboard with supplementary stiffness elements are higher than the corresponding eigenfrequencies of the paperboard without supplementary stiffness elements. This indicates the stiffening effect of the supplementary stiffness elements for the lower eigenmodes.

The experimental results reveal that the paperboard with supplementary stiffness elements of bio-degradable strip type is more ecological: the introduction of additional stiffness elements to the composition of the paperboard in the packaging material has an effect of additional rigidity of the structure and thus reduces the amplitudes of resonance vibrations.

The obtained eigenmodes enable during process of design of elements of packages determination by means of nondestructive methods not only the vibrational characteristics of the package wall, but also the strength parameters of the material of the package such as the Poisson’s ratio and the modulus of elasticity. This enables to make the process of design of such packages simpler and easier.

The packages made from paperboard having glued strips of bio-degradable materials in their walls are more stable and thus in the process of transportation they experience lower levels of damage.

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