649. The influence of parameters of consecutive speed control humps on the chaotic vibration of a 2-DOF nonlinear vehicle model

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Abstract. This paper is aimed at analyzing the chaotic vibration of a vehicle passing the consecutive speed control humps (SCHs) on a highway. A consecutive SCHs-speed coupling excitation function is presented. The chaotic vibration of nonlinear vehicle is studied by numerical simulation under a 2-DOF nonlinear vehicle suspension model. The chaotic vibration excited by the consecutive SCHs with different parameters is analyzed. Simulation results demonstrate that the chaotic motion may occur as the vehicle moves over a series of the consecutive SCHs. Furthermore, chaotic motion can be inhibited reasonably and effectively by proper adjustment of parameters of the consecutive SCHs.

Keywords: chaotic vibration, consecutive SCHs, nonlinear vehicle model, bifurcation.

Introduction

As a special type of road safety facility, the speed control hump plays an important role for preventing traffic accidents [1-4]. Noise and annoying vibrations caused by vehicle passing over the consecutive SCHs at a high velocity would make drivers slow down to a certain extent. At present, there are some shapes of the consecutive SCHs in the special sections on the highway, such as at tunnel portal, curve or slope. By now only few results on the dynamic characteristics in nonlinear 2-DOF quarter vehicle model excited by the consecutive SCHs were reported [5, 6]. The chaos and bifurcation may appear in a vehicle system when the vehicle moves over a bumpy road at a high speed [7-9]. The chaotic variation may cause early damage to road and the components of vehicle and bring physical injury to crew member [10].

In these studies a vehicle usually can be simulated by means of three typical models depending upon the aim of numerical study. A quarter-vehicle model (1-DOF or 2-DOF system) for studying heave (vertical) motion [5, 6, 7, 10], a half-vehicle model (4-DOF system) for studying the heave and pitch motions [8], and full-vehicle model (7-DOF system) for studying the heave, pitch and roll motions [9]. Comparing to the full-vehicle model and the half-vehicle model, the quarter-vehicle model is the simplest to analyze and yet can reasonably predict the response of the system. Therefore many researchers often rely on this model. In practice, the consecutive SCHs excitation is of constant low-amplitude and wide range frequency, and the higher the velocity of the vehicle, the higher the excitation from road surface. The magnitude of the vehicle response in the vertical plane is mainly moderated by the height of the consecutive SCHs, while the most important variation with speed is set by its profile and width [5, 6]. There was a disparity between the parameters of the nonlinear vehicle suspension model in [5] and reality. Zheng et al [6] has given the region of the speed in chaotic vibration of vehicle only. What is more important, however, the reasonableness of the consecutive SCHs parameters' designing and adjusting had been not analyzed in [5] and [6].

In this paper we present the investigation of the chaotic vibration in a 2-DOF nonlinear vehicle suspension system with dynamic excitation brought by the periodic consecutive SCHs on the highway. Numerical simulations based on the differential equation are conducted to analyze the chaotic vibration of a vehicle. The results indicate that the chaotic vibration exists when the vehicle passed over the consecutive SCHs. And, chaotic motion can be suppressed by using suitable consecutive SCHs parameters.

Dynamic excitation brought by the periodic consecutive SCHs

The width S and the duty cycle ϕ of the consecutive SCHs are not fixed. The road roughness can be negligible relative to the consecutive SCHs. The geometric shape can be described as a heaped rectangular wave approximately because the width of the consecutive SCHs is much larger than the height of the consecutive SCHs. The input of the suspension system is influenced by both the consecutive SCHs parameters and the speed. The periodic combined SCHs-speed excitation model is established as in Fig. 1.

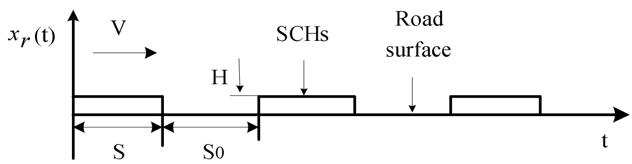

Fig. 1. The combined SCHs-speed excitation function

When the vehicle passing over a series of SCHs with height A, width S and interval S_0 at the speed of V, the exciting frequency of the vehicle and the duty cycle are expressed by $f = V / (S + S_0)$ and $\phi = S / (S + S_0)$ respectively. Then, the periodic consecutive SCHs is expressed by:

$$X_r(t) = \frac{H}{2}(square(2\pi\phi \frac{V}{S} t, 100\phi) + 1) \tag{1}$$

Nonlinear vehicle suspension model and motion differential equation

The vehicle suspension model consists of a vehicle body, unsprung mass, spring, suspension and tire. The longitudinal view of the 2-DOF quarter-vehicle simplified model is illustrated in Fig. 2.

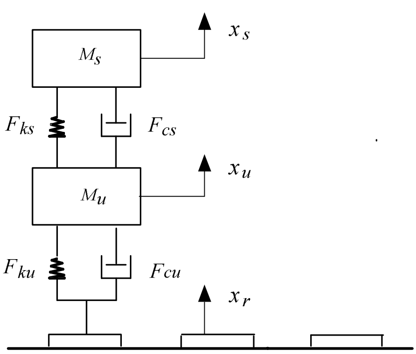

Fig. 2. 2-DOF vehicle suspension model

In Fig. 2, the quarter-vehicle body is represented by a 2-DOF rigid cuboid with M_s, the unsprung mass is M_u, F_{ks} and F_{ku} are respectively nonlinear suspension spring force and nonlinear tire spring force, F_{cs} and F_{cu} are respectively nonlinear suspension damper force and nonlinear tire damper force, x_s and x_u are the displacement of M_s and displacement of M_u, respectively, road disturbance variation x_r is described as a heaped rectangular wave approximately with oscillation amplitude A and period $1/f$. The differential equation of motion of the system in this condition is given by:

$$\begin{cases} M_s \ddot{x}_s = -F_{ks} - F_{cs} - M_s g \\ M_u \ddot{x}_u = F_{ks} + F_{cs} - F_{ku} - F_{cu} - M_u g \end{cases} \tag{2}$$

where $\ddot{x}_s$ and $\ddot{x}_u$ is the vertical acceleration of the quarter-vehicle body and the tire respectively, F_{ks}, F_{ku}, F_{cs} and F_{cu} are assumed to have the following nonlinearity characteristics [5, 6, 8].

$$\begin{cases} F_{ks} = k_1(x_s - x_u - \delta_s) + k_2(x_s - x_u - \delta_s)^2 + k_3(x_s - x_u - \delta_s)^3 \\ F_{cs} = c_1(\dot{x}_s - \dot{x}_u) + c_2(\dot{x}_s - \dot{x}_u)^2 \\ F_{ku} = 100^{n-1} k_u \,\mathrm{sign}(x_u - x_r - \delta_u)\,|x_u - x_r - \delta_u|^n \\ F_{cu} = c_u(\dot{x}_u - \dot{x}_r) \end{cases} \tag{3}$$

where $\dot{x}_s$ and $\dot{x}_u$ is the vertical velocity of the quarter-vehicle body and the tire, $\dot{x}_r$ is described as the rate of change of SCHs, n is referred to as the nonlinear coefficient of the tire. The damping coefficient c_u is expressed by:

$$c_u = \begin{cases} c_3, & \dot{x}_u - \dot{x}_r \geq 0 \\ c_4, & \dot{x}_u - \dot{x}_r < 0 \end{cases} \tag{4}$$

Thus, the equations for equilibrium state of vertical direction can be expressed as:

$$\begin{cases} -M_s g = k_1(-\delta_s) + k_2(-\delta_s)^2 + k_3(-\delta_s)^3 \\ -(M_s + M_u) g = 100^{n-1} k_u \,\mathrm{sign}(-\delta_u)\,|-\delta_u|^n \end{cases} \tag{5}$$

where δ_s and δ_u respectively denote static deformation of the suspension spring and tire with k_1, k_2, k_3 and k_u. From (5) we have:

$$\delta_u = \left\{ \frac{(M_s + M_u) g}{100^{n-1} k_u} \right\}^{1/n} \tag{6}$$

$$\delta_s = \frac{\sqrt[3]{U_1 + U_2}}{6k_3} - \frac{6k_1k_2 - 2k_2^2}{3k_3\sqrt[3]{U_1 + U_2}} + \frac{k_2}{3k_3} \tag{7}$$

where:

$$U_1 = -36k_1k_2k_3 + 108M_s g k_3^2 + 8k_2^3$$

$$U_2 = 12\sqrt{3}k_3\sqrt{4k_1^3k_3 - k_1^2k_2^3 - 18k_1k_2k_3M_sg + 27(M_sg)^2k_3^2 + 4M_sgk_2^3}$$

The dynamic behavior of the vehicle

Assume $x_1 = x_s$, $x_2 = \dot{x}_s$, $x_3 = x_u$, $x_4 = \dot{x}_u$, and the state equations of vehicle suspension system can be formulated as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -(F_{ks} + F_{cs})/M_s - g \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = (F_{ks} + F_{cs} - F_{ku} - F_{cu})/M_u - g \end{cases} \tag{8}$$

Numerical values of the system parameters are $M_s = 315.00kg$, $M_u = 30.00kg$, $k_1 = 12394.00N/m$, $k_2 = -73696.00N/m^2$, $k_3 = 3170400.00N/m^3$, $k_u = 187000.00N/m$, $c_1 = 1385.40N/(m/s)$, $c_2 = 524.28N/(m/s)^2$, $c_3 = 8.00\ N/(m/s)$, $c_4 = 5.00N/(m/s)$, $n = 1.2$ and $g = 9.81N/kg$. In order to guarantee the stationarity, safety and decelerating effectively of vehicle passing over the consecutive SCHs, strict limitation regarding the height of the consecutive SCHs is needed. Usually, H shall be limited to between $3mm$ to $15mm$. Here, supposes $H = 5mm$, $S = 500mm$ and $S_0 = 500mm$. The dynamics of the vehicle suspension system are first analyzed through bifurcation diagram, which is obtained by plotting the speed of the combined SCHs-speed excitation versus the displacement of body, the phase plane diagram, the Poincaré map and PSP (period sampling peak-to-peak value) diagram in single speed. PSP diagram is particularly well suited to describe vehicle vibration resulted from the consecutive SCHs because PSP diagram can reflect the amplitude information of vibration in detail. In simulation, the initial condition is set as $[x_s, \dot{x}_s, x_u, \dot{x}_u] = [-\delta_s, 0, -\delta_u, 0]$. The bifurcation diagram of vehicle body with the parameter V from $40.00km/h$ to $70.00km/h$ is shown in Fig. 3. Simulation results reveal that the system presents complicated dynamical characteristic.

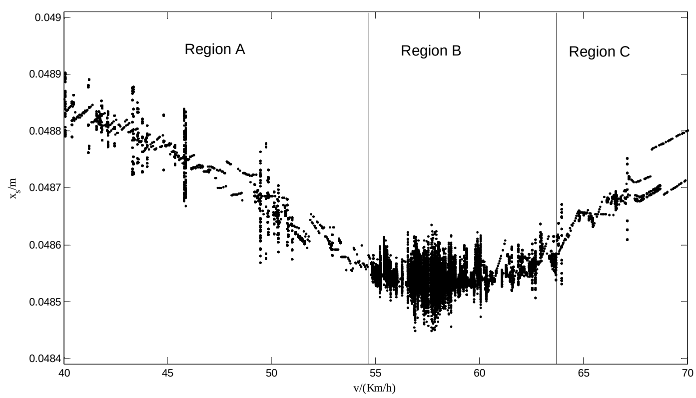

Fig. 3. Bifurcation diagram of x_s obtained by varying V

As can be observed from Fig. 3, the system response can be classified into three categories: region A (from $40.00km/h$ to $54.60km/h$), region B (from $54.60km/h$ to $63.80km/h$)

and region C (from $63.80 km/h$ to $70.00 km/h$). The speed $V = 50.60 km/h$, $58.60 km/h$ and $66.98 km/h$ were taken separately to analyze the dynamic response of the vehicle model. The phase plane diagram, the Poincaré map and PSP diagram for different speed values are shown in Figs. 4-6. When $V = 50.60 km/h$ in region A and $V = 66.98 km/h$ in region C, the system acquires period-N motion. When $V = 58.60 km/h$, the chaotic vibration occurs in region B as illustrated in Fig. 5. In general, the system perform period-N (the major period-one motion) motion first. When speed V increase to $54.60 km/h$, the system is out of the balance, chaotic vibration will be occur in the system. As the speed V continues to increase to region C, the system enters into stable period-N motion (the major period-one motion).

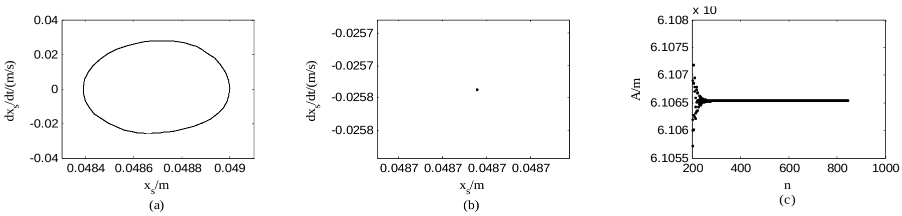

Fig. 4. Periodic-one motion for $V = 50.60 km/h$

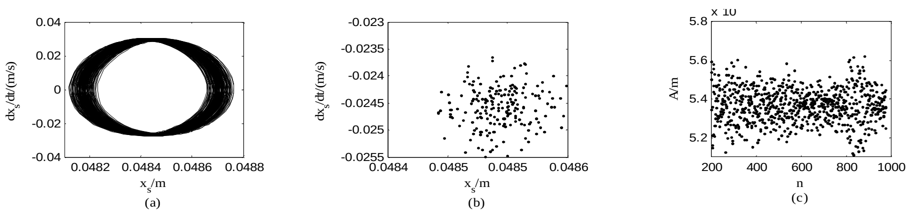

Fig. 5. Chaotic motion for $V = 58.60 km/h$

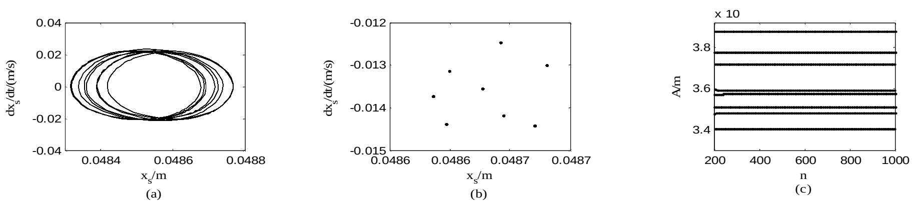

Fig. 6. Periodic-N (N=8) motion for $V = 66.98 km/h$

Parameter adjustment for the consecutive SCHs

The paper assumed the vehicle speed to vary between $50.00 km/h$ and $70.00 km/h$. The bifurcation diagram of the vehicle body in Fig. 3 demonstrates that chaotic vibration will occur in the system during $50.00 km/h$ to $70.00 km/h$. The chaotic vibration in the system here should be suppressed reasonably because it will aggravate the comfort and security. Therefore, it is important to study the parameter adjustment of the consecutive SCHs for inhibiting the chaotic vibration effectively. Adjustable parameters include the width S and the duty cycle ϕ of the consecutive SCHs.

The width S of the consecutive SCHs. The width S of the consecutive SCHs is an important parameter. The width S affects the dynamic response of the vehicle through exerting influence on the frequency of the consecutive SCHs excitation. When the width $S = 650 mm$, the bifurcation diagram of vehicle body with the parameter V from $50.00 km/h$ to

$80.00km/h$ is provided in Fig. 7. As Fig. 7 indicates the chaotic vibration in vehicle system exists in region B. The vehicle body of the system perform period-N (the major period-one motion) motion in speed limit sections (in region A) $50.00km/h$ to $70.00km/h$. Therefore, chaotic motion can be prevented effectively by proper adjustment of the width of the consecutive SCHs.

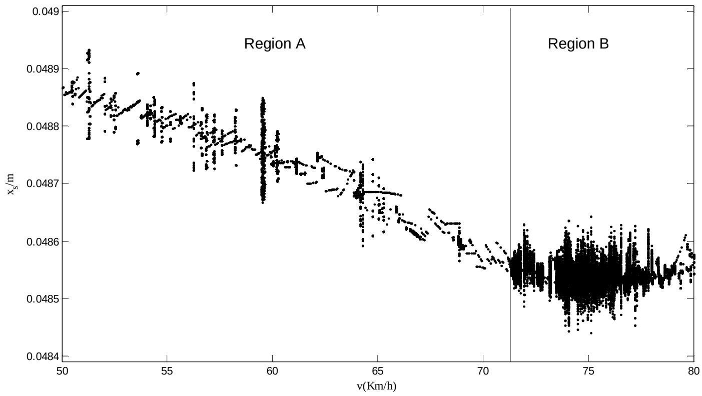

Fig. 7. Bifurcation diagram of x_s obtained by varying V

The duty cycle ϕ of the consecutive SCHs. The duty cycle ϕ of the consecutive SCHs is also a very important parameter. The duty cycle ϕ can change the dynamic response of the vehicle through influence on the shape of the consecutive SCHs excitation. Suppose the duty cycle $\phi = 42.4\%$, the bifurcation diagram of vehicle body with the parameter V from $50.00km/h$ to $80.00km/h$ is illustrated in Fig. 8. It reveals that the system acquires period-N motion (the major period-one motion) in speed limit sections (in region B) $50.00km/h - 70.00km/h$. Therefore, chaotic motion can be eliminated availably via regulating the duty cycle of the consecutive SCHs appropriately.

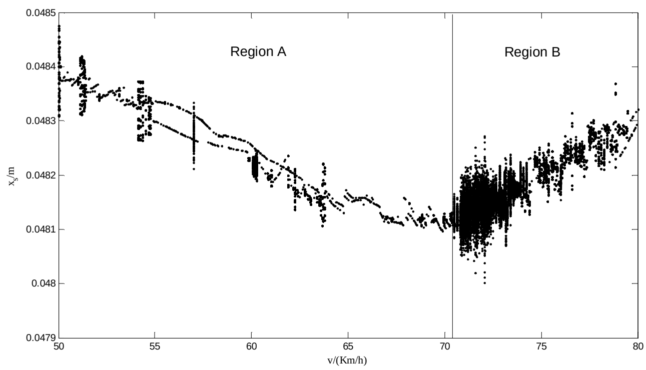

Fig. 8. Bifurcation diagram of x_s obtained by varying V

Rationality analysis. Considering the above analysis, the width S and the duty cycle ϕ of the consecutive SCHs can suppress chaotic vibration effectively in vehicle system in speed limit section $50.00km/h$ to $70.00km/h$. However, we must consider the quality of speed control because the consecutive SCHs is a special road excitation. Therefore, parameter adjustment must be controlled to a reasonable extent in order to ensure reasonable modification of oscillation amplitude.

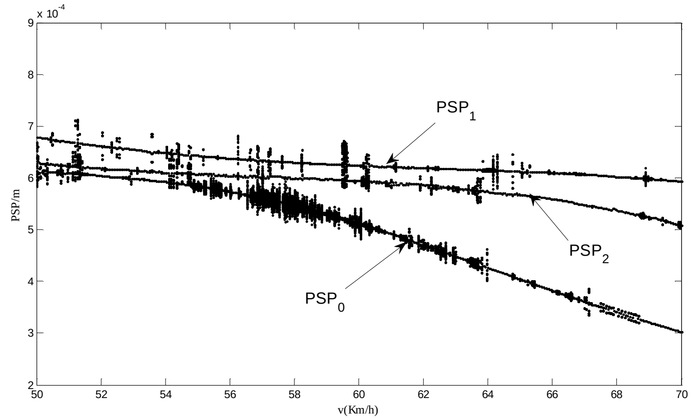

Fig. 9. Bifurcation diagram of PSP obtained by varying V

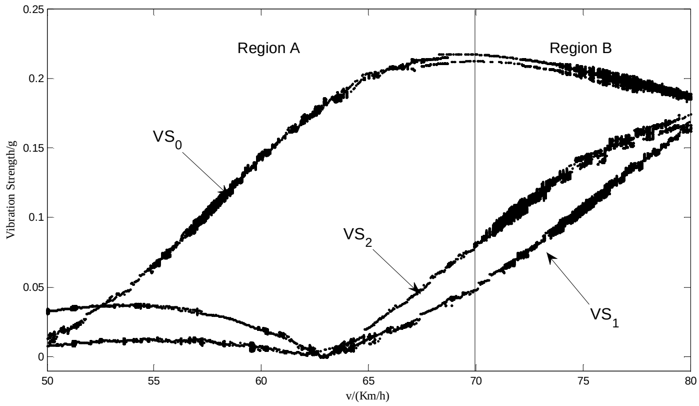

Fig. 10. Bifurcation diagram of VS obtained by varying V

The bifurcation diagram of PSP with the parameter V from $50.00km/h$ to $80.00km/h$ is illustrated in Fig. 9. Where PSP_0 is the PSP of vehicle body with the primal consecutive SCHs parameters, PSP_1 is the PSP of vehicle body with the width $S = 650mm$, PSP_2 is the PSP of vehicle body with the duty cycle $\phi = 42.4\%$. Fig. 9 indicates that the PSP of vehicle body after appropriate adjustments of the consecutive SCHs parameters (S or ϕ) is slightly more stable than that before parameters adjustment.

The bifurcation diagram of vibration strength (VS) with the parameter V from $50.00km/h$ to $80.00km/h$ is illustrated in Fig. 10, where VS_0 is the vibration strength of vehicle body with the primal consecutive SCHs parameters, VS_1 is the vibration strength of vehicle body with the width $S = 650mm$, VS_2 is the vibration strength of vehicle body with the duty cycle $\phi = 42.4\%$. Fig. 10 indicates that vibration strength of vehicle body after appropriate adjustments of the parameters of the consecutive SCHs (S or ϕ) is much lower than that before. When the vehicle passes through the consecutive SCHs over the speed limit

($70.00 km/h$), the vehicular vibration increases rapidly. Obviously, adjusting the parameters can improve the performance and actual effect of speed management of the consecutive SCHs.

On the basis of the above analysis, the chaotic vibration in vehicle system can be suppressed effectively and rationally by modifying the consecutive SCHs parameters.

Conclusions

Through analyzing the consecutive SCHs at a tunnel portal on the highway, the combined SCHs-speed excitation model and 2-DOF nonlinear vehicle suspension model is established. The dynamic behavior of the vehicle system was analyzed by numerical simulation. Results show that reasonable speed range can be acquired by identifying chaotic vibration via bifurcation diagram, the phase plane diagram, Poincaré map and PSP diagram. Further, the chaotic vibration taking place in the speed limit can be effectively suppressed by adjusting the consecutive SCHs parameters in a proper manner. Experimental measurements of the consecutive SCHs parameters in a real system are left for future research.

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References

[1] **Kokowski P., Makarewicz R.** Predicted effects of a speed bump on light vehicle noise, Applied Acoustics 67 (6) (2006) 570–579.

[2] **Salau T. A., Adeyefa A. O., Oke S. A.** Vehicle speed control using road bumps, Transport 19 (3) (2004) 130–136.

[3] **Oke S. A., Salau T. A., Adeyefa A. O.** Vehicle speed control using road bumps: Part 2, Transport 20 (3) (2005) 99–105.

[4] **Yang Y., Han Z., Wang B., Liu C.** Vibration analyses caused by vehicles running across the speed control hump, Journal of Vibration Engineering 20 (5) (2007) 502–506, (in Chinese).

[5] **Liu F., Liang S., Zhu Q., Xiong Q.** Effects of the consecutive speed humps on chaotic vibration of a nonlinear vehicle model, ICIC Express Letters 4 (5) (2010) 1657–1664.

[6] **Zheng J., Liang S., Zhu Q., Li C. G., Xiong Q.** Chaotic vibration of vehicle suspension excited by consecutive speed control hump, Journal of Highway and Transportation Research and Development 28 (1) (2011) 132–137, (in Chinese).

[7] **Li S., Yang S., Guo W.** Investigation on chaotic motion in hysteretic non-linear suspension system with multi-frequency excitations, Mechanics Research Communications 31 (2) (2004) 229–236.

[8] **Zhu Q., Ishitobi M.** Chaos and bifurcations in a nonlinear vehicle model, Journal of Sound and Vibration 275 (2004) 1136–1146.

[9] **Zhu Q., Ishitobi M.** Chaotic vibration of a nonlinear full-vehicle model, International Journal of Solids and Structures 43 (2006) 747–759.

[10] **Litak G., Borowiec M., Friswell M. I., Przystupa W.** Chaotic response of a quarter car model forced by a road profile with a stochastic component, Chaos, Solitons and Fractals 39 (5) (2009) 2448–2456.