607. The sound wave displacement-based ultrasonic meter dependence on various atmospheric factors

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Annotation. The dependence of speed of ultrasound in air on various climatic factors operating in the atmosphere has a significant impact on the accuracy of the ultrasonic flow measurement method. The article examines how to analyse, calculate and form mathematical formulas for air speed dependencies on all foreseen atmospheric factors that can affect the accuracy and results of measurements. The results obtained will be used for creating a mathematical model in MATLAB environment. The examination of a great number of sources suggests that a mathematical model of the speed of sound evaluating various influencing factors in a wide range has not been created until now. The article provides a partial analysis on this issue that would be applicable only for a partial application of the model used for an acoustic impulse meter.

Key words: Ultrasonic meter, sound wave displacement, speed meter, flow meter, spead of sound.

Introduction

The performance of the method examined in a present article is based on a fact that ultrasound (as well as air) waves are affected by air flow, and as a result they change their direction depending on the strength of air flow.

An impulse signal sent by a transmitter to a matrix of an ultrasound receiver at speed u is affected by the air flow moving at speed v in a pipe (Jozonis V., Stankūnas J., 2009). As a result a realignment of the signal is α (Fig.1). Figure 2 shows that air speed is equal to:

$$\mathrm{tg}\,\alpha = v/u \quad \text{so} \quad v = u \times \mathrm{tg}\,\alpha\,. \qquad (1)$$

To calculate air speed v we should know the angle α and the ultrasound speed u. The dependence of ultrasound speed u on atmospheric factors will be analyses in the present article.

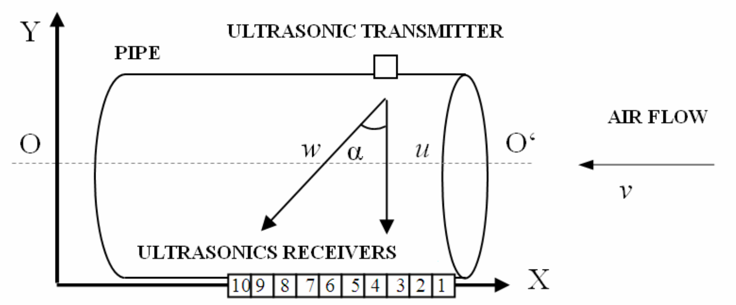

Fig. 1. Ultrasound meter model

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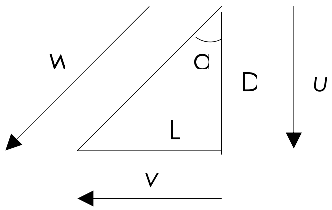

Fig. 2. Triangle of meter velocities

General equations

The theoretical expression for the speed of sound c in an ideal gas is (Laplace formula, Tamašauskas A., 1986):

$$c = \sqrt{\frac{\gamma P}{p}}\,, \tag{2}$$

where P is ambient pressure, p is gas density, γ – ratio of the specific head of gas at constant pressure to that at constant volume. Value of γ depends on a number of degrees of freedom.

γ = 1,67	for monatomic molecules
γ = 1,40	for diatomic molecules
γ = 1,33	for triatomic molecules

The number of degrees of freedom depends on complexity of molecules. Since air is composed of diatomic molecules, the speed of sound in air is:

$$c = \sqrt{\frac{1{,}4P}{p}}\,. \tag{3}$$

In accordance with values examined empirically by other authors, the speed of sound c in dry air is equal to:

$$c = 331.45 \pm 0{,}05 m/s\,, \tag{4}$$

when the speed of sound is constant at 0°C and 1 atm (760 mm Hg) with 0,03% mol - carbon dioxide.

Impact of temperature

Substituting the equation of state of air of an ideal gas ($PV = RT$) and a definition of density p (mass per unit volume), equation (3) may be written as:

$$c = \sqrt{\frac{1{,}4RT}{M}}\,, \tag{5}$$

where R is the universal gas constant, T – absolute temperature, and M – molecular weight of the gas.

The equation (5) shows temperature dependence and indicates that pressure does not depend on the speed of sound. When pressure rises, the density rises as well. Due to this reason, any changes occur when pressure changes. However, that applies only in a case when temperature remains constant. Density changes that have no impact on pressure are caused by temperature changes. Humidity also has an impact on density and causes changes to the speed of sound.

While R and M are constants, we can prove that initially the speed of sound depends on temperature:

$$c \cong C_O \sqrt{\frac{T}{273}}, \tag{6}$$

where T is temperature in kelvins (K), and C_0 is equal to the standard speed of sound according to conditions defined.

It is obvious that the speed of sound rises when the square root of absolute temperature rises. After converting the temperature to degrees Celsius and making some arithmetical simplifications we get the following equation:

$$c = 331{,}45\sqrt{1+\frac{t}{273}}, \tag{7}$$

where t is the temperature in degrees Celsius. The graph of equation (7) is provided in Fig. 3.

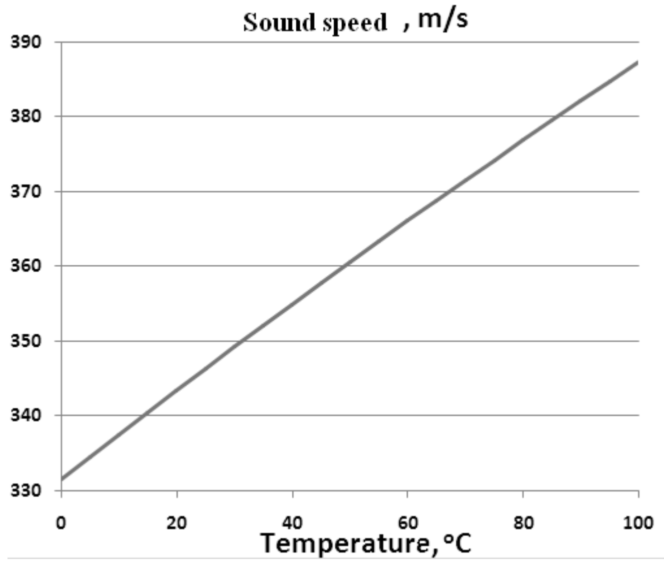

Fig. 3. Speed of sound (m/s) versus temperature (^oC)

The data above is presented in a more practical way in Fig. 4, where it shows graphically how the speed of sound changes percentage-wise as temperature increases.

Impact of humidity

The analysis provided above is based on an assumption that air is dry. Based on the equation (2), humidity has an impact on air density and the speed of sound in air. Dry air is less dense than humid air, and value of p (2) is lower in the equation. Accordingly, the value of the speed of sound decreases as well. Furthermore, decrease of the speed of sound is caused by a specific warmth ratio lowered due to humidity. In this case the decrease of density is dominant, and as a result humidity rises when the speed of sound decreases.

A limited amount of information on practice and correlation between relative humidity and the speed of sound tests is available in literature. A detailed analysis of direct correlation between relative humidity and sound speed percentage increase is provided in the book by A. Pierce (Pierce A. D., 1981).

The equation (7) is applicable only when air is dry. It is necessary to change two expressions in order to determine a precise impact of humidity (water vapor) on the speed of sound. These are the specific-head ratio (1.4 for dry air) and *M*, the average molecular weight of the different types of molecules in the air. The values of *R* (universal gas constant) and *T* (absolute temperature) remain unchanged.

The specific-heat ratio γ can be expressed as an exact fraction if *d* is equal to the number of degrees of freedom for air molecules. In this case the following equation can be formed:

$$\gamma = \frac{d+2}{d} \qquad (8)$$

Dry air is mainly composed of molecules consisting of two atoms. Because of that this type of air is called diatomic gas characterized by 5 degrees of freedom, three translational and two rotational degrees; respectively, in case of dry air $d = 5$ and $\gamma = 1{,}40$.

We will describe *h* as a part which is equal to a number of molecules in the air. The number of approximate degrees of freedom for water (6 degrees of freedom) that goes to one molecule rises: $5 + h$. Now we can rewrite the equation (9) and include humidity impact on the air:

$$\gamma_W = \frac{7+h}{5+h}\,. \qquad (9)$$

In a case when humidity increases, an approximate molecule weight of air decreases. To prove that at the beginning we have to calculate the value of *M*. Below a composition of dry air is presented:

78 % nitrogen (molecular weight = 28)
21 % oxygen (molecular weight = 32)
1 % argon (molecular weight = 40)

The total molecular weight is equal to:

$$M = (0{,}78)(28) + (0{,}21)(32) + (0{,}01)(40) = 29 \qquad (10)$$

Due to water (its molecular weight is 18) approximate molecular weight decreases: 29–(29–18)*h*, or

$$M_W := 29 - 11\,h. \qquad (11)$$

Two expressions of the equation (5) are changed in the equations (9) and (11). Occurrence of water vapor has an impact on them. Both expressions are added to the part of water molecule *h*. Relative humidity *RH* (expressed as a percentage) has the following expression:

$$h = \frac{0{,}01 RH e(t)}{p}\,, \qquad (12)$$

where *p* equals ambient pressure ($1{,}013 \times 10^5$ *Pa* = 1 atm reference pressure), and *e(t)* is the vapor pressure of water at temperature *t.* Values of *e(t)* according to temperature in Celsius scale are given below:

Table 1. Dependence of water vapor pressure on temperature

t, ^oC	5	10	15	20	30	40
$e(t)$,*Pa*	872	1228	1705	2338	4243	7376

We have to take ratio of sound speed in humid and dry air, subtract 1 and multiply by 100 in order to express percentage of speed increase caused by relative humidity.

While the expressions of sound speed in dry and humid air have the same constants *(R* and *T),* their ratio will disappear and we will get the following expression:

$$\frac{c_W}{c_d} = \frac{\sqrt{\dfrac{\gamma_W}{M_W}}}{\sqrt{\dfrac{\gamma_d}{M_d}}} = \frac{\sqrt{\dfrac{\gamma_W}{M_W}}}{\sqrt{\dfrac{1.4}{29}}} = 4.5513\sqrt{\frac{\gamma_W}{M_W}} . \tag{13}$$

When we subtract 1 and multiply by 100, we get:

$$\text{Increase of the speed of sound} = 4.5513\sqrt{\frac{\gamma_W}{M_W}} - 100 \tag{14}$$

Figure 5 presents the equation (14) formed as a function for relative humidity of six temperature values. Fig. 5 shows the percentage of the increase of the speed of sound caused by relative humidity; the temperature values are given for accurate determination of relative humidity. The results obtained in the equation (14) are provided in Table 2.

Combined effects of temperature and relative humidity

Summarizing Fig. 3, Fig. 4, and the results provided in two tables a dependence of the speed of sound on temperature and relative air humidity may be formed. That is given in Table 3.

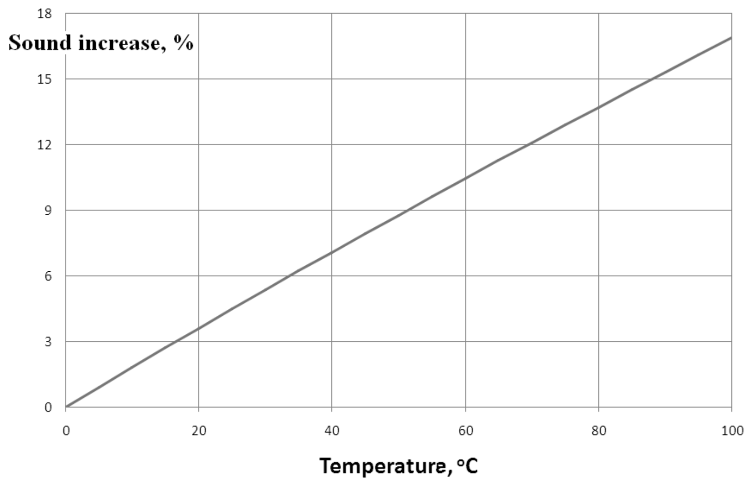

Fig. 4. Increase of the speed of sound depending on temperature (expression by percentage in dry air)

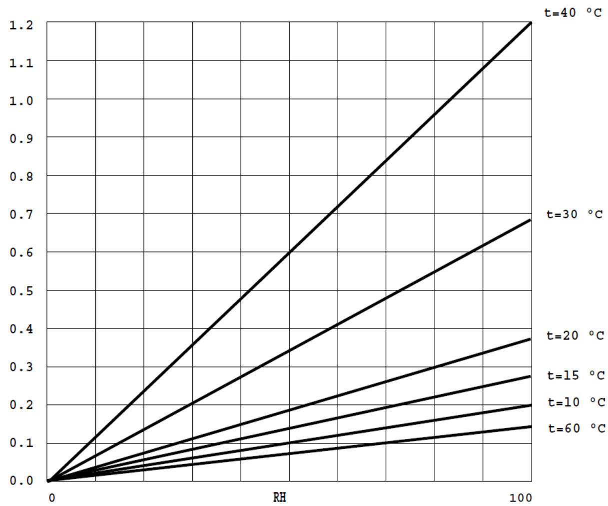

Fig. 5. Relative humidity versus percentage change in the speed of sound as a function of temperature

Table 2. Increase of the speed of sound expressed by percentage when changes of relative humidity expressed in percentage (in this case temperature is not being evaluated)

(°C)	10	20	30	40	50	60	70	80	90	100
5	0.014	0.028	0.042	0.056	0.070	0.083	0.097	0.111	0.125	0.139
10	0.020	0.039	0.059	0.078	0.098	0.118	0.137	0.157	0.176	0.196
15	0.027	0.054	0.082	0.109	0.136	0.163	0.191	0.218	0.245	0.273
20	0.037	0.075	0.112	0.149	0.187	0.224	0.262	0.299	0.337	0.375
30	0.068	0.135	0.203	0.272	0.340	0.408	0.477	0.546	0.615	0.684
40	0.118	0.236	0.355	0.474	0.594	0.714	0.835	0.957	1.08	1.20

Table 3. Increase of the speed of sound expressed as a percentage when changes of relative humidity expressed as a percentage and changes of temperature

(°C)	0	30	40	50	80	100
5	0.91	0.952	0.966	0.980	1.02	
10	1.81	1.87	1.89	1.91	1.97	2.01
15	2.71	2.79	2.82	2.85	2.93	2.98
20	3.60	3.71	3.75	3.79	3.90	3.98
30	5.35	5.55	5.62	5.69	5.90	6.03
40	7.07	7.43	7.54	7.66	8.03	8.27

Effects of frequency, pressure and CO_2

The dependence of the speed of sound in air on carbon dioxide CO_2 expressed in value h_c in percent at the rate 0-1 % and when temperature rate is 0-30 °C at 101,325 kPa is given in the following approximation formula:

$$c(h_C, {}^o t) = c_o(1{,}0000974 + 10^{-7} \cdot {}^o t - h_C(0{,}003091 + 2{,}7 \cdot 10^{-6} \cdot {}^o t)) . \qquad (15)$$

The dependence of the speed of sound in air on temperature has been determined empirically by A. H. Hodge. The dependence is provided in a table in the book by D. Kei ir T. Lebi and shown as a graph in the book by L. Bergman.

As we can see from the table, the speed of sound rapidly rises when pressure starts rising. The latter conclusion applies when it comes to temperatures higher than 250 ^oK. At the beginning of a case at lower temperatures the speed of sound in air decreases when pressure rises. Then the speed of sound passes the "break" point and begins to increase.

G. P. Howell and C. L. Morfey present the dependence of the speed of sound in air in the frequency range of 200-500Hz. Fig. 2 of their book shows that due to oxygen relaxation process, 20^{o}C and 1 atm, the speed of sound increases by about 0,1m/s. The increase in the speed of sound due to increased frequency is related to air humidity too. In a case when humidity is higher, the speed of sound increases when frequency is higher (Howell G. P. and Morfey C. L., 1987).

Table 4. Dependence of the sound of speed on pressure

Pressure (atm)	1	10	20	50	100
Relative sound speed	1	1,003	1,008	1,024	1,064

Conclusions

It has been estimated that the maximum variation range for the speed of sound is between 30 and 50 ^{o}C. The cubic meter measurement range is from -40 up to +60 °C. We can conclude that factors such as temperature as well as humidity, pressure, frequency and contamination have a significant effect.

The maximum speed of sound is mostly affected by temperature fluctuations that can vary by about 17% at temperature range of 0÷100^{o}C, while humidity in the same range can vary by about 2%.

It is necessary to control and evaluate the effects of the pressure, carbon dioxide and humidity while performing accurate measurements of the dependence of the speed of sound on temperature.

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