597. Fundamental Understanding of the Dynamic Measurement Equation as the Principle of Certain Measurements

G. Abramchuk
Expert, NSERC of Canada
e-mail: *george.abramchuk@sympatico.ca*

Abstract. The paper presents a place of a dynamic measurement equation in hierarchy of experimental investigations of dynamically-functioning nonlinear structures in conditions of nonlinearity and chaos.

Keywords: dynamic measurements, non-linear structural dynamics, chaos, dynamic error and uncertainty

Introduction and setting of the task

It is obvious fact now that the demand for enhanced and reliable performance of vibrating structures in terms of weight, comfort, safety, noise and durability is ever increasing. The need for designing lighter, more flexible, and consequently, more nonlinear structural elements working in medium of increasing speeds is claimed in creation world. It follows that the demand to utilise nonlinear or even strongly nonlinear structural components is increasingly present in engineering applications. Of course, nonlinearity is generic in nature, and linear behaviour is an exception. The inapplicability of various concepts of linear theory is obvious. It is rather paradoxical to observe that linear behaviour is frequently taken for granted in structural dynamics. The highly individualistic nature of nonlinear systems and the basic principles that apply to linear systems are no longer valid in the presence of nonlinearity. One is forced to admit that there is no general analysis method that can be applied to all dynamic systems in all instances. Nevertheless, linear or linearized models consider in preference in the real majority of previous and current publications which are concerned with accurate analysis of dynamic measurements; they cannot be useful, therefore, cannot bring desired results. The results of incorrect dynamic measurements at dynamically functioning structures (DFS) based on linear or linearized models can cause the difficulties that often are misunderstood and unreasonable, and cannot bring the reliable solutions. Current decade was marked by growth of professional interest to creation of the theory of behaviour of nonlinear structures and systems. Different nature of nonlinearity makes structural dynamics egoistic. Unfortunately, there is affection to describe behaviour of the DFS as time-invariant and deterministic for concrete excitation conditions, and the system response always considers as the same without any uncertainty, often does not give objective results in strong approach. What way is the optimal way?

One of the basic elements of a considered problem is *the behaviour of dynamic measurement equation* (DME) as one of the established elements in hierarchy of dynamic measurements of the DFS for creation of flexible measuring system, metrology, consequently, for research, testing, monitoring, and other applications. One of next important steps is understanding that the 'smoothing analysis' of the measuring information does not give possibility to carry on real estimation of the effects generated by behaviour of a nonlinear dynamic system as well as estimation of nonstationary process that are so necessary for extraction of trustworthy information.

The known publications addressed to problems of dynamic measurements in structures with changing nonlinearity are dedicated to develop understanding how it has to be in order to absorb the results of the measurements, to optimize, and to correct measurements as well as to build the experiments or testing, based on nonchanging measurement equations. Hence, the problem addressed to dynamic measurements requires special attention. This article is dedicated to the general analysis of methodical features of dynamic measurements at DFS with expressed nonlinear dynamics or nonlinear dynamic processes, when the DME plays one of the main roles.

Dynamic measurement equations in medium of nonlinear dynamics of Markov's object or processes and general solutions for dynamic measurements

It is known that the solution of a dynamic measuring problem begins from the construction of a priori physical-mathematical or conceptual models directed to the specific aim of metrology, research, and designing as well as testing and certification. The models of the concrete structure cannot be exhaustive. There always exist physical, informational, and model uncertainties in modeling and simulation of complex engineering structures. In real words, the adopted model unavoidably differs to some extent from the physical model of a structure and, consequently, the results of modeling contain dynamic error which, as a rule, is a function of time. The depth of knowledge based on the model has to minimize the functional of dynamic uncertainty and methodical dynamic error. In order to complete considering stage of the theory of measurements, one has to use not only physical and mathematical instruments but results of special experiments to construct a priori structural model for physical structures, one has to solve the specific problems of the estimation of the dynamic metering error what is the result of model validation, model evaluation, and model estimation.

Numerous methods have been considered for identification of nonlinear dynamic structural (NDS) models which may be suggested to be effective instrument for creation of the DME (because of the nature the models are highly individualistic and egoistic).

A NDS model for hierarchical validation of dynamical structures, for creation of the DME has to be considered. In most cases, this model describing a structure is based on different types of nonlinearities such as geometrical, inertial, vibro-contact, and boundary variable condition nonlinearities as well as nonlinearities created by material behaviour, damping dissipation, damaged-structure nonlinearity, small degradation of joints, [1], and others, which are functions of time. To solve such class of problems, perspective approaches has to be used, for instance those considered in [2-8], where used methodologies consist of the following stages: (i) a measuring model to specify the relationships of the latent constructs to the observed variables; (ii) a computational model to map response, to input variables, and to quantify the relationships of the latent variables to the predicted variables, and (iii) a structural model to identify the relationships among the unobserved latent variables, thus relating the computational model output to the high-level data, and relating the lower-level data to the high-level data. The hierarchical Bayesian inference network associated with simulation based on Markov Chain Monte Carlo approach and Gibbs sampler is employed in [9] to represent the NDS model and to estimate the model parameters. Methodology provides more accurate modeling of the hierarchical validation problem. The nonlinear relationships in referred methodology can effectively represent the nonlinearities which exist in a physical model of a DFS in light of the validation data. What kind of model may be used for creation of the DME, or may not be used.

It is known that the value of changing nonlinearity is determined by amplitudes of excitations and by load rates as functions in time. A state of a DFS may be described by a state equation

$$Y(t) = [B(t) + n(t)]\, X(t), \tag{1}$$

where Y (t) is output responses, B (t) is the transformation operator, n (t) is the operator for total errors and uncertainties, and X (t) is the input impact i.e. measurand impact, respectively. As a consequence, a dynamic measurement equation may be

$$y_t = C_t X_t + \varepsilon_t, \tag{2}$$

where y_t represents results of measurements, C_t is the certain functional, X_t is input impact, and ε_t denotes total errors and uncertainties, respectively. The C_t may be the vector, the scalar, or presented in view of components. It is an example. Different ways can be used to solve this part of a regarding problem. However, the essence is the same.

Creation of high-accuracy DFS requires of very scrupulous synthesis and of analysis of a DME.

It is obvious that the members of measurement equation are not always determined by direct measurements. Being members of the main DME, they often can be determined as a result of indirect measurements by the local DMEs. As an accessible example, this can be indirect measurements of parameters of motion (acceleration, vibration and shock), dynamic temperature and pressure, etc. including medical diagnostics. DME joins all members in order to extract useful and necessary information, to predict behaviour of a structure in extreme conditions. Extracted information may be essentially different in dependence of measurement methodology used in the basis of a measurement problem, dynamics of the structure, generated by nonlinearities, and the changing “momentary” complete dynamic characteristics of the structure [10] that is not obvious. Certainly, behaviour of nonlinear structures or the processes in structure are subordinated to multifactor influences. Investigators study real simulation, i.e. the nonstationary chaotic response based on results of verification and validation of the DME. Uncertainty and credibility are estimated for different input conditions. It is addressed to diagnosis, experimental uncertainty, fidelity-testing for prediction, as well as to sensitivity, to variability, to uncertainty, to lack-of-knowledge that is the necessary and important integral parts of the complex works.

Analysis cannot give the results if a priori information was incorrect. If a measurement channel does not give possibility to distinguish behaviour of the DFS in different conditions, dynamic behaviour of the structure is perceived as the widely dim process which does not yield to intelligible approximation. However, investigators do not know about it without detailed analysis of the DME. Fuzziness considers dynamic uncertainty. Results of approximated measurements are carried to uncertainty. In real words, it is not the case. Linear approximation of the DME looses sense if it was used for chaotic or nonstationary processes.

One has to receive possibility to understand what kind of variations of the measuring information [11] generated by behaviour of a Markov’s structure may lead to variation of the dynamic measurement equation during measuring procedures. One has to be sure that measurements are correctly built when they are based on the DME, effects associated with behaviour of nonlinear dynamic system and their influence on total measurement uncertainty and metering error are controlled. The DME considers nonsmoothing analysis. Integral analysis distorts the reality. Responses of a DME have to be analysed theoretically with needed accuracy.

Everyone seeks to have a stable object to disturbances, to the internal and external sources of nonlinearity as well as to transient dynamics. Naturally, an unchanged DME is the desired purpose to reach the necessary level of accuracy. Use of the “static” measurement equation for cases with high level of the latent dynamic error often deforms the real picture of processes or behaviour of a DFS that makes the further processing of the measuring information senseless.

A changing DME shows what transform may be used to absorb useful information - STFT, WT, HHT, Sparse and compressible signal transform or their combinations, and so on. For an information- measuring system, it is suitable to take into consideration the transition from the *dynamic measurement equation* to the *equation of measurement processes* (EOMP) representing all main operations of detailed procedures. The EOMP establishes interaction between the quantities in measuring processes, provides possibility to choose or to create not only information-signal processing techniques. There are problems of dynamic measurements of a DFS in condition when a DME incorporating aims for established accuracy (error and uncertainty) changes during measuring procedures [12] (for instance, there are both high-speed dynamics and quickly proceeding processes which can be in a considered structure). For it, evaluation of the dynamic uncertainty and error establishes the "momentum" DME for the measurements of the concrete referred processes; they are not received by integral methods.

Effects of chaotic dynamics have to be considered and their influence to measurement and methodical uncertainty. Different DMEs are created to be used in analysis and in evaluation of potential dynamic measurements. Any measurable quantity characterizing the object of investigation is connected with changes of parameters of an object. Uniqueness of dynamic measurements is in it as well. Investigators should approximate processes. It is not the correct way. There are a lot of arguments are for it.

Time-dependence of dynamic uncertainty - an example

The results presented in Fig.1 are illustrated by one example of transformation of the object of investigations from the object with concentrated parameters to the object with distributed parameters by wave processes in an object.

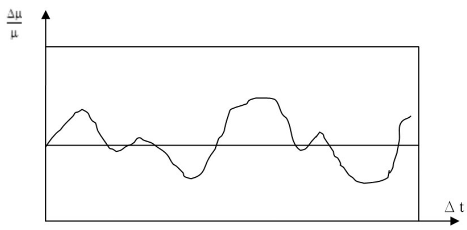

Fig. 1. Dependence of fractional uncertainty Δμ to average of uncertainty value μ as of the function of time t

However, it cannot be for situation when one follows to a DME when parameters of an object changes every time. One has to deal with a momentary DME, time-dependence of uncertainty as well as with methodical error.
If one has the DME and the equation of measurement processes, it means, one implies that the specifications for of measurement result accuracy are given. In fact, it determines how completely one incorporates all influencing factors in measurements.

Concluding remarks

The paper overviews a place of a DME, the equation of measurement processes, local DME as well as a state equation in hierarchy of dynamic measurements of processes or behaviour of a dynamically-functioning nonlinear structures in chaotic medium in order to obtain highly needed results. Influencing factors on the DME were analysed. It is demonstrated that dynamic characteristics of a measurement channel during dynamic measurements may be distorted on account of the inertia of members of the DME. In practice, one generates the slightest dynamic measuring right up to the rounding-off error that results in complete loss of physical sense in the final results.

It is truly a difficult job that requires considerable efforts and expenses. If one does not perform all the complex described operations one cannot achieve significant results. Nevertheless, the expenses will grow.

Nonlinearity and chaos reflection in the DME require simulation and scrupulous analysis of all components which bear responsibility for reliability of results conversely determining subsequent critical solutions.

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