596. Amplitude of Radial Oscillations and Imbalance Parameters of the Rotating Rotor

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Abstract. The paper proposes a method of continuous definition of imbalance magnitude of a rotating rotor in process of its operation on the basis of analysis of amplitude of the radial fluctuations, which is measured (identified) as a distance between rotation and symmetry axes of a rotor.

Keywords: rotor imbalance, amplitude of radial oscillations, radial displacement, axis of rotation, symmetry axes.

The amplitude of radial oscillations of a rotating rotor may be determined in the context of continuous systemic definition of parameters [1-3] as distance between axes of rotation and symmetry of a rotor (Fig. 1) in the form [4-6]:

$$A_0 = \sqrt{R^2 + r_3^2 - 2Rr_3 \cos(\alpha - \beta)}\,, \tag{1}$$

where radius of a rotor

$$R = \frac{1}{2}\sqrt{\frac{r_2^2 + r_3^2 - 2r_2 r_3 \cos\chi_{23}}{1 - \dfrac{r_1^2 - r_1 r_2 \cos\chi_{12} - r_1 r_3 \cos\chi_{13} + r_2 r_3 \cos\chi_{23}}{\sqrt{\left(r_1^2 + r_2^2 - 2r_1 r_2 \cos\chi_{12}\right)\left(r_1^2 + r_3^2 - 2r_1 r_3 \cos\chi_{13}\right)}}}}\,; \tag{2}$$

$$\alpha = \arcsin\sqrt{\frac{r_1^2 - r_1 r_2 \cos\chi_{12} - r_1 r_3 \cos\chi_{13} + r_2 r_3 \cos\chi_{23}}{\sqrt{\left(r_1^2 + r_2^2 - 2r_1 r_2 \cos\chi_{12}\right)\left(r_1^2 + r_3^2 - 2r_1 r_3 \cos\chi_{13}\right)}}}\,; \tag{3}$$

$$\beta = \arcsin\frac{r_2 \sin\chi_{23}}{\sqrt{r_2^2 + r_3^2 - 2r_2 r_3 \cos\chi_{23}}}\,; \tag{4}$$

r_1, r_2 r_3 – radiuses-vectors of points A, B, C on a surface of the rotor, which are controlled by sensors D_1, D_2, D_3 accordingly (Fig. 1);

χ_1, χ_2, χ_3 – angles between radiuses-vectors r_1, r_2 r_3.

D_1, D_2, D_3 – sensors which supervise distances s_1, s_2, s_3 up to opposite points A, B, C on surfaces of a rotor and also linear speeds V_1, V_2, V_3 of these points.

Geometrical construction, presented in Fig. 1, b, allows to compose and solve the equations system in which radiuses-vectors r_1, r_2 r_3 and angles χ_1, χ_2, χ_3 between them are decision variable, and directly measured by sensors D_1, D_2, D_3 magnitudes s_1, s_2, s_3, V_1, V_2, V_3 are coefficients changing at a rotor rotation [2, 6].

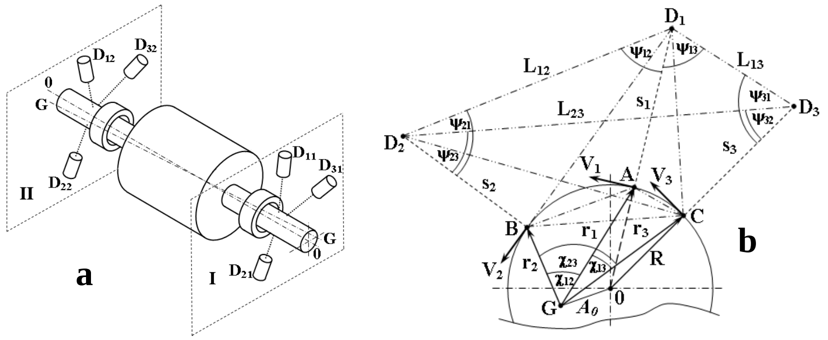

Fig. 1. Distance definition between axes of rotation and symmetry of a rotor
a – sensors disposition; b – rotor section by a plane of sensors disposition

It is considered that due to rotor imbalance its rotation occurs around the axis which passes through the center of masses, and the amplitude of radial oscillations A_0 is equal to mass eccentricity ε (i.e., $A_0 = \varepsilon$). We will further demonstrate that such statement is fair only for angular rotor speed ω, which is significantly smaller than resonant value Ω (at condition $\omega << \Omega$).

In the sufficiently large values of angular speed ω displacement of the mass center of the rotor concerning the symmetry axis, which is caused by its imbalance, leads to additional inertial displacement λ of the rotation axis concerning the mass center [7-10]. In this case the amplitude of radial fluctuations can be represented in the following form:

$$A_0 = \varepsilon + \lambda \ . \tag{5}$$

Magnitude of inertial displacement λ for harmonic oscillations can be determined as [7, 8, 10]:

$$\lambda = \frac{\varepsilon\omega^2 sign\left(\Omega^2 - \omega^2\right)}{\sqrt{\left(\Omega^2 - \omega^2\right)^2 + 4\mu^2\omega^2}} \ , \tag{6}$$

where μ is the damping coefficient of oscillations.

We may find from (5) an allowance expression (6):

$$A_0 = \varepsilon\left[1 + \frac{\omega^2 sign\left(\Omega^2 - \omega^2\right)}{\sqrt{\left(\Omega^2 - \omega^2\right)^2 + 4\mu^2\omega^2}}\right]. \tag{7}$$

The analysis of expression (7) indicates that under condition of $\Omega > \omega$ magnitude

$$\lambda = \frac{\varepsilon\omega^2 sign\left(\Omega^2 - \omega^2\right)}{\sqrt{\left(\Omega^2 - \omega^2\right)^2 + 4\mu^2\omega^2}} > 0 \ ,$$

and, consequently

$$A_0 = \left(\varepsilon + \lambda\right) > \varepsilon \ . \tag{8}$$

It follows from (8), in the area of under-resonant frequencies amplitude A_0 of radial oscillations exceeds mass eccentricity of a rotor.

Under condition of $\Omega > \omega$ magnitude

$$\lambda = \frac{\varepsilon\omega^2 sign\left(\Omega^2 - \omega^2\right)}{\sqrt{\left(\Omega^2 - \omega^2\right)^2 + 4\mu^2\omega^2}} < 0 ,$$

and, consequently

$$A_0 = \left(\varepsilon + \lambda\right) < \varepsilon . \tag{9}$$

It follows from (9), in the area of after-resonant frequencies amplitude A_0 of radial oscillations becomes smaller mass eccentricity of a rotor.

Let's define limiting values of inertial displacement λ and amplitudes A_0 of radial oscillations.

$$\lim_{\omega\to 0}\lambda = \lim_{\omega\to 0}\frac{\varepsilon\omega^2 sign\left(\Omega^2 - \omega^2\right)}{\sqrt{\left(\Omega^2 - \omega^2\right)^2 + 4\mu_r^2\omega^2}} = 0 ; \tag{10}$$

$$\lim_{\omega\to 0} A_0 = \lim_{\omega\to 0}\varepsilon\left[1 + \frac{\omega^2 sign\left(\Omega^2 - \omega^2\right)}{\sqrt{\left(\Omega^2 - \omega^2\right)^2 + 4\mu_r^2\omega^2}}\right] = \varepsilon . \tag{11}$$

It follows from (10) and (11), under condition $\omega \to 0$ and, accordingly, $\lambda\big|_{\omega\to 0} \to 0$, amplitude A_0 of radial oscillations approaches to magnitude of mass eccentricity ε of a rotor:

$$A_0\big|_{\omega\to 0} \to \varepsilon . \tag{12}$$

Magnitude $sign\left(\Omega^2 - \omega^2\right) < 0$ at infinitely large increase of angular speed ω (and, hence, under condition $\Omega < \omega$), and we may find from (6) and (7):

$$\lim_{\omega\to \infty}\lambda = \lim_{\omega\to \infty}\frac{\varepsilon\omega^2 sign\left(\Omega^2 - \omega^2\right)}{\sqrt{\left(\Omega^2 - \omega^2\right)^2 + 4\mu_r^2\omega^2}} = -\varepsilon ; \tag{13}$$

$$\lim_{\omega\to \infty} A_0 = \lim_{\omega\to \infty}\varepsilon\left[1 + \frac{\omega^2 sign\left(\Omega^2 - \omega^2\right)}{\sqrt{\left(\Omega^2 - \omega^2\right)^2 + 4\mu_r^2\omega^2}}\right] = 0 . \tag{14}$$

Dependence of inertial displacement λ (a) and amplitude A_0 of radial oscillations (b) from angular speed ω and mass eccentricity ε is presented in Fig. 2.

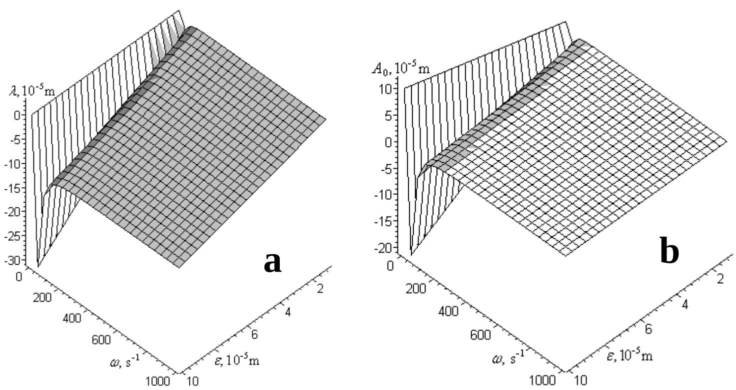

Fig. 2. Dependence of inertial displacement λ and amplitude A_0 of radial oscillations from angular speed ω and mass eccentricity ε

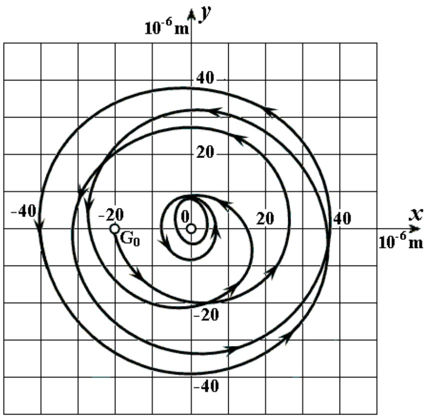

Fig. 3. Displacement of the rotation axis of a high-speed rotor at its acceleration. 0 – position of the symmetry axis; G_0 – position of the rotation axis $\omega \to 0$

It follows from (13) and (14), under condition $\omega \to \infty$ and, accordingly, $\lambda\big|_{\omega\to\infty} \to -\varepsilon$, amplitude A_0 of radial oscillations has approximation to zero value. It testifies to approximation of rotation axis of a rotor to its symmetry axis (so-called effect of a self-centering of a high-speed rotor).

The trajectory of point G_0 of crossing of rotation axis of the high-speed rotor (ω_{st}>1000 s^{-1}) with a supporting plane 0*xy* (at that the point 0 is a point of crossing of the symmetry axis of a rotor with plane 0*xy*) is provided in Fig. 3 [11, 12]. Conditions $t \to 0$ and $\omega \to 0$ occur at the starting moment. According to (10) and (11), the distance $0G_0$ is distance between rotation axis and symmetry axis (amplitude of radial oscillations A_0), is equal to mass eccentricity ε of a rotor. The increase of angular speed in limits $\Omega > \omega > 0$ entails increase of amplitude of radial oscillations: $A_0 > \varepsilon$. After a resonance passage the amplitude of radial oscillations A_0 are reduced and at exceeding by angular speed of a resonant threshold (the condition $\omega > \Omega$) it is become less of mass eccentricity ε.

We may define mass eccentricity of a rotor by comparison of expressions (1) and (7):

$$\varepsilon = \frac{\sqrt{R^2 + r_3^2 - 2Rr_3\cos(\alpha-\beta)}}{1 + \dfrac{\omega^2 sign\left(\Omega^2-\omega^2\right)}{\sqrt{\left(\Omega^2-\omega^2\right)^2 + 4\mu^2\omega^2}}}, \tag{15}$$

where magnitudes R, α, and β are defined by expressions (2), (3) and (4).

Momentary value of angular speed w of a rotor can be determined in the form:

$$\omega = \frac{V_1}{r_1} = \frac{V_2}{r_2} = \frac{V_3}{r_3}, \tag{16}$$

where r_1, r_2, r_3 are momentary values of radiuses-vectors of supervised points A, B, C on a rotor surface and V_1, V_2, V_3 are linear speeds of these points which are measured directly by sensors D_1, D_2, D_3 [6, 10].

The damping coefficient μ of oscillations may be defined in the form [13]:

$$\mu = \delta\frac{\Omega^2-\omega^2}{2\omega}\sqrt{\frac{1}{\pi^2-\delta^2}}, \tag{17}$$

where logarithmic decrement of oscillation attenuation

$$\delta = \ln \frac{A_{1res}}{A_{2res}}, \tag{18}$$

A_{1res} and A_{2res} are the amplitudes of two consecutive oscillations which are divided by period T and measured at a resonance passage (oscillations, which are approximate to the free radial oscillations of a rotor).

Dependence of the damping coefficient μ on angular speed ω and logarithmic decrement δ attenuations of fluctuations is given in Fig. 4.

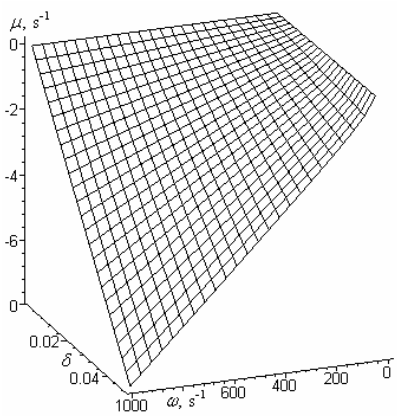

Fig. 4. Variation of damping coefficient μ as a function of angular speed ω and logarithmic decrement of attenuation δ

We find from expression (15) by taking into account (16) and (17):

$$\varepsilon = \frac{\sqrt{R^2 + r_3^2 - 2Rr_3\cos(\alpha - \beta)}}{1 + \dfrac{V_3^2\sqrt{(\pi^2 - \delta^2)}}{\pi\left(\Omega^2 r_3^2 - V_3^2\right)}}. \tag{19}$$

The analysis of expression (19) indicates that mass eccentricity of a rotating rotor can be measured continuously, in process of its operation, at continuous measurement of linear speeds V_1, V_2, V_3 of supervised points A, B, C on a rotor surface and identification of radiuses-vectors r_1, r_2, r_3 of these points[1)] [6].

If the rotor oscillations are periodic nonharmonic oscillations, the value of inertial displacement λ with consideration of Fourier series may be determined in the form [10]:

$$\lambda = \sum_{k=1}^{\infty} \frac{\varepsilon k^2\omega^2 sign\left(\Omega^2 - \omega^2\right)}{\sqrt{\left(\Omega^2 - k^2\omega^2\right)^2 + 4\mu^2 k^2\omega^2}}, \tag{20}$$

where k is the harmonic order of Fourier series.

We may find from equation (5) with regard to expression (20):

[1)] The choice of magnitudes r_3 and V_3 in formula (19) is determined by a casual choice of geometrical construction (Fig. 1, b). Other identical geometrical constructions define a choice of magnitudes r_1 and V_1, or magnitudes r_2 and V_2.

$$A_0 = \varepsilon\left[1 + \sum_{k=1}^{\infty}\frac{k^2\omega^2 sign\left(\Omega^2 - \omega^2\right)}{\sqrt{\left(\Omega^2 - k^2\omega^2\right)^2 + 4\mu^2 k^2\omega^2}}\right]. \tag{21}$$

In the case of periodic nonharmonic oscillations mass eccentricity ε of a rotor may be determined from comparison of expressions (1) and (21):

$$\varepsilon = \frac{\sqrt{R^2 + r_i^2 - 2Rr_i\cos(\alpha - \beta)}}{1 + \sum_{k=1}^{\infty}\frac{k^2V_i^2\sqrt{\left(\pi^2 - \delta^2\right)}}{\pi\left(\Omega^2 r_i^2 - k^2V_i^2\right)}}, \tag{22}$$

Where i is the number of one of three radiuses-vectors of points A, B, C on a rotor surface and V_i is linear speed of corresponding point.

It follows from the presented study that continuous measurement (identification) of mass eccentricity is provided with system of integrated measurement of mechanical parameters of a rotor [1-3] and does not require application of the special measuring elements and devices.

References

[1]. **Sokol V. M.** Mathematical Identification's Methods of Dynamic Parameters and Characteristics of Rotors' Systems. Proceedings of Institute for Advanced Studies, Arad (Israel), 2006, Issue 6, 19-34. ISBN 965-90599-5-7.

[2]. **Sokol V. M.** Complex system of continuous measurement of dynamic parameters of rotors. Proceedings of the International Scientific Conference «The Modern Achievements of Science and Education», Netania (Israel), September 9 – 17, 2007, P. 96-101. ISBN 966-330-025-6.

[3]. **Sokol V. M.** Continuous Measurement of Mechanical Rotors Parameters – the System Approach. System Research and Open Systems Management, Haifa (Israel), 2008, Issue 4, P. 74-79. ISSN 1565-8147.

[4]. **Sokol V. M.** Some problems of the differentiated vibration measurement of rotors systems. Journal of Vibroengineering, 2009, 11 (3), 392-399. ISSN 1392-8716.

[5]. **Sokol V. M.** Systemic Measurement of a Rotor Parameters. A Method of Vibration Measurement. Proceedings of 3-rd International Scientific Conference on Modern Achievements of Science and Education, Tel Aviv (Israel), September, 16-23, 2009, P. 12-16. ISBN 978-966-336-070-2.

[6]. **Sokol V. M.** Geometrical Fundamentals of Continuous Systemic Parameters Measurement of the Rotating Rotor. Proceedings of Scientific Conference on Researches in the Field of Control and Diagnostics, Arad (Israel), December, 2009, P. 40-48. ISBN 978-965-90599-8-0.

[7]. **Sokol V. M.** Determination of rotors moment of inertia during its balancing. Automation and modern technologies, 1995, 7, 22-24. ISSN 0869-4931.

[8]. **Sokol V. M.** Dynamic model of high-speed rotor with regard to its imbalance. Automation and modern technologies, 1996, 6, 23-26. ISSN 0869-4931.

[9]. **Goldin A. S.** Vibration of rotary machines. Moscow: Mashinostroenie, 1999. ISBN 5-217-02927-7.

[10]. **Sokol V. M.** The moment of inertia and oscillation of a statically unbalanced rotor. Proceedings of Institute for Advanced Studies. Arad (Israel), 2003, Issue 3. 7-31. ISBN 965-90599-0-6.

[11]. **Sokol V. M.** To a question on identification of stochastic mechanical parameters of a rotor. Proceedings of Institute for Advanced Studies. Arad (Israel), 2005, Issue 5, 49-55. ISBN 965-90599-3-0.

[12]. **Pchelin I. K., Schneider A. G.** Dynamics of the twisting mechanism with gas-magnetic support of a rotor. Technology of the textile industry, 1988, 3 (183), 88-92.

[13]. **Sokol V. M.** Force characteristics of dynamically unbalanced revolving rotors (mathematical model). Proceedings of VII International Conference on Improvement of Quality, Reliability and Long Usage of Technical Systems and Technological Processes, Sharm al Sheikh (Egypt), December 7-14, 2008, P. 39-44. ISBN 966-330-048-5.