

# 572. Cargo pendulum vibration damping inside vehicle hull

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**Abstract.** A model is proposed to study the dynamic response of cargo pendulum vibrations. Equations of motion are obtained by Lagrange equations using Lagrange multiplier. Two independent bumpers of the securing cargo are considered in the model. Bumpers forces are represented by a special stiffness and damping as a function of phase coordinates. Investigation was performed for: a) transient motion from free initial conditions; b) stationary motion excitation from constrained string or hull bumpers vibrations; c) joint transient and excitation motion take into account random parameters. Results of investigations may be used for cargo fastening inside trailer, ship or airplane.

**Keywords:** cargo pendulum motion, vehicle hull vibrations, nonlinear vibrations, vibro-shock motion, transient motion of pendulum

## Introduction

During transportation cargos are subjected to various vibrations and acceleration forces. These forces are caused by many factors from road vehicles engine and transmission systems, uneven roads, the shunting of rail cars, cargo handling at terminals and docks and the pitching, yawing and rolling of a ship at sea [1]. One of the important types of cargo vibration system is pendulum motion that imitates ship-mounted cranes (Fig. 1.). Simulation and experimental results of the special controlled cranes for different operating conditions and payload masses are investigated in many research works, e.g. [2-4]. Analysis of publications revealed that motion of cargo with additional shocks is not fully investigated [5]. Therefore, this is the object of research in this paper, where traditional models of simple pendulum with rigid hoisting cable and a lumped mass at the end of cable are observed (Fig. 1.). Oscillations in machines are invariably nonlinear. This is either because of inertial coupling effects between different motions of the moving components, material and constitutive phenomena giving rise to stiffness modifications, nonlinear dissipation mechanisms, large deflections, or, as in most case - some sort of combination of all of these. The net effect of nonlinear vibrations is that at best the machine may well behave a little differently from the way the designer intended, or at worst, in a manner which renders it completely unsuitable for the job. The extent of such problems depends on the nature and the scale of the nonlinearities that are present but it is safe to say that nonlinear oscillations can rarely be completely overlooked in design and analysis of precision machinery.

The unifying theme in this paper is pendulum motion, firstly in the case of a mobile gantry crane for container stacking where we wish to minimize such motion and converge on a target, and then secondly in the case of a vibration absorber in which we choose to initiate pendulum motion within a special absorber, for the purposes of vibration minimization. The third example involves the potential for pendulum motion at a very much larger scale and summarizes the main control problem that is likely to be encountered in a fully deployed momentum exchange

propulsion tether operating in space. The paper discusses the general mathematical issues that pertain to pendulum motion in each of the three cases [6]. Static stiffness of tower cranes is studied by using the proposed formulations and finite element method. A reasonable control value based on theoretical calculation and finite element method must be obtained and verified via collected field data. The results obtained from the finite element analysis must then be compared with the collected field data and that by the proposed formula. Corresponding to theoretical formulations and field data, it is deemed that the results of finite element analysis are closer to the actual data [7].



Fig. 1. <http://www.pmel.noaa.gov/vents/nemo1999/images/elevatorrecovery2.jpg>

### Equations of pendulum motion

Pendulum relative motion along sphere as mathematical point has constraint (1):

$$f(x, y, z) = 0 \text{ or } x^2 + y^2 + (h - z)^2 - L^2 = 0, \quad (1)$$

where  $x, y, z$  – coordinates of center mass;  $L$  – theoretical length of cable till pendulum centre;  $h$  – constant (Fig. 2.).

For motion investigation it is convenient to use first form of Lagrange differential equations with Lagrange multiplier  $\lambda$ . For this reason normal reaction  $N$  of constraint is equal (2):

$$\bar{N} = \lambda \cdot \text{grad}(f(x, y, z)), \text{ or } \bar{N} = \lambda \cdot \begin{Bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{Bmatrix}, \quad \bar{N} = \lambda \cdot \begin{Bmatrix} 2x \\ 2y \\ -2(h - z) \end{Bmatrix}. \quad (2)$$

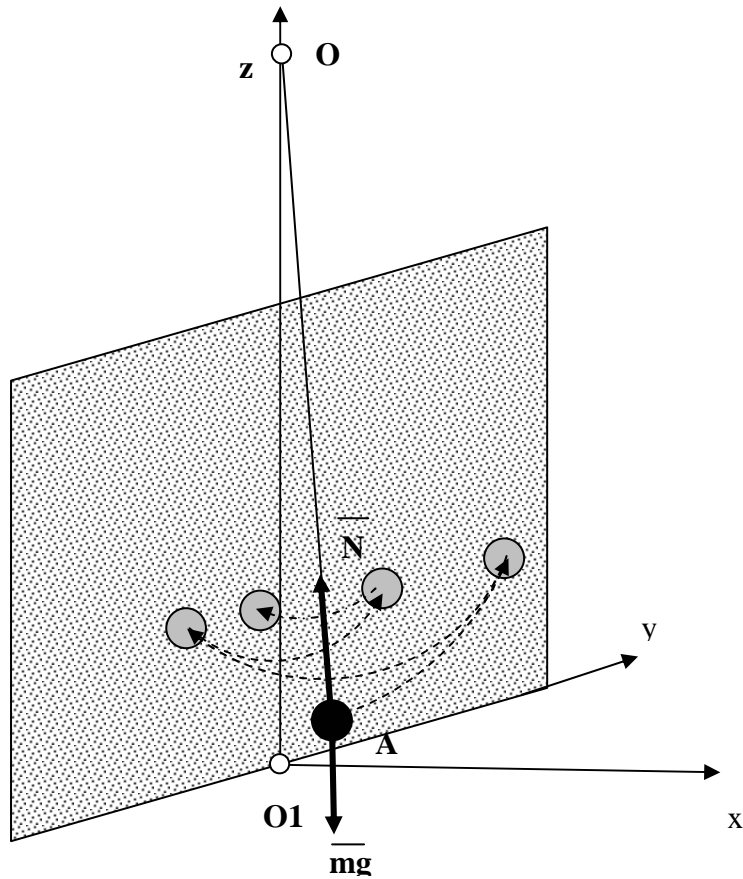


Fig. 2. Mathematical model of pendulum with one bumper

Pendulum has two degree of freedom. To use  $x$  and  $y$  like independent coordinates from equation (1) must be finding:

$$x \cdot \dot{x} + y \cdot \dot{y} - (h - z) \cdot \dot{z} = 0;$$

$$\dot{x}^2 + x \cdot \ddot{x} + \dot{y}^2 + y \cdot \ddot{y} + \dot{z}^2 - (h - z) \cdot \ddot{z} = 0. \quad (3)$$

Therefore from Newton's second law and equations (1), (2) and (3) accelerations  $\ddot{x}$ ,  $\ddot{y}$  may be determined in the form:

$$\ddot{x} = \ddot{x}(x, \dot{x}, y, \dot{y}, t); \quad (4)$$

$$\ddot{y} = \ddot{y}(x, \dot{x}, y, \dot{y}, t).$$

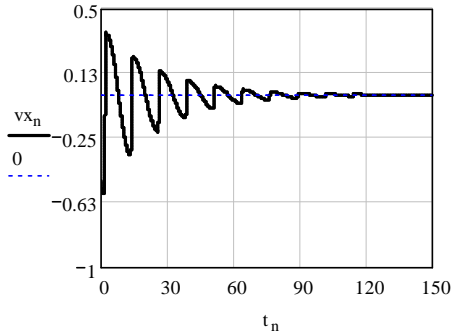
### Single-bumper pendulum model

Differential equations of motion are:

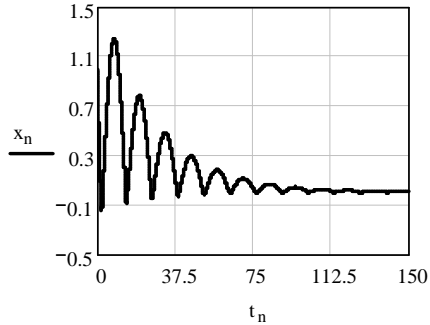
$$\begin{aligned}
 m(\ddot{x} + A1) &= B1(x, \dot{x}) \cdot 0,5 \cdot (1 - \text{sign}(x + \Delta1)) + 2 \cdot \lambda \cdot x; \\
 m(\ddot{y} + A2) &= 2 \cdot \lambda \cdot y; \\
 m(\ddot{z} + A3) &= -m \cdot g - 2 \cdot \lambda \cdot (h - z),
 \end{aligned}
 \tag{3}$$

where  $m$  – mass;  $g$  – acceleration of free fall;  $A1$ ,  $A2$  and  $A3$  – translation acceleration of reference system along  $x$ ,  $y$ ,  $z$  axis (Fig. 2.);  $B1(x, \dot{x}) \cdot 0,5 \cdot (1 - \text{sign}(x + \Delta1))$  – reaction of bumper interaction along  $x$  axis;  $\Delta1$  – gap along  $x$  axis.

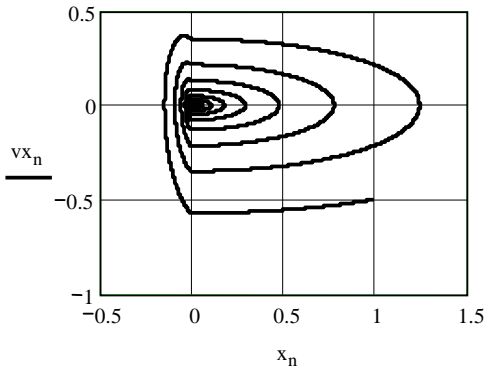
For the single-bumper model the results of investigation (3) are presented in Figs. 3-6. The results indicate that (for vibration damping) bumper must be soft and gap negative (then vibration damping is efficient).



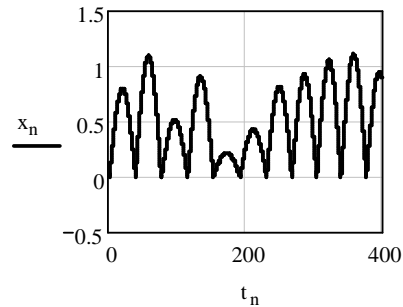
**Fig. 3.** Displacement of mass along  $x$  axis in time domain without hull vibration



**Fig. 4.** Velocity of mass along  $x$  axis in time domain without hull vibration



**Fig. 5.** Motion in phase plane



**Fig. 6.** Displacement of mass along  $x$  axis in time domain with hull vibration

### Double-bumper pendulum model

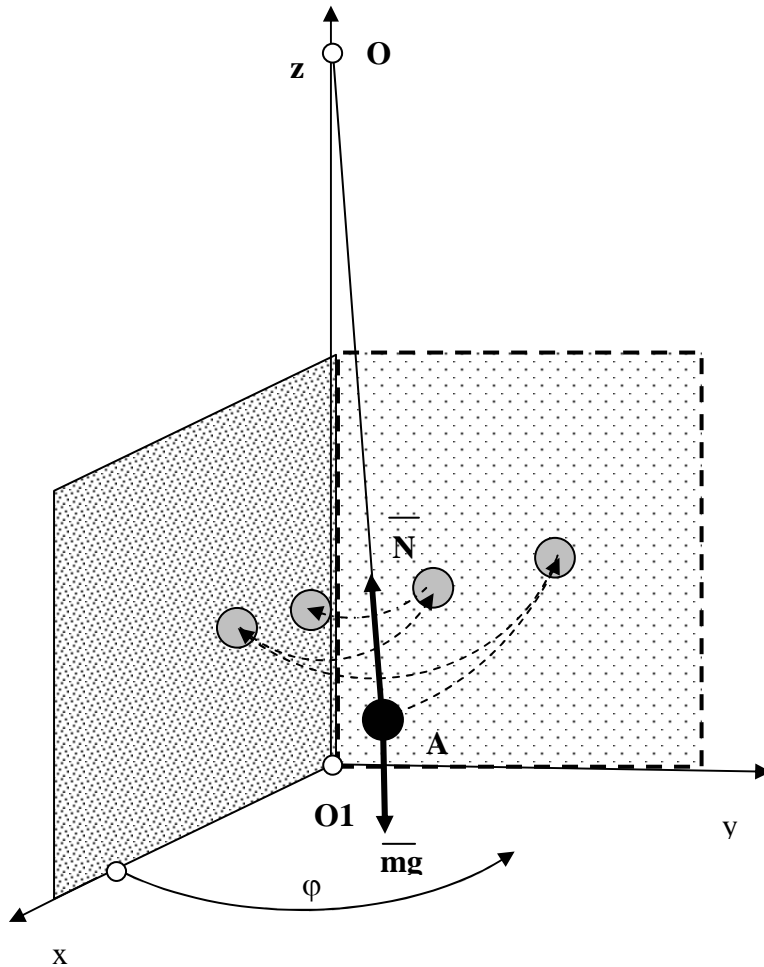
Double-bumper model is depicted in Fig. 7. It includes two independent bumpers.

There is not a right angle between bumpers ( $\varphi \neq \pi/2$ ) and forces are represented by a special stiffness and damping functions like equations (3).

Differential equations of motion are:

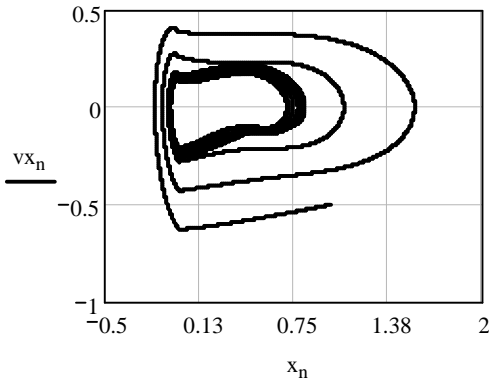
$$\begin{aligned}
 m(\ddot{x} + A1) &= B1(x, \dot{x}) \cdot 0,5 \cdot (1 - \text{sign}(x + \Delta 1)) + B12(x, \dot{x}, y, \dot{y}, \varphi) \cdot 0,5 \cdot (1 + \\
 &+ \text{sign}(\arccos \frac{x}{\sqrt{x^2 + y^2}} - \varphi) + 2 \cdot \lambda \cdot x; \\
 m(\ddot{y} + A2) &= B22(x, \dot{x}, y, \dot{y}, \varphi) \cdot (1 + \text{sign}(\arccos \frac{x}{\sqrt{x^2 + y^2}} - \varphi) + 2 \cdot \lambda \cdot y; \\
 m(\ddot{z} + A3) &= -m \cdot g - 2 \cdot \lambda \cdot (H - z),
 \end{aligned}
 \tag{4}$$

where  $B12(x, \dot{x}, y, \dot{y}) \cdot 0,5 \cdot (1 + \text{sign}(\arccos \frac{x}{\sqrt{x^2 + y^2}} - \varphi))$  - bumper interaction from right side along  $x$  axis;  $B22(x, \dot{x}, y, \dot{y}) \cdot 0,5 \cdot (1 + \text{sign}(\arccos \frac{x}{\sqrt{x^2 + y^2}} - \varphi))$  - bumper interaction from right side along  $y$  axis.

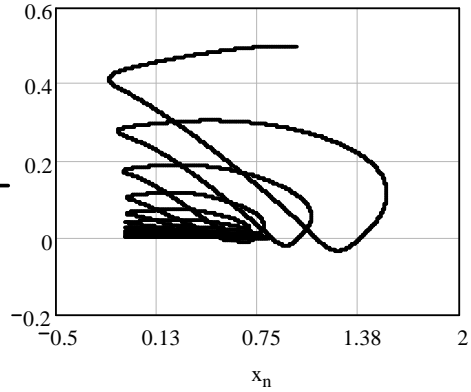


**Fig. 7.** Mathematical model of pendulum with two bumpers

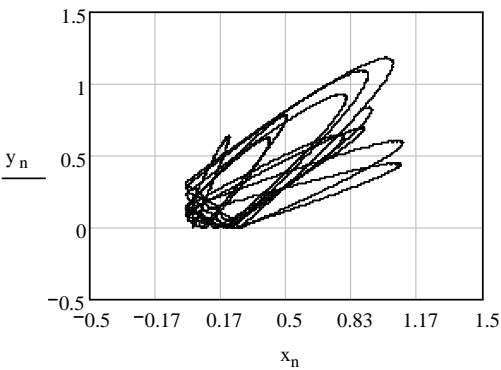
Results of modeling are presented in Figs. 8-11.



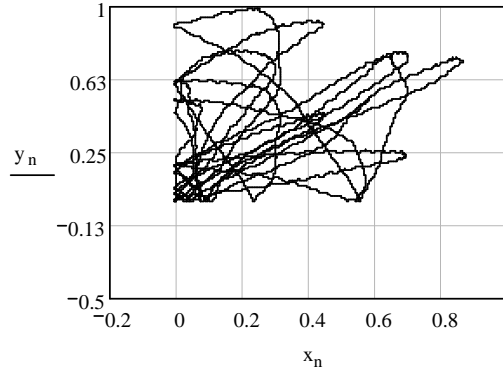
**Fig. 8.** Transient motion in phase plane  $vx$ - $x$  with  $\varphi = \frac{\pi}{2}$  and vibrations along  $x$  axis



**Fig. 9.** Displacement in plane  $x - y$  for Fig. 8.



**Fig. 10.** Motion in plane  $x$ - $y$  with both bumpers harmonic vibration



**Fig. 11.** Motion in plane  $x$ - $y$  with bumpers random vibration

## Conclusions

Mathematical modeling and optimization of geometric parameters such as length of string and gap between bumpers and pendulum demonstrates that processes of transient motion are very short when gap is “negative”. Additionally, results indicate that angle  $\varphi$  can be less than  $\pi/2$ . Special harmonic frequency can induce vibro-shock motion with large amplitude. Recommendations arising from this research work may be used for cargo fastening inside trailer, ship or airplane.

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