552. Clusters of submerged subharmonic isles with rare attractors in nonlinear machine dynamics

M. Zakrzhevsky[1], V. Yevstignejev [2]
[1] Institute of Mechanics, Riga Technical University
1 Kalku Str., Riga, LV-1658, Latvia
e-mail: *mzakr@latnet.lv*
[2] Institute of Mechanics, Riga Technical University
1 Kalku Str., Riga, LV-1658, Latvia
e-mail: *vljevst@latnet.lv*

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Abstract. This work is devoted to new nonlinear effects in the machine dynamics. This article presents a new type of subharmonic solutions which in the bifurcation diagrams have the form of the isles, all solutions of which are unstable except for small ranges of the bifurcation parameter at the edges of the isles. It is shown that such isles can form clusters, which we assume are typical topological formation for a wide class of systems.

Keywords: nonlinear machine dynamics, rare attractors, subharmonic regimes.

Introduction

A machine as a mechanical system may experience unexpectedly excessive vibrations, whose nature remains unknown. Rare attractors (RA) refer to this type of phenomena [1-5]. This article presents a new type of subharmonic solutions which in the bifurcation diagrams have the form of the isles with rare attractors, i.e. all solutions of which are unstable except for small ranges of the bifurcation parameter at the edges of the isles. Such isles can form clusters, which we assume are typical topological formation for all typical widely used nonlinear models.

Clusters of submerged subharmonic isles will be illustrated by the examples of forced vibrations in three nonlinear models - bilinear, trilinear with clearance and pendulum. These models are used for study the dynamics of offshore structures, suspended bridges, valves, gears and others.

We consider the following mathematical model

$$\ddot{x} + b\dot{x} + f(x) = h1\cos\omega t, \tag{1}$$

where *x* – generalized coordinate; *b* – coefficient of linear dissipation; *f(x)* – elastic force, which has three different characteristics: bilinear, trilinear with clearance and pendulum; *h*1, *ω* – amplitude and frequency of excitation.

The model (1) was studied by means of bifurcation analysis based on the method of complete bifurcation groups, which allows to find rare attractors [6-8].

Submerged subharmonic isles

It is known that one of the typical elements of the bifurcation diagrams for the dissipative driven systems is subharmonic isles. These isles differ in topology. In the simplest case a subharmonic isle consists of one stable and one unstable solution branches. The topology of submerged subharmonic isle is as follows (see Fig. 1): all periodic solutions Pn, 2Pn, 4Pn, … are unstable except for small ranges of the bifurcation parameter at the edges of the isle, i.e. except periodic tip rear attractors. Structure of tip rear attractor is shown in Fig. 2: on one side there is fold bifurcation, on the other the period doubling transition to chaos. So, between two tip rare attractors there is infinite number of unstable periodic solutions, so called unstable periodic infinitium (UPI). It is a well-known fact that the presence of the UPI characterizes the parameter region with chaotic behavior – chaotic attractor and/or chaotic transient [1-8].

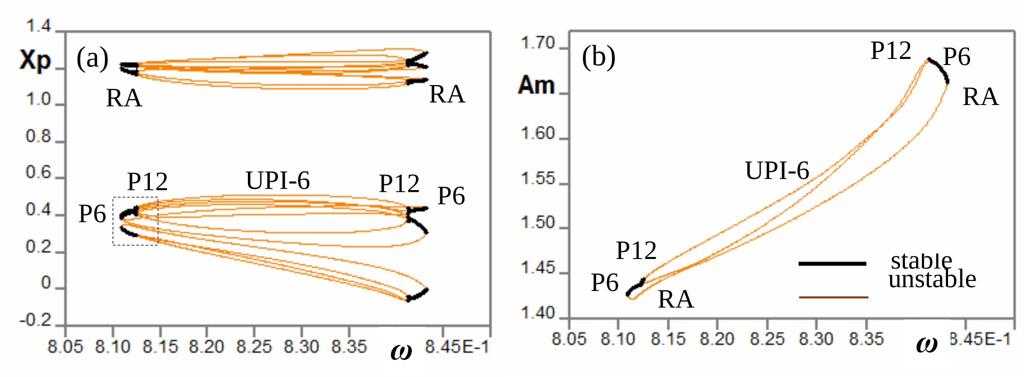

Fig. 1. Topology of submerged subharmonic isle in example of system with bilinear elastic characteristic and linear dissipation at harmonic excitation: (a) coordinate x_p of fixed point and (b) amplitude of vibrations Am of periodic regimes vs frequency of excitation ω

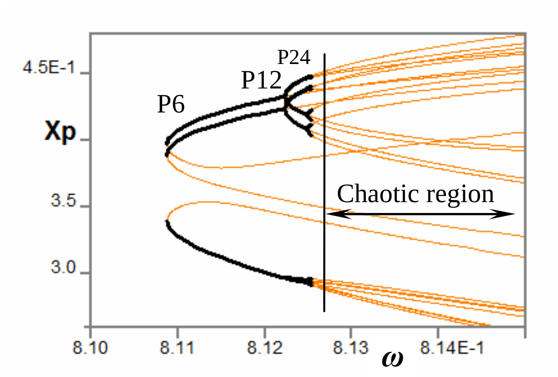

Fig. 2. A typical fragment of the bifurcation diagram corresponding to the tip rare attractor with infinite number of unstable periodic regimes (UPI), i.e. chaotic region. Fragment from Fig. 1a

A systematic search of rare attractors of different types is possible only by the newly developed method of complete bifurcation groups.

Method of complete bifurcation groups

The method of complete bifurcation groups consist in direct numerical modeling of originate existing nonlinear model, that is, without its simplification. Under method of complete bifurcation groups we understand complex of approaches to analysis of dynamic systems, which involves the following procedures: at fixed system parameters – search of all periodic stable and unstable regimes and bifurcation subgroups with unstable periodic infinitiums (UPI) on plane of states, constructing the regimes' basins of attraction on plane of states; at varying system parameters – constructing the bifurcation diagrams and bifurcation maps. Special importance in the method is for continuation of parameter solution (in one-parameter task) along solution branch of definite regime (not along parameter), and this allows to find new, previously unknown stable regimes in all typical widely used nonlinear models. Method of complete bifurcation groups is realized in the software NLO and SPRING.

Bilinear system

Elastic force *f(x)* for bilinear system has the following form

$$f(x) = \begin{cases} c1x & if\ \ x \le d \\ c2x & if\ \ x > d \end{cases}, \tag{2}$$

where $c1$, $c2$ – stiffness coefficients of nonlinear elastic characteristic on linear sub-regions, d – coordinate of break point.

Cluster of five submerged subharmonic isles is illustrated in Fig. 3. In the same bifurcation region there is also a simple subharmonic isle 5T of one stable and one unstable solution branches.

Trilinear system

Elastic force *f(x)* for trilinear symmetrical system has the following form

$$f(x) = \begin{cases} c2x - (c1 - c2)d\ if\ \ x < -d \\ c1x\ if\ -d \le x \le d \\ c2x - (c2 - c1)d\ if\ \ x > d \end{cases}, \tag{3}$$

where $c1$, $c2$ – stiffness coefficients of nonlinear elastic characteristic on linear sub-regions, d – coordinate of break point.

Cluster of nine submerged subharmonic isles is illustrated in Fig. 4.

Pendulum system

Restoring force *f(x)* for pendulum system has the following form

$$f(x) = a1 \sin \pi x, \tag{4}$$

where $a1$ – coefficient which includes pendulum length and gravitational constant.

Cluster of three submerged subharmonic isles is shown in Fig. 5. In the same bifurcation region there is also a complex subharmonic isle 3T with its own unstable periodic infinitium.

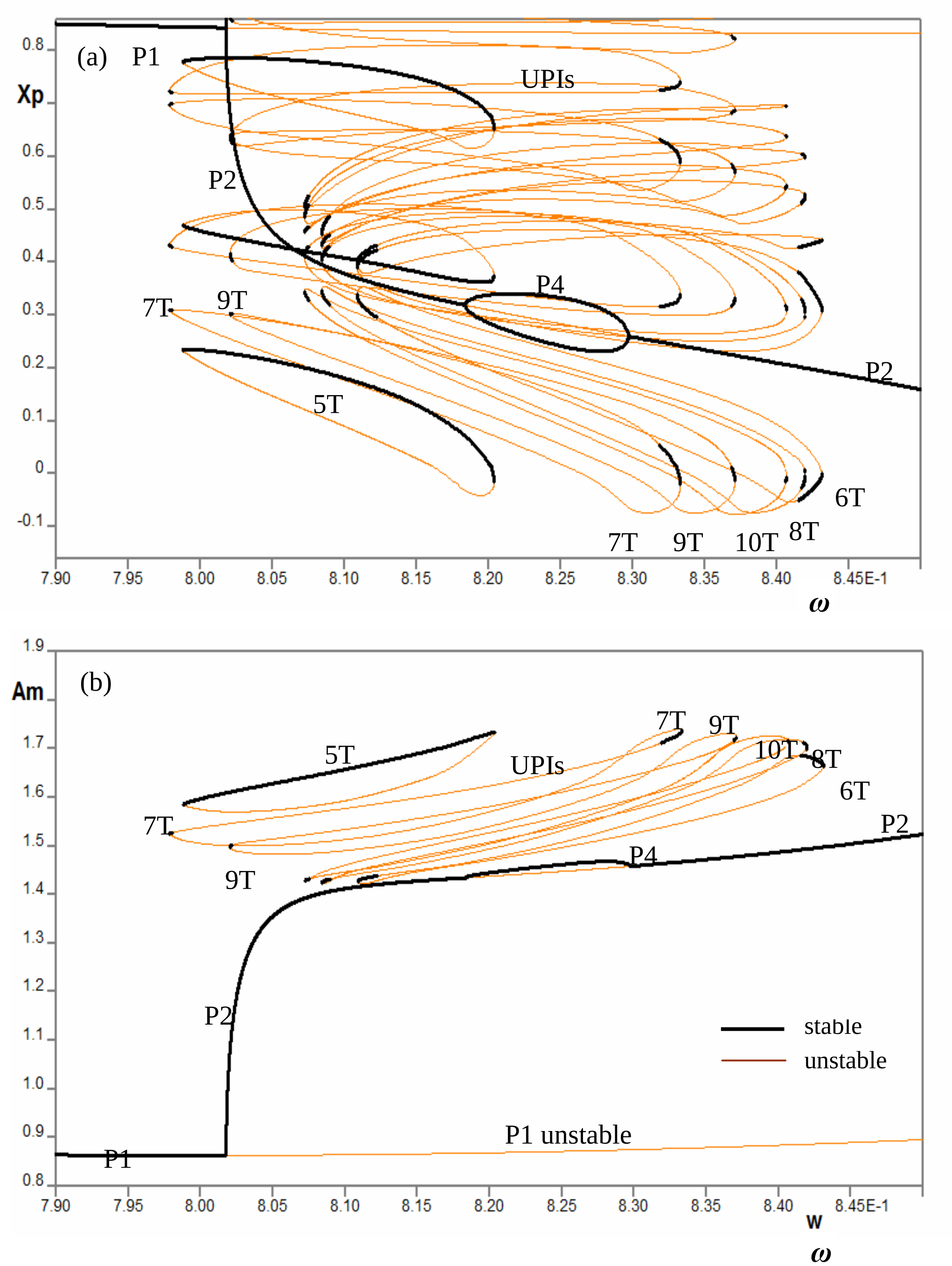

Fig. 3. Complete bifurcation diagrams of 7 bifurcation groups 1T, 5T-10T. Bifurcation groups of submerged subharmonic isles 6T-10T form cluster. Bilinear system with linear dissipation and harmonic excitation (Eqs. 1, 2): (a) coordinate x_p of fixed point and (b) amplitude of vibrations Am of periodic regimes vs frequency of excitation ω. Parameters: $c1 = 1$, $c2 = 4$, $d = 0$, $b = 0.01$, $h1 = 1$, $k = 7$, ω = var.

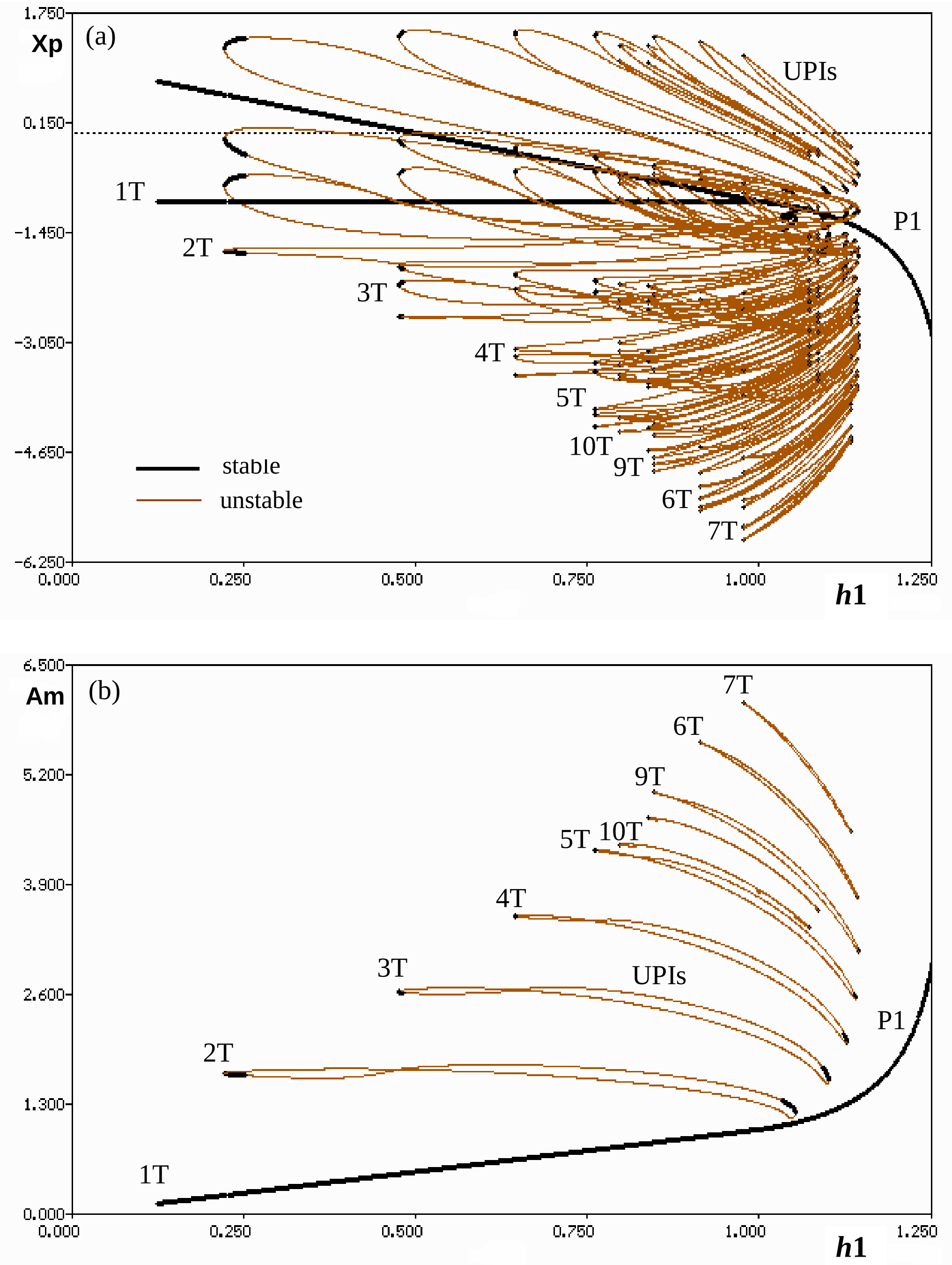

Fig. 4. Complete bifurcation diagrams of 10 bifurcation groups 1T-10T. All isles and 1T are twins. Bifurcation groups of submerged subharmonic isles 2T-10T form cluster. Trilinear symmetrical system with clearance, linear dissipation and harmonic excitation (Eqs. 1, 3): (a) coordinate x_p of fixed point and (b) amplitude of vibrations Am of periodic regimes vs amplitude of excitation $h1$. Parameters: $c1 = 0$, $c2 = 1$, $d = 1$, $b = 0.04$, $\omega = 1$, $k = 7$, $h1$ = var.

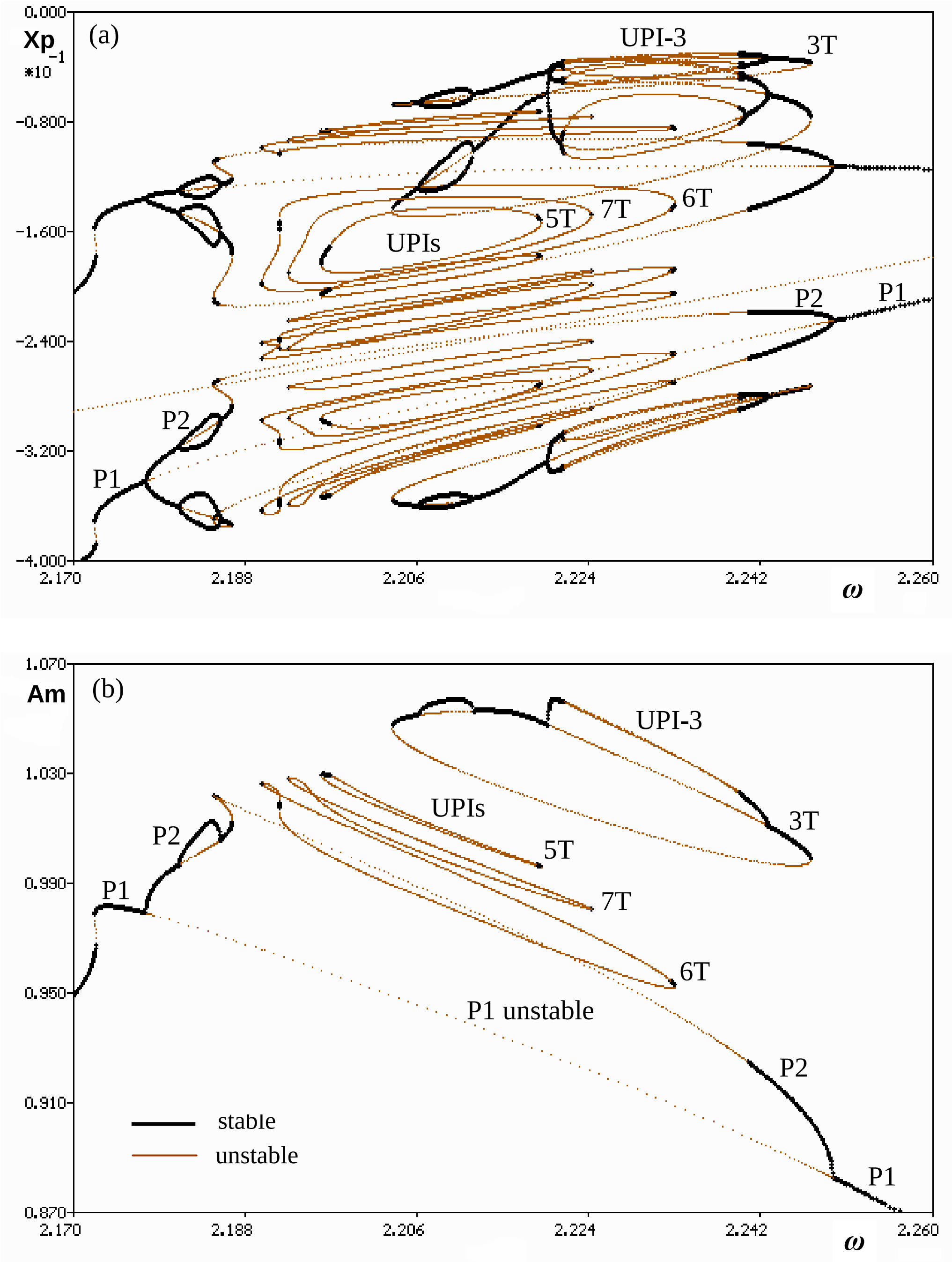

Fig. 5. Complete bifurcation diagrams of 5 bifurcation groups 1T, 3T, 5T-7T. Bifurcation groups of submerged subharmonic isles 5T-7T form cluster. Harmonically driven strongly damped pendulum system (Eqs. 1, 4): (a) coordinate x_p of fixed point and (b) amplitude of vibrations Am of periodic regimes vs frequency of excitation ω. Parameters: b = 0.7, $a1$ = - 1, $h1$ = 4, k = 6-7, ω = var.

Conclusions

In this work we considered novel nonlinear effects in the machine dynamics. The paper presented a new type of subharmonic solutions, which in the bifurcation diagrams have the form of the isles, all solutions of which are unstable except for small ranges of the bifurcation parameter at the edges of the isles. It is shown that such isles can form clusters, which we assume are typical topological formation for a wide class of systems. In the same parameter region along with cluster of submerged subharmonic isles there can be also simple or complex subharmonic isles and other complex structures.

Acknowledgments

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