544. Bending – rotational vibrations of a strip of cardboard

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Abstract. The eigenmodes of vibration of the plane cardboard which is considered as an element of the package made from cardboard are analyzed in this paper. It is assumed that a strip of cardboard is loaded in its plane and thus a problem of plane stress is solved. Then the eigenmodes of bending – rotational vibrations of a strip of cardboard as of a beam by taking into account the stresses obtained from the previous two – dimensional problem of plane stress are obtained and analyzed. Graphical representation of bending and of rotation for the first eigenmodes is obtained. Experimental investigations using time averaged projection moiré are performed and the correspondence between the experimental and numerical results is discussed. The obtained results are used in the process of design of elements of packages.

Keywords: cardboard, finite elements, plane stress, beam, bending – rotation, vibrations, eigenmodes, experimental setup, projection moiré, time averaged moiré.

Introduction

In recent years the investigations of mechanical characteristics of the materials of packages become especially important. The analysis of geometrical and strength parameters of those packaging materials for static loads is performed in [1, 2, 3, 4, 5, 6]. Related problems of analysis of vibrations and stability are solved in [7, 8, 9]. The strength characteristics of the packages first of all depend on the material used for their production. The mechanical characteristics of paper and the influence of various types of defects of the paper to the eigenmodes of the paper were analyzed in the research papers [10, 11]. The packages in the time period of their exploitation experience the action of the force of tension because of the weight of the packaged material as well as the compression loads because of the action of other packaged materials located above the analyzed package which occur during the transportation of the packaged materials. Thus it is important to investigate the effect of those loads on vibrations not only for the package as a whole, but also for a separate plane of a package which may be analyzed separately. The packages experience the action of vibrations in the process of transportation [12, 13, 14]. The purpose of this research paper is to investigate the eigenmodes of bending and rotational vibrations of the plane element of the cardboard package which is analyzed as a strip of cardboard.

The model for the analysis of vibrations of a strip of cardboard is proposed on the basis of the material described in [15, 16, 17]. It is assumed that a strip of cardboard is loaded in its plane. The static problem of plane stress by assuming the displacements at the boundary of the analyzed strip of cardboard to be given is solved. Then the eigenmodes of bending – rotational vibrations of a strip of cardboard as of a beam are calculated. The eigenmode of the strip of cardboard is represented by two graphical relationships: one for bending and another for rotation.

Model for the analysis of bending – rotational vibrations of a strip of cardboard

In the theoretical study of cardboard sheet dynamics the following characteristics of paper have been adopted: cardboard sheet was 0.2 m width and 0.2 m length, Poisson's ratio ν=0.37, the modulus of elasticity E=5.6 GPa, the density of the paper ρ=735 kg/m^3, the thickness h=0.00034 m. Further x, y and z denote the axes of the system of coordinates. First the static problem of plane stress is analyzed. The element has two nodal degrees of freedom: the displacements u and v in the directions of the axes x and y. Further ξ and η denote the local coordinates of the two – dimensional finite element. The strip of cardboard is represented by a single row of Lagrange quadratic rectangular elements with the value of the local coordinate η=0 coinciding with the x axis. The vector of displacements $\{\delta\}$ is determined by solving the system of linear algebraic equations. Then the bending – rotational vibrations of a strip of cardboard as of a beam coinciding with the x axis are analyzed by taking into account the stresses obtained from the two – dimensional problem.

The stiffness matrix has the form:

$$[K]=\int [B]^T [D][B]hdxdy, \tag{1}$$

where h is the thickness of the strip of cardboard and:

$$[B]=\begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \dots \\ 0 & \frac{\partial N_1}{\partial y} & \dots \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \dots \end{bmatrix}, \tag{2}$$

where N_i are the shape functions of the two – dimensional finite element and:

$$[D]=\begin{bmatrix} \frac{E}{1-\nu^2} & \frac{E\nu}{1-\nu^2} & 0 \\ \frac{E\nu}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix}, \tag{3}$$

where E is the modulus of elasticity and ν is the Poisson's ratio.

Then the eigenproblem of bending – rotational vibrations of a strip of cardboard as of a beam is solved. The element has three nodal degrees of freedom: the transverse displacement of the strip of cardboard w and the rotations Θ_x and Θ_y about the axes of coordinates x and y. The mass matrix has the form:

$$[\bar{M}]=\int [N]^T \begin{bmatrix} \rho h & 0 & 0 \\ 0 & \frac{\rho h^3}{12} & 0 \\ 0 & 0 & \frac{\rho h^3}{12} \end{bmatrix} [N]dx, \tag{4}$$

where ρ is the density of the material of the strip of cardboard and:

$$[N]=\begin{bmatrix} \bar{N}_1 & 0 & 0 & \dots \\ 0 & \bar{N}_1 & 0 & \dots \\ 0 & 0 & \bar{N}_1 & \dots \end{bmatrix}, \tag{5}$$

where $\bar{N}_i$ are the shape functions of the one – dimensional finite element.

The stiffness matrix has the form:

$$[\bar{K}]=\int\left([\bar{B}]^T \frac{h^3}{12}[D][\bar{B}]+[\bar{\bar{B}}]^T \frac{Eh}{2(1+\nu)1.2}[\bar{\bar{B}}]+[G]^T[M_\sigma][G]\right)dx, \tag{6}$$

where:

$$[\bar{B}]=\begin{bmatrix} 0 & 0 & \frac{d\bar{N}_1}{dx} & \dots \\ 0 & 0 & 0 & \dots \\ 0 & -\frac{d\bar{N}_1}{dx} & 0 & \dots \end{bmatrix}, \tag{7}$$

$$[\bar{\bar{B}}]=\begin{bmatrix} \frac{d\bar{N}_1}{dx} & 0 & \bar{N}_1 & \dots \end{bmatrix}, \tag{8}$$

$$[G]=\begin{bmatrix} \frac{d\bar{N}_1}{dx} & 0 & 0 & \dots \\ 0 & \bar{N}_1 & 0 & \dots \end{bmatrix}, \tag{9}$$

$$[M_\sigma]=h\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}, \tag{10}$$

where the stresses on the x axis σ_x, σ_y, τ_{xy} are determined from:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}=[D][B]\{\delta\}. \tag{11}$$

Results of analysis of bending – rotational vibrations of the strip of cardboard

The displacements at the boundary of the analyzed strip of cardboard are given and they produce the loading vector. The following boundary conditions are assumed: on the left boundary it is assumed that u=v=0; on the right boundary it is assumed that u=0 and v=0 for the lower node and u=0.00002 m and v=0 for the upper node, assuming linear variation of the displacement u on this boundary. For bending – rotational vibrations all generalized displacements at both ends of the strip of cardboard are assumed equal to zero.

The first eigenmode is presented in Fig. 1. The second eigenmode is presented in Fig. 2. The third eigenmode is presented in Fig. 3.

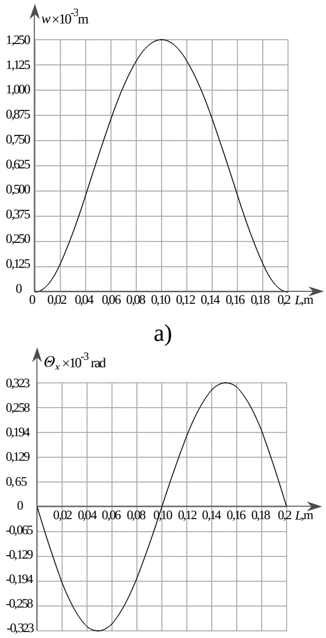

Fig. 1. The first eigenmode: a) bending, b) rotation.

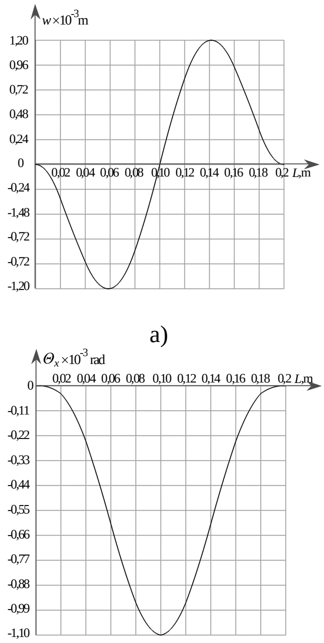

Fig. 2. The second eigenmode: a) bending, b) rotation.

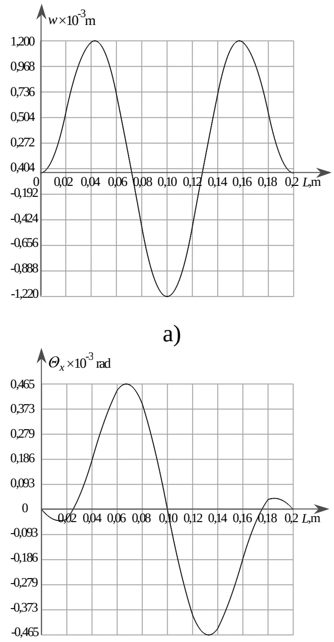

Fig. 3. The third eigenmode: a) bending, b) rotation.

Experimental investigations

The structure of the package may be considered to consist from six planes: four side walls, one upper wall and one bottom wall. The side planes of the package may be analyzed as separate elements of the package. Thus the sheet of cardboard may be investigated as a separate element of the structure of the package. The dynamical characteristics of the side plane of the cardboard package were determined by using the method of projection moiré. The analysis was performed by using the method of experimental investigations presented in [10, 11]. For those investigations the cardboard MC Mirabell was used. The technical characteristics of this cardboard are presented in Table 1. For the samples of the investigated material the square list of paper 20×20 cm was used. The obtained results of experimental investigations are presented in Fig. 4 – Fig. 6.

Table 1. Technical Characteristics of Cardboard Mirabell

No.	Type of cardboard	Grammage, g/m²	Thickness, μm	Rigidity: *L&W*[1], Nm (5°)		Rigidity: TABER[2], Nm (15°)		Modulus of elasticity *E*, MPa		Poisson's ratio ν	
				MD[3]	*CD*[4]	*MD*	*CD*	*MD*	*CD*	*MD*	*CD*
1	MC Mirabell	250	340	15.4	7.0	7.5	3.4	5,6	2,0	0,37	0,12

[1] *- Moment measured by the device L&W, which is necessary for bending of the sample of material by an angle of 50°*

[2] *- Moment measured by the device TABER, which is necessary for bending of the sample of material by an angle of 150°*

[3] *- MD (machine direction) longitudinal direction of production of paperboard*

[4] *- CD (cross machine direction) transverse direction of production of paperboard*

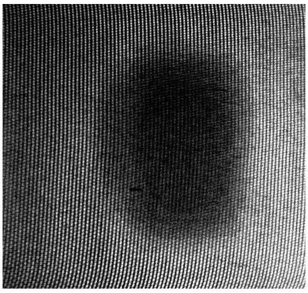

Fig. 4. The first eigenmode of the cardboard Mirabell (frequency of vibrations 74 Hz, amplitude 2×10^{-5}m)

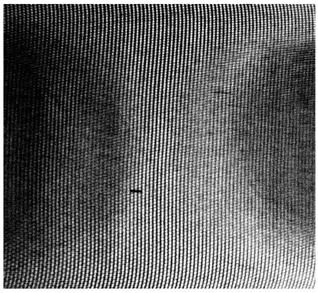

Fig. 5. The second eigenmode of the cardboard Mirabell (frequency of vibrations 82 Hz, amplitude 2×10^{-5}m)

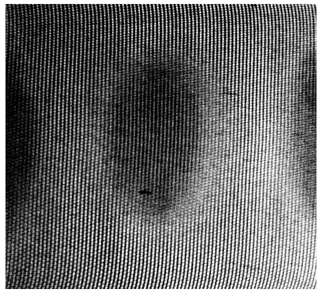

Fig. 6. The third eigenmode of the cardboard Mirabell (frequency of vibrations 90 Hz, amplitude 2×10^{-5}m)

The eigenmodes of the cardboard Mirabell obtained by using the method of time averaged projection moire correspond to the numerical model based on the assumption that the side wall of the package is analyzed as a plate. Such a model was presented in detail in the previous research papers. In this paper the strip of cardboard is analyzed numerically and the advantage of the numerical model presented in this research paper is the possibility to represent the eigenmode by the two graphical relationships: one for bending and another one for rotation. From the graphical relationships one can determine the values of the represented quantities much more precisely than from the two dimensional contour plots presented in the previous papers. But this model is applicable only for a strip of cardboard which is long and narrow. Otherwise, as seen from the investigations presented in this research paper, the correspondence of experimental and numerical results can be observed only for some of the eigenmodes: that is for such eigenmodes in which the behavior in the direction of the width of the strip of cardboard can be represented by bending and rotation with sufficient accuracy.

Also one is to have in mind that the numerical model is for isotropic material, while the experiments were conducted for the orthotropic cardboard. But for such problems the orthotropic qualities of the material are of secondary importance (the calculations are performed using the averaged values of the physical parameters using the isotropic model), while the length to width ratio is more essential for the successful application of the proposed numerical model.

Conclusions

The model for the analysis of bending – rotational vibrations of a strip of cardboard is proposed. It is assumed that a strip of cardboard is loaded in its plane. The static problem of plane stress by assuming the displacements at the boundary of the analyzed strip of cardboard to be given is solved. Then the eigenmodes of bending – rotational vibrations as of a beam by taking into account the stresses on the x axis obtained from the previous two – dimensional problem of plane stress are calculated. Graphical representations of bending and of rotation for the first eigenmodes are obtained.

The eigenmodes of the cardboard Mirabell obtained by using the method of time averaged projection moire correspond to the numerical model based on the assumption that the cardboard is analyzed as a plate. In this paper the strip of cardboard is analyzed numerically and the advantage of the numerical model presented is the possibility to represent the eigenmode by the two graphical relationships: one for bending and another one for rotation. From the graphical relationships one can determine the values of the represented quantities much more precisely than from the two dimensional contour plots presented in the previous papers. But this model is applicable only for a strip of cardboard which is long and narrow. Otherwise the correspondence of experimental and numerical results can be observed only for some of the eigenmodes: that is for such eigenmodes in which the behavior in the direction of the width of the strip of cardboard can be represented by bending and rotation with sufficient accuracy.

The obtained experimental and numerical results are used in the process of design of packages and choosing the materials for their production.

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