533. Investigation of vibrations caused by the inaccuracy of transmission of the mechatronic system

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Abstract. The questions connected with dynamics of mechatronic system at certain radial misalignment of shafts are investigated in offered work. As object of research is the system of two shafts connected by elastic centrifugal ring coupling.

It is established, that in coupling, connecting radially misaligned shafts, an internal moment of resistance to rotation is arising. Using principle of d'Alamber for rotary movement, and it is worked out the differential equation describing rotation of the second shaft, and received solutions show, that rotational fluctuations arise in rotating misaligned system. The frequency of fluctuations is double rotation frequency.

The results of investigation were applied for vibration reducing of mechatronic systems.

Keywords: centrifugal ring coupling, misaligned shafts, evenness rotation.

Introduction

The rotor system consists of many synchronously rotating links. Due to shafts misalignment, of the links non-balanced parts, manufacture and assembly errors and variation of power supply, the links rotate irregularly. The above factors cause an increased dynamic load in machines and mechanisms, which intensifies rotary vibration. When summed, these factors reach rather high values.

The research was carried out with the aim of decreasing rotary vibrations and the forces which generate them. One of the most effective ways of decreasing vibrations is an improvement of structural elements of machines and their replacement with structures resistant to vibrations. For this purpose, effective rotary motion transmission and stabilisation devices in the form of various clutches and vibration dampers can be used. Vibration protection in coupling devices manifests itself as suppression of vibration of constituent links of the elements [1, 2].

Lately, two ways have been used to avoid undesirable harmful vibrations, i.e. the development of devices with low activity vibrations and installation of special structural units which suppress and absorb vibrations in the machines.

Different methods are applied to research of oscillatory processes in such systems [3, 4].

The Object of the Investigation

One of methods of a reduction of an unevenness of a rotational movement is based on usage of elastic centrifugal ring couplings in the mechanical system. It was already described in [1].

Herein, we'll discuss the dynamic processes in such drive, when shafts being misaligned in radial direction are connected using the above–mentioned couplings.

The construction of the simplest elastic centrifugal ring coupling is presented at Fig. 1.

The coupling consists of the driving half – coupling 1 and the driven half – coupling 2. The terminals 3 and 4 of the half – couplings are connected with the elastic steel ring 5, usually made of a wire wound into a circle. When the driving half – coupling is loaded with the moment of rotation, a slight angular shifting of the half – couplings in respect of each other takes place on their rotation because of elastic deformations of the ring 5, and the shape of the ring becomes similar to an ellipsis. On a rotation of the system, the forces of elasticity of the ring and the centrifugal forces of its distributed mass seek for a restoration of the initial round shape of the ring and simultaneously reduce the deformation of the coupling. So, links of two types (elastic and centrifugal) participate in the transmission of a rotational movement.

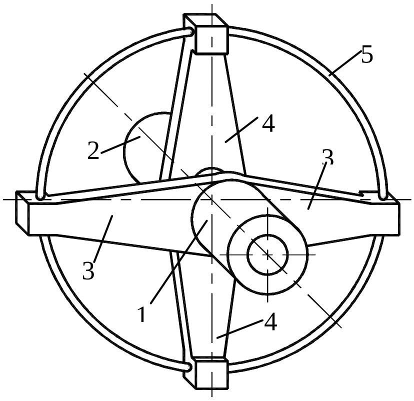

Fig. 1. The construction of the simplest elastic centrifugal ring coupling

Convention of Shafts Misaligned in Radial Direction

The above–described situation is characteristic for a connection of ideally aligned shafts. In most cases, the axes of the shafts do not coincide, in case of radial misalignment of connected shafts, the shape of the deformed elastic ring will be another (Fig. 2); the reactions P_x and P_y appear in the fitting points of the terminals of the half–couplings.

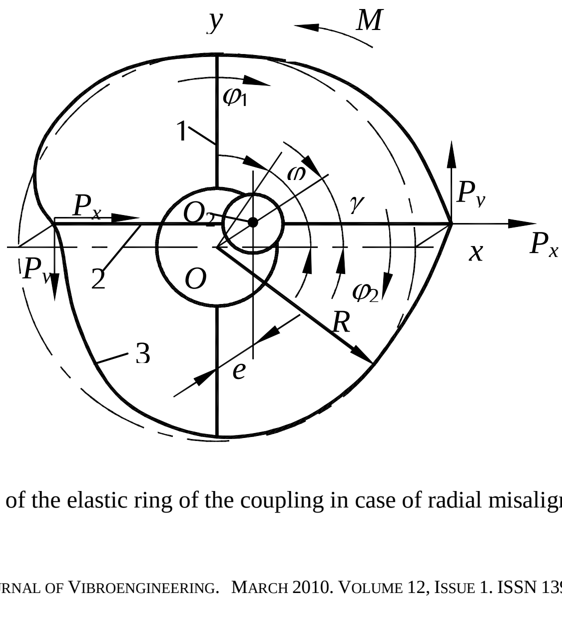

Fig. 2. Deformation of the elastic ring of the coupling in case of radial misalignment

In this case, potential energy of the deformed elastic ring is found from the following expression:

$$\Pi = \sum_n \int_0^{\frac{2\pi}{n}} \frac{M_l R d\varphi}{2EI} + \sum_n \int_0^{\frac{2\pi}{n}} \frac{N_a R d\varphi}{2EF}, \quad (1)$$

where M_l – the moment of bending in the ring; N_a – the axial force in it; R – the radius of the bend of the ring; EI, EF – the stiffness of the ring for bending and lengthening; φ - an angular coordinate; n – the number of ring segments between the terminals of half – couplings.

Making use of [5], we find the values of the reactions P_x and P_y as the functions of radial misalignment e. On a rotation of the system, an internal moment of resistance to rotation M_p appears in the coupling that is found from the equation of moments around the point O_2:

$$M_p = 157 \frac{EI}{R^3} e^2 \cos 2\gamma. \quad (2)$$

Taking into account that the angle of rotation γ of the coupling in respect of the direction of misalignment may be interpreted as

$$\gamma = \frac{\varphi_1 + \varphi_2}{2}, \quad (3)$$

where φ_1, φ_2 – angles of rotation of first and second half–couplings 1, 2.

On an investigation of the impact of misalignment of shafts upon the dynamics of rotating system, we suppose that rotation of the first shaft is even. In such case, the generalized coordinates that describe the movement of the system will be the following:

$$\begin{aligned} \varphi_1 &= \alpha_1 + \omega t, \\ \varphi_2 &= \alpha_2 + \omega t + x_e, \end{aligned} \quad (4)$$

where α_1, α_2 – the initial angles of rotation, ω – the angular speed, t – time, x_e – the value (angle) that characterizes the unevenness of rotation of the second shaft.

Using d'Alamber's principle, from equations (2), (3) and (4) we find:

$$\ddot{x}_e - 157 \frac{e^2}{I_2} \frac{EI}{R^3} \cos\left(\alpha_1 + \alpha_2 + 2\omega t + x_e\right) = 0, \quad (5)$$

where: I_2 – the moment of inertia of the second shaft.

The Results of the Solution

Bearing in mind that the value of misalignment e is small in comparison with other sizes, and, in addition, it is raised to the second power in the expression, we find the solutions of the system and finally

$$\varphi_2 = \alpha_2 + \omega t + \frac{39,25\, e^2\ EI}{\omega^2 I_2 R^3} \sin\left(2\omega t + \delta_0\right) = \alpha_2 + \omega t + B \sin\left(2\omega t + \delta_0\right), \tag{6}$$

where δ_0 – constant.

The last member of the expression (6) indicates that rotation of the second shaft is uneven, its angle of rotation is supplemented with a periodic component of the double frequency of rotation and the amplitude of this periodical part decreases on increasing of the angular rotational speed. This indicates an increase of an evenness of the movement.

The presence of the angular rotational speed in the denominator of the expression of the amplitude indicates that the small parameter method cannot be applied for a solution of the equation (5) in all cases (when $\omega \to 0$, the amplitude in (6) is growing up to the infinity).

The problem on an application of the expression (6) may be settled on a base of the following considerations. The positive angle of shifting of the half–couplings in respect of each other upon an impact of the moment of resistance to rotation M_p may be found from the following expression:

$$x_e = \frac{M_p}{c_s}, \tag{7}$$

where c_s – the rotational stiffness of the coupling.

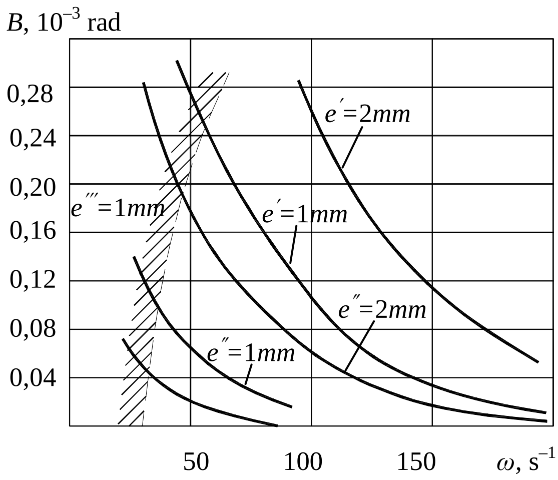

Fig. 3. The dependence of the amplitude of rotational vibrations of the second shaft on the angular rotational speed, when the diameter of the wire of the ring d = 3 mm, $'$ - R = 0,1 m, $''$ - R = 0,2 m, $'''$ - R = 0,3 m

Knowing that $c_s = 87\frac{EI}{R^3}$ [5], we find maximal size of $B = B_{max}$ and determinating maximal size of M_P (it depends on the proportionate limit of spring steel), we find the lower limit value of the angular speed applicable to the expression (6) (the stroked zone in Fig. 3).

Conclusions

1. Elastic centrifugal ring couplings may be successfully usable for a connection of misaligned shafts in mechatronic systems.
2. The frequency of the additional periodical component of the coordinate of unevenness of rotation is equal to the double frequency of rotation of the system.
3. If two misaligned shafts are connected with an elastic centrifugal ring coupling, an evenness of the rotation increases with an increase of the rotational speed.

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