524. Optimization of vibrator motion with air flow excitation

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Abstract. In the daily life and techniques people constantly interact with continuous media such like air or water. In this paper motion of a vibrator with constant air or water flow excitation is considered. Firstly, motion of the vibrator with constant air or water flow velocity excitation is investigated. The main idea is to determine optimal control law for variation of additional area of vibrating object within limits. The criterion of optimization is time required to move object from initial position to the final one. The maximum principle is used for solution of the high-speed problem. It is demonstrated that optimal control action is on bounds of area limits. Examples of synthesis of real mechatronic systems are given.

Keywords: motion control, air, water excitation, optimal control, adaptive control, synthesis, adaptive systems, energy utilization.

Introduction

Motion of a vibrator with two degrees of freedom and constant air flow $\overline{V}_0$ excitation is investigated (Fig. 1.). System consists of masses m_1, m_2 with springs c_1, c_{12} and dampers b_1, b_{12}. The main idea is to find out optimal control law for variation of additional area *A(t)* of vibrating mass m_2 within limits (1):

$$A_1 \leq A(t) \leq A_2, \tag{1}$$

where A_1 - lower level of additional area of mass m_2; A_2 - upper level of additional area of mass m_2, t - time.

The criterion of optimization is time *T* required to move object from initial position to the end. First of all, in order to understand process of fluid excitation and optimal solution of control problem we consider system with one degree of freedom when mass m_1 is very large (massive base): $m_1 >>> m_2$.

To simplify the equation it is easy (for system with one degree of freedom) to miss indexes of motion description. Then the differential equation is (2):

$$m\,\ddot{x} = -c\,x - b\,\dot{x} - u(t)\cdot(V_0 + \dot{x})^2, \tag{2}$$

where $u(t) = A(t) \cdot k$, $m = m_2$ – mass, $\ddot{x} = \ddot{x}_2$ – acceleration, $x = x_2$ – displacement of object, $\dot{x} = \dot{x}_2$ – velocity of object, $c = c_{12}$ – stiffness of spring, $b = b_{12}$ – damping coefficient, V_0 – constant velocity of wind, *A(t)* – area variation, $u(t)$ – control action (3), *k* – constant.

It is required to determine the control action $u = u(t)$ for displacement of a system (2) from initial position $x(t_0)$ to the end position $x(t_1)$ in minimal time (criterion K) K=T, if area *A(t)* has limit (1).

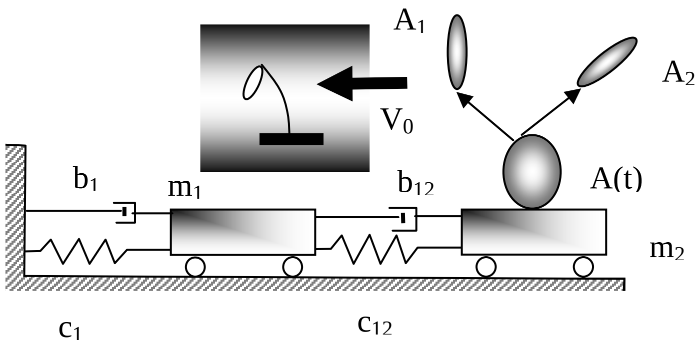

Fig. 1. Scheme of model with area *A(t)* control

Solution of optimal control problem for system with one degree of freedom

High-speed problem must be solved for system excitation at any time [1 - 9]:

$$K = \int_{t_0}^{t_1} 1 \cdot dt .$$

To assume $t_0 = 0; \quad t_1 = T$, we have $K = T$.

System (2) transforms to:

$$x_1 = x; \quad \dot{x}_1 = x_2 \quad \text{or}$$

$$\dot{x}_1 = x_2; \quad m\,\dot{x}_2 = -c\,x - b\,\dot{x} - u(t) \cdot (V_0 + \dot{x})^2,$$

and Hamiltonian is (3) [1 – 3]:

$$H = \psi_0 + \psi_1 x_2 + \psi_2 \left(\frac{1}{m} \cdot \left(-cx_1 - bx_2 - u(t) \cdot (V_0 + x_2)^2\right)\right), \tag{3}$$

here $H = \psi \cdot X$, where (4)

$$\psi = \begin{Bmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{Bmatrix};\cdot\ X = \begin{Bmatrix} 0 \\ x_2 \\ \frac{1}{m}\cdot[-cx_1 - bx_2 - u(t)\cdot(V_0 + x_2)^2] \end{Bmatrix}. \tag{4}$$

Scalar multiplication of two last vector functions ψ and X in any time (Hamiltonian *H* [3]) must be maximal [2 – 9]. To have such maximum, control action *u(t)* must be within limits $u(t) = u_1;\quad u(t) = u_2$, depending only on the sign of function ψ_2 (5) (see, for example, [3 – 6]):

$$\begin{aligned} &H = \max H, \\ &if \quad \psi_2 \cdot (-u(t)\cdot(V_0 + x_2)^2) = \max \end{aligned} \tag{5}$$

Therefore if $\psi_2 > 0,_1$ the $u(t) = u_1$ and if $\psi_2 < 0$, the $u(t) = u_2$, where $u_1 = A_1 \cdot k$ and $u_2 = A_2 \cdot k$, see (1). Examples of very simple control action (with one and three switch points) are shown in Fig. 2, 3.

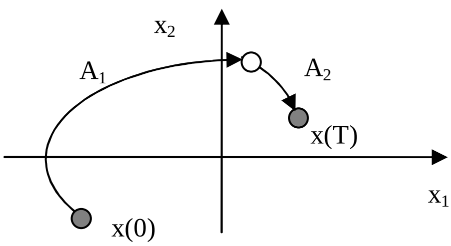

Fig. 2. Optimal control with one switch point

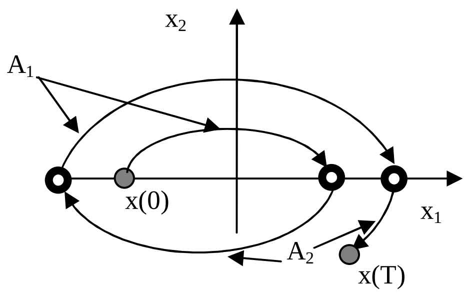

Fig. 3. Optimal control with three switch points when $x_2 = 0$

We will not consider in this paper how to find switches points (e.g., $\psi_2 > 0$, or $\psi_2 < 0$, [3 – 9]. But the main conclusion of optimal control law is that value of area at any time must be on bounds $A(t) = A_1$ or $A(t) = A_2$ (5). In real systems it allows synthesizing of quasi-optimal control actions (see, for example, [10 - 13]). Additionally, here must be mentioned that optimal control in time domain *u(t)* (like programming control) in real nonlinear systems without feedback often are unstable. Therefore in this case the task of synthesis of new real control systems includes step of forming control like mixed function of phase coordinates and time $u(t) = u(x_1, x_2, ,t)$ (see, for example, [10 – 13]).

Solution of optimal control problem for system with two degrees of freedom

The equation of motion may be described as (6):

$$\begin{aligned} m_1\ddot{y} &= -c_1 y - c_{12}(y-z) - b_1\dot{y} - b_{12}(\dot{y}-\dot{z}); \\ m_2\ddot{z} &= c_{12}(y-z) + b_{12}(\dot{y}-\dot{z}) - u(t)\cdot(V_0+\dot{z})^2, \end{aligned} \tag{6}$$

where $y, \dot{y}, \ddot{y} -$ displacement, velocity and acceleration of mass m_1; $z, \dot{z}, \ddot{z} -$ displacement, velocity and acceleration of mass m_2. To use new variables (phase coordinates) $x_1 = y,\quad x_2 = \dot{x}_1 = \dot{y},\quad x_3 = z,\quad x_4 = \dot{x}_3 = \dot{z}$ the system (6) may be written as first order differential equation (7):

$$\begin{aligned} \dot{x}_1 &= x_2; \\ \dot{x}_2 &= \frac{1}{m_1}[-c_1x_1 - c_{12}(x_1-x_3) - b_1x_2 - b_{12}(x_2-x_4)]; \\ \dot{x}_3 &= x_4; \\ \dot{x}_4 &= \frac{1}{m_2}[c_{12}(x_1-x_3) + b_{12}(x_2-x_4) - u(t)\cdot(V_0+x_4)] \end{aligned} \tag{7}$$

In this system with two degrees of freedom the Hamiltonian is as follows [3 - 8]:

$$\begin{aligned} H = \psi_0 + \psi_1x_2 + \psi_2(\frac{1}{m_1}\cdot\left(-c_1x_1 - c_{12}(x_1-x_3) - b_1x_2 - b_{12}(x_2-x_4)\right) + \psi_3x_4 + \\ + \psi_4(\frac{1}{m_2}(c_{12}(x_1-x_3) + b_{12}(x_2-x_4) - u(t)\cdot(V_0+x_4)). \end{aligned} \tag{8}$$

Optimal control law is of the same structure than solution (5):

$$\begin{aligned} &H = \max H, \\ &if \quad \psi_4\cdot(-u(t)\cdot(V_0+x_4)^2) = \max. \end{aligned} \tag{9}$$

The main conclusion of optimal control law (9) for system with two degrees of freedom is the same like for the system with one degree of freedom: value of area at any time must be on the bounds (1): $A(t) = A_1$ or $A(t) = A_2$.

Real control action synthesis

For realizing optimal control actions (in general case) system of one degree of freedom needs a feedback system with two adapters: one for displacement measurement and another - for velocity measurement. There is a simple case of control existing with only one adapter when motion changes directions, as shown in Fig. 3, [12]. It means that control action is similar to negative dry friction and switch points are along zero velocity line. In that case equation of motion for large velocity $|V0| \geq |\dot{x}|$ and dry friction is (10):

$$m \cdot \ddot{x} = -c \cdot x - b \cdot \dot{x} - F \cdot sign(\dot{x}) + U(\dot{x}), \tag{10}$$

where $U(\dot{x}) = -\left[k \cdot (V0 + \dot{x})^2 \cdot A_1 \cdot \dfrac{1 + sign(\dot{x})}{2}\right] - \left[k \cdot (V0 + \dot{x})^2 \cdot A_2 \cdot \dfrac{1 - sign(\dot{x})}{2}\right],$ and m – mass; c, b, F, k, V_0 – constants. Examples of modeling are provided in Fig. 4 – Fig. 7.

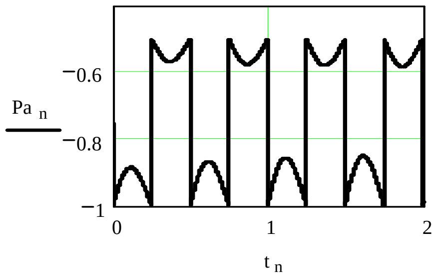

Fig. 4. Full control action (10) $Pa_n = U(\dot{x})$ in time t_n domain (SI system)

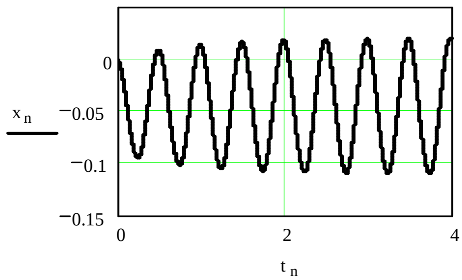

Fig. 5. Displacement x_n in time t_n domain (SI system)

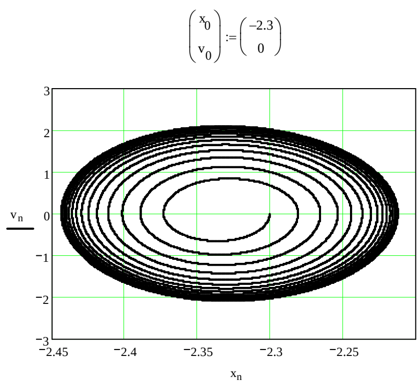

Fig. 6. Motion in phase plane (x = x_n; $\dot{x}$ = v_n) with initial conditions inside of limit cycle

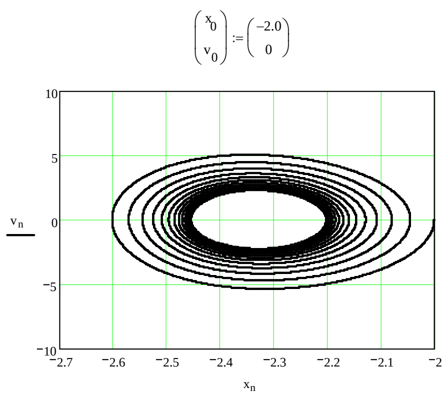

Fig. 7. Motion with initial conditions outside of limit cycle

An attempt to find more than one limit cycle was investigated in complicated system with cubic resistance force and dry friction (11). Answer is positive: for a system with non-periodical excitation (e.g. constant velocity V_0 of air or water flow) there can be more than one limit cycled (Fig. 8, 9). Both cycles are separated by different initial conditions.

$$m \cdot \ddot{x} = -c \cdot x^3 - b \cdot \dot{x} - F \cdot sign(\dot{x}) - \left[k \cdot (V0 + \dot{x})^2 \cdot A_2 \cdot \frac{1 - sign(\dot{x})}{2}\right] - \\ -\left[k \cdot (V0 + \dot{x})^2 \cdot A_1 \cdot \frac{1 + sign(\dot{x})}{2}\right], \tag{11}$$

For system with two degrees of freedom (6) the same control action U was investigated (see (10, 11)):

$$U(\dot{z}) = -\left[k \cdot (V0 + \dot{z})^2 \cdot A_1 \cdot \frac{1 + sign(\dot{z})}{2}\right] - \left[k \cdot (V0 + \dot{z})^2 \cdot A_2 \cdot \frac{1 - sign(\dot{z})}{2}\right], \tag{12}$$

Results of modeling are given in Fig. 10 – 13.

$$\begin{pmatrix} x_0 \\ v_0 \end{pmatrix} := \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

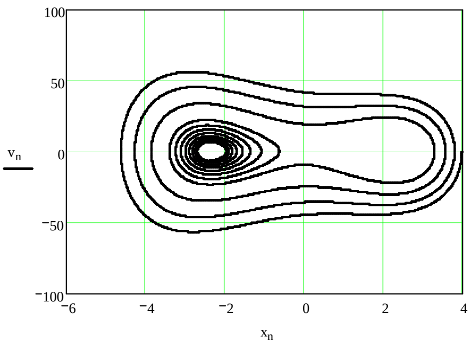

Fig. 8. Motion in phase plane for left side limit cycle

$$\begin{pmatrix} x_0 \\ v_0 \end{pmatrix} := \begin{pmatrix} 4 \\ 10 \end{pmatrix}$$

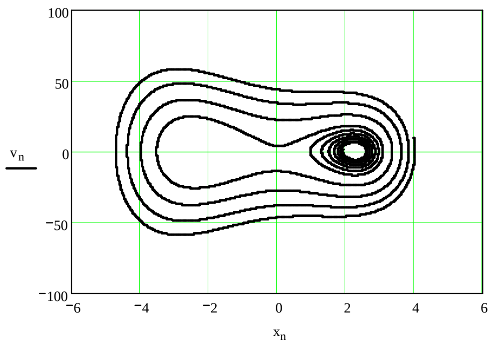

Fig. 9. Motion in phase plane for right side limit cycle

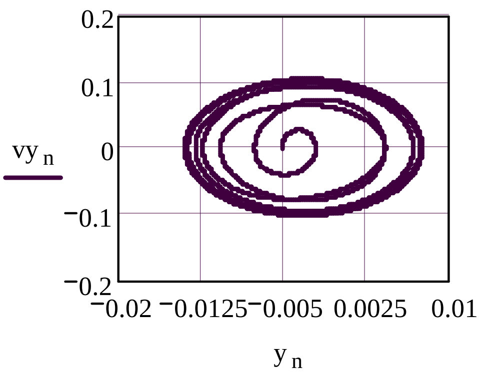

Fig. 10. Motion of mass m_1 in phase plane from small initial conditions

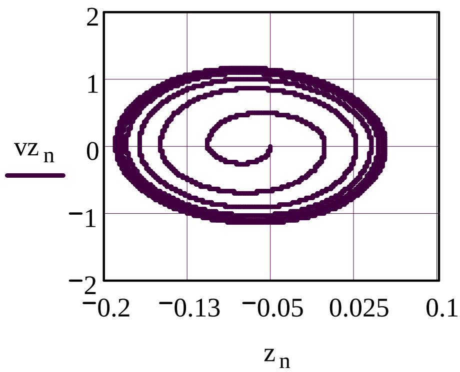

Fig. 11. Motion of mass m_2 in phase plane from small initial conditions

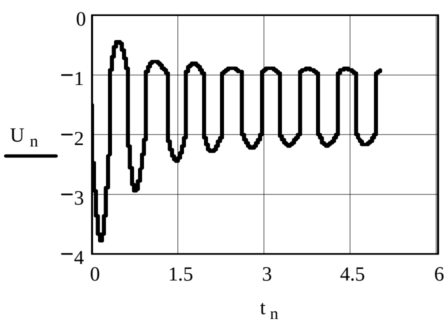

Fig. 12. Control action in (12) time domain

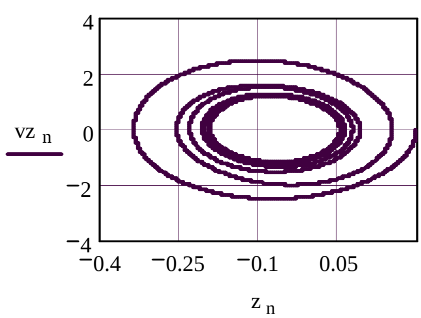

Fig. 13. Motion of mass m_2 in phase plane from large initial conditions

It is demonstrated that adaptive systems are very stable because air excitation and damping forces depend on velocity in second degree.

At the end of this study some experiments inside wind tunnel are considered. Variable speed motor driven unit downstream the working section permits continuous control of airspeed between 0 and 26 ms^{-1}. Experiments confirm that airflow excitation is very efficient.

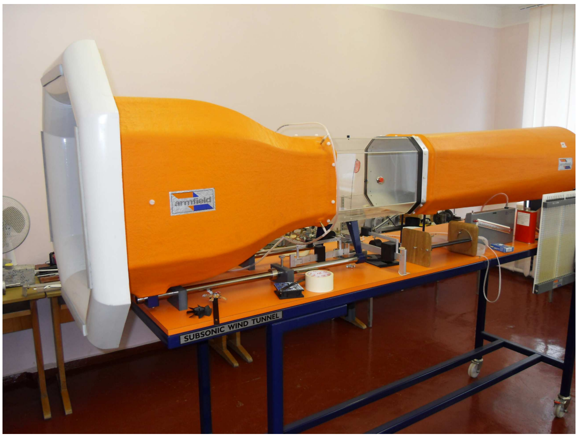

Fig. 14. Wind tunnel

Conclusion

Air or water flow may be used for excitation of objects motion by means of vibration technique. Control of object area allows development of very efficient mechatronic systems. Algorithm synthesis of strongly nonlinear mechanical systems includes tasks of optimization to obtain principally new vibration systems. For realization of such systems adapters and controllers must be employed. For this reason very simple control actions have solutions with use of sign functions.

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