511. Vibration of beams and rotating shafts under thermal effects

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Abstract. Temperature variations can significantly change the dynamic characteristics of structures. In the presented paper we have studied the clamped isotropic rotor under thermal effects. We have studied aluminium and copper beam. Because the expansion is restrained by the clamping, the beam undergoes internal stresses, which induce changes in dynamic characteristics. We have also expanded and developed Marques and Inman model as well as Bahzad and Bastami model. We have taken into account the compound influence of thermal force, axial force in rotating shaft and gyroscopic effect. The behavior of rotating shaft is studied in the temperature range of 250-700 K. The influence of temperature-dependent material properties was considered primarily with respect to temperature variations. The presented research work demonstrates a very good agreement between experimental data and results from our vibrational model. On the basis of the proposed analytical model it is possible to determine the vibrational characteristics in very wide range of temperatures. The reported paper is the first one in scientific literature to consider collectively the combined influence of temperature, gyroscopic effects and rotor speeds on shaft and beam vibrations.

Keywords: beam vibrations, rotor vibrations, temperature effects, mechanics, thermomechanics

Introduction

Mechanics and thermodynamics are two fundamental sciences needed for the computation of combined thermo-mechanical problems. At first sight, it seems these are two completely different scientific disciplines. Both of them have made progress of unimaginable dimensions using mathematics and experimental techniques. The impression is that thermodynamics helps compute physical properties required in mechanics of solids. Engineering devices very often operate under diverse thermal and mechanical conditions. In internal combustion engines, rocket systems, movement of the satellites, etc. the conditions are particularly temperature-sensitive. At the same time the mentioned phenomena are also very complex systems regarding mechanics. Thermodynamic effects are frequently ignored in research, which may yield totally incorrect results. Literature [1-2] shows that even the slightest temperature change leads to huge alteration of the clamped beam vibration properties.

In this paper we have developed a dynamic thermo-vibrational model for the clamped-clamped (Fig. 1), the supported-simply supported isotropic beam (Fig. 2), clamped-simply supported beam (Figure 3) and clamped-free beam (Figure 4) under the next assumptions [1]:

-the temperature is uniform over the beam
-the beam is uniform, with constant cross-section and made of isotropic material
-the elastic limit of the material is not exceeded
-the material properties are constants over temperature range.

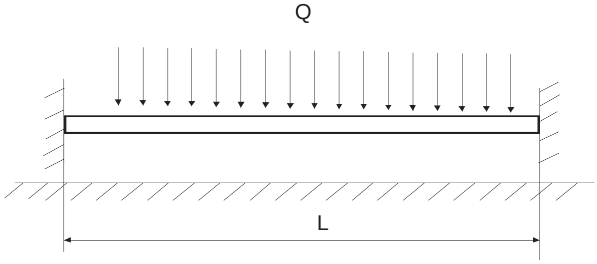

Fig. 1. Isotropic clamped-clamped beam

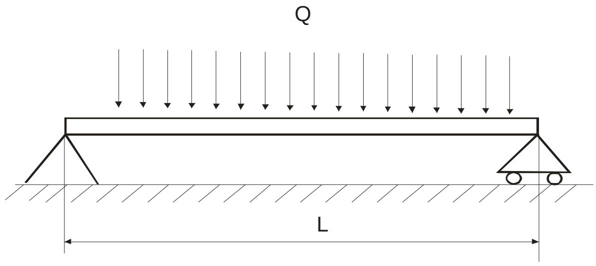

Fig. 2. Supported- simply supported beam

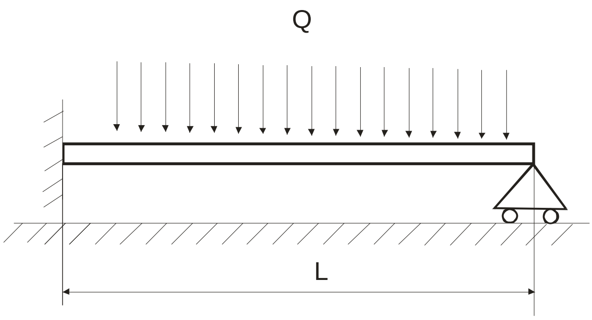

Fig. 3. Fixed-simply supported beam

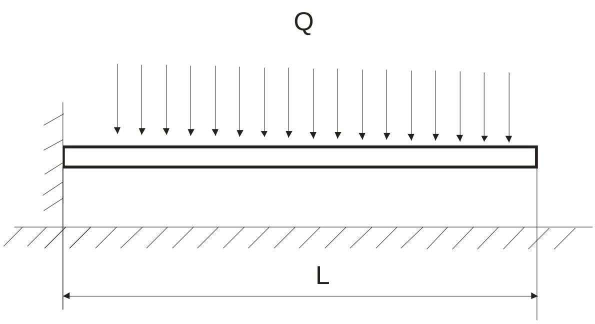

Fig. 4. Fixed-free beam

Let us assume that the support is homogenous, having the same temperature over its entire length. As a result of thermal expansion, an additional axial force F_T occurs:

$$F_T = \alpha\theta EA \tag{1}$$

In equation (1) α is the linear thermal extension coefficient, θ is the temperature difference between the actual and initial or reference temperature. The equation by means of which we can resolve the problem using the axial force is as follows according to Wear, Timoshenko and Young [4]:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + F_T\frac{\partial^2 w(x,t)}{\partial x^2} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = 0\,, \tag{2}$$

where E means Young modulus, I area moment of inertia, A area, ρ density of material, t time and w the displacement. Using the method of separation of variables $\mathrm{w}(\mathrm{x},\mathrm{t}) = \mathrm{X}(\mathrm{x})\Omega(\mathrm{t})$ and introducing the new functions, Equation (2) can be written down in a slightly less complicated way:

$$c^2\frac{X''''(x)}{X(x)} + 2\gamma\frac{X''(x)}{X(x)} - \frac{\ddot{\Omega}(t)}{\Omega} = \omega^2\,, \tag{3}$$

where the partial derivatives have been replaced with total derivatives.

$$\ddot{\Omega}(t) + \omega^2\Omega(t) = 0 \tag{4}$$

$$X''''(x) + 2\gamma X''(x) - \beta^4 X(x) = 0 \tag{5}$$

In Equation (5), the new symbols represent the following functional relations:

$$\beta^2 = \frac{\omega}{c}, c^2 = \frac{EI}{\rho A},\ \gamma = \frac{F_T}{2EI} \tag{6}$$

Thus, a general solution to Equations (4) and (5) are ($\lambda = \sqrt{\beta^4 + \gamma^2}$) [1-4]:

$$X(x) = C_1\cos\left(\sqrt{\lambda+\gamma}\,x\right) + C_2\cosh\left(\sqrt{\lambda-\gamma}\,x\right) + C_3\sin\left(\sqrt{\lambda+\gamma}\,x\right) + C_4\sinh\left(\sqrt{\lambda-\gamma}\,x\right) \tag{7}$$

$$\Omega(t) = A\sin(\omega t) + B\cos(\omega t) \tag{8}$$

In the equation (7) the value of λ (where is hidden the influence of angular frequency ω) and three of four constants of integration C_1, C_2, C_3 and C_4 are determined from the boundary conditions. The fourth constant is possible to find in the combination with the constants A and B in Equation (8). For a given beam at defined temperature the values by λ depend upon the boundary conditions [5-9]. Using boundary conditions, the following solutions can be analytically computed ($\Gamma = L^2\gamma, \Lambda = L^2\lambda$):

a) Clamped-clamped beam:

$$\sqrt{\Lambda^2-\Gamma^2}\{\cos(\Lambda+\Gamma)\cosh(\Lambda-\Gamma)-1\}+\Gamma\sin(\Lambda+\Gamma)\sinh(\Lambda-\Gamma)=0 \tag{9}$$

b) Supported-simply supported beam

$$\sin(\Lambda+\Gamma)=0 \tag{10}$$

c) Clamped-simply supported beam:

$$\frac{1}{\sqrt{\Lambda+\Gamma}}tg\left(\sqrt{\Lambda+\Gamma}\right)=\frac{1}{\sqrt{\Lambda-\Gamma}}tgh\left(\sqrt{\Lambda-\Gamma}\right) \tag{11}$$

d) Clamped-free beam:

$$\sqrt{\Lambda^2-\Gamma^2}\begin{pmatrix}\sqrt{\Lambda^2-\Gamma^2}\left(\Lambda^2+\Gamma^2\right)-(-\Gamma+\Lambda)^{3/2}(\Lambda+\Gamma)^{3/2}Cos(\Lambda+\Gamma)\cosh(-\Gamma+\Lambda)+\\ \Gamma(\Gamma-\Lambda)(\Lambda+\Gamma)\sin(\Lambda+\Gamma)\sinh(-\Gamma+\Lambda)\end{pmatrix}=0 \tag{12}$$

With the known angular frequencies ω_n of individual modes of vibration is possibly to calculate X_n and Ω_n of individual modes of vibration. To determine the solution for the displacement we have to solve the equation [5-9]:

$$w(x,t)=\sum_{i=1}^{\infty}\left(A_n\sin(\omega_n t)+B_n\cos(\omega_n t)\right)X_n(x), \tag{13}$$

where the modal shapes can be shown to be orthogonal:

$$\int_0^l X_n(x)X_m(x)dx=0 \ \text{ for } \ n\neq m \tag{14}$$

The model presented in our paper is fully analytical, but if compared with the measured results it points to a large deviation from reality [1, 2]. The biggest problem of this model is that in the mathematical model in question the clamped wall can fully withstand the beam for the beam to have a constant length all the time. The above assumption is not realistic. As a result, a new model was designed to reduce to at least to some extent the huge differences between the analytical results and the measured values.

The dynamic model for beams under thermal stresses

Fig. 3 and 4 illustrate a new rheological model for the clamped and the simply supported beam. To this end, a spring is added with the spring constant K. The model slightly differs from the model presented in paper [1], where the authors Marques and Inman integrated additional torsion springs into the rheological model.

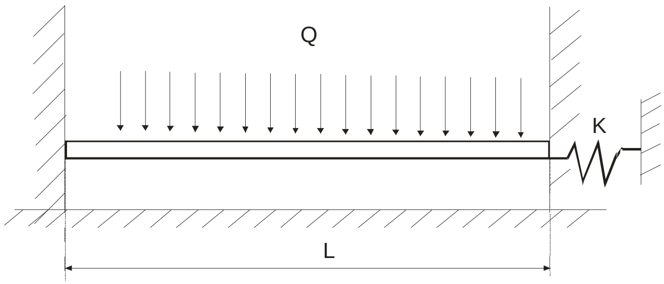

Fig. 5. New model of clamped beam

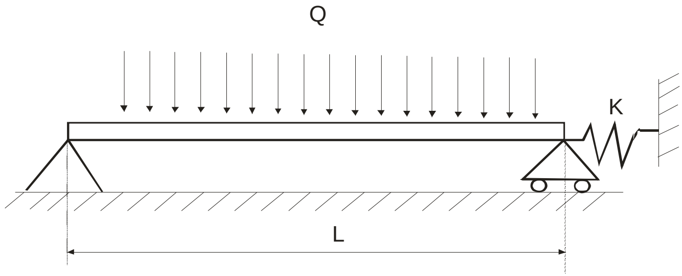

Fig. 6. New model of simply supported beam

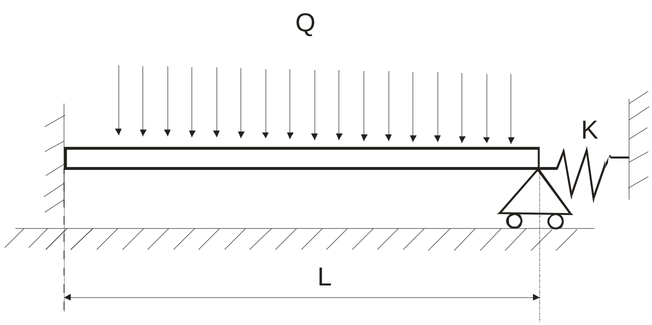

Fig. 7. New model for clamped-simply supported beam

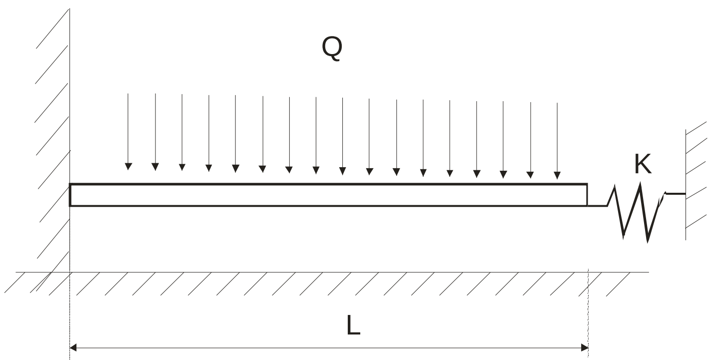

Fig. 8. New model for clamped-free beam

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In this case, force F_T can be computed in the following way [3]:

$$\varepsilon EA = \alpha\theta EA - K\delta\,,\ \ K\delta = EA\left(\alpha\theta - \frac{\delta}{L}\right),\ \ \delta = \frac{EAL\alpha\theta}{KL + EA} \tag{15}$$

The reaction force computation can be as follows:

$$F_T = K\delta = \frac{EAL\alpha\theta}{L + \dfrac{EA}{K}}, \tag{16}$$

where the modulus of elasticity E and linear expansion coefficient α are temperature dependent functions. By means of the boundary conditions, Equations (5) and (6) also apply now.

An aluminium beam with the dimensions indicated in Table 1 were used for the computation.

Table 1. Fundamental constants for aluminium beam

	Beam
Length (m)	$6.35\cdot10^{-2}$
Width (m)	$2.04\cdot10^{-2}$
Thickness (m)	$1.62\cdot10^{-3}$
Young modulus (N/m^2)	$6.9\cdot10^{10}$
Volume expansion coefficient (1/K)	$24\cdot10^{-6}$ K^{-1}
Spring constant (N/m)	$1.553\cdot10^{5}$
Density (kg/m^3)	2780

Vibration of cylindrical shafts under thermal effects

In the case of rotor vibration we have used the rheological model as it was presented for beams (Figure 9 and 10):

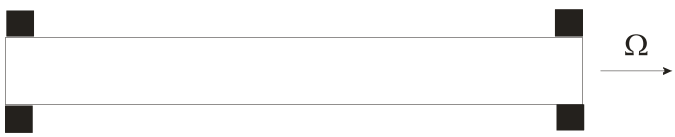

Fig. 9. Vibration of cylindrical shaft

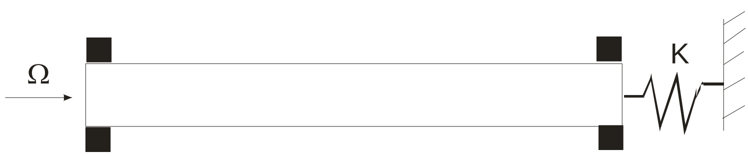

Fig. 10. New rheological model of cylindrical shaft

If we consider cylindrical shafts, the existence of axial force changes the equation of lateral vibration. The equation of lateral vibration of Euler-Bernoulli beam in the presence of axial force P together with temperature effects can be written as [10,11]:

$$EI\frac{d^4X}{dx^4}+\left(F_t-P\right)\frac{d^2X}{dx^2}-\rho A\omega^2X=0 \tag{17}$$

Upper equation we can express also with the next expression:

$$X''''(x)+2\gamma X''(x)-\beta^4X(x)=0 \tag{18}$$

In Equation (10), the new symbols represent the following functional relations:

$$\beta^2=\frac{\omega}{c},c^2=\frac{EI}{\rho A},\ \gamma=\frac{F_t-P}{2EI} \tag{19}$$

If we use 3-D linear elasticity relations we obtain:

$$P=\upsilon\rho I_p\Omega^2 \tag{20}$$

Supported-simply supported beam

$$\sin\left(\Lambda+\Gamma\right)=0 \tag{21}$$

Where the I_p is polar moment of inertia, ν presents Poisson ratio and Ω is rotational speed of rotating shaft.

Influence of gyroscopic effect in combination with temperature effect [10,11]:

$$EI\frac{d^4X}{dx^4}+\left(F_t-P_G\right)\frac{d^2X}{dx^2}-\rho A\omega^2X=0$$
$$P_G=\rho I\omega\left(2\Omega-\omega\right) \tag{22}$$

Table 2. Fundamental constants of steel shaft

	Shaft
Length (m)	1
Diameter (m)	0.01
Young modulus (N/m^2)	$2.07\cdot10^{11}$
Volume expansion coefficient (1/K)	$1.3\cdot10^{-5}$ K^{-1}
Spring constant (N/m)	$1.553\cdot10^{5}$
Density (kg/m^3)	7800
Poisson number	0.33333

Results and discussion

The presented mathematical model was used to calculate thermodynamic properties of state of pure aluminium. Figures 11-14 present the prediction of angular frequency ω in dependence of mode of vibration and temperature difference. The comparison of analytical results and experimental data is presented in the paper [1]. The provided results demonstrate relatively good agreement between analytical models and experimental data [3]. The detailed analysis indicates that small changes of temperature cause significant changes of natural frequencies for beams.

Table 1 contains the main data on the beam. An aluminium beam was chosen for analysis. The aluminium beam is very interesting, particularly due to relatively high expansion coefficients. Figure 11 presents the results for the angular frequency of vibration modes for the clamped-clamped beam. Figure 12 contains the results for the oscillation frequency of vibrational modes for the supported-simply supported aluminium beam. Figures 13 and 14 provides results of oscillation frequency for clamped-free and clamped-simply supported beam. The detailed analysis reveals that also small changes of temperature induce significant changes of natural frequencies for beams. The proposed models provide the basis for future research of micro- beams. Figures 15-17 illustrate the influence of temperature, rotor speed and gyroscopic effect on angular frequencies up to the sixth mode. The analysis demonstrates that we have to take into account at high rotor speeds also thermal and gyroscopic effects in case we need very accurate dynamic calculations. The influence of temperature effects on vibrational characteristics depends on boundary conditions. The results are very sensitive to the structure of beam or rotor material.

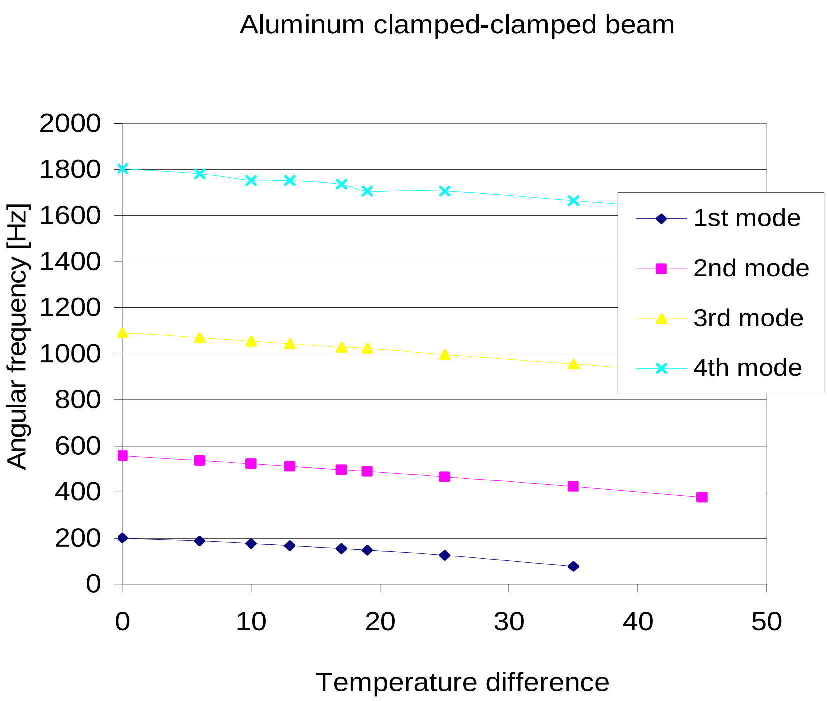

Fig. 11. Aluminium clamped-clamped beam

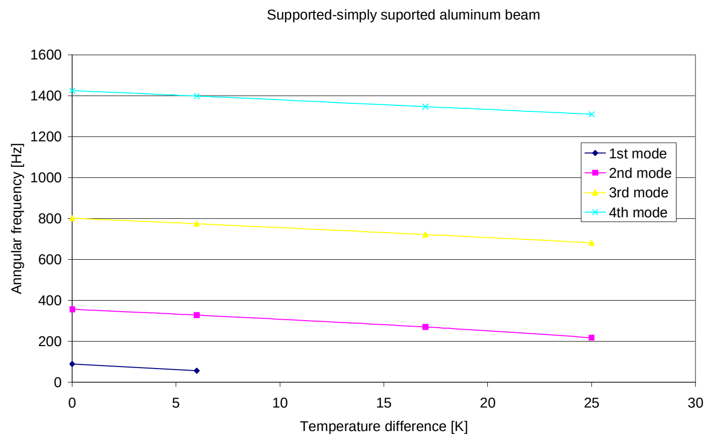

Fig. 12. Supported-simply supported aluminium beam

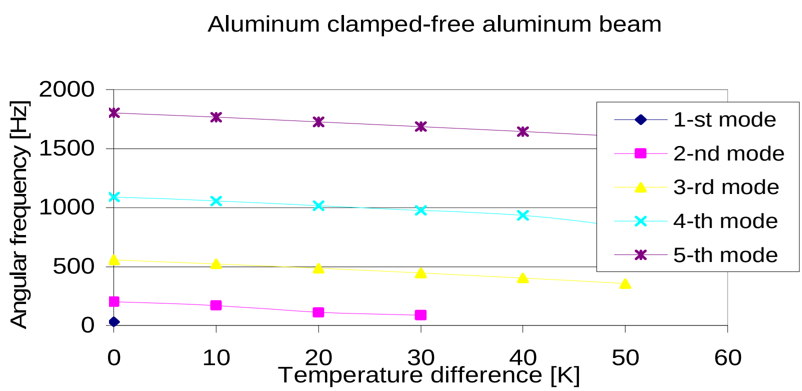

Fig. 13. Aluminium clamped-free beam

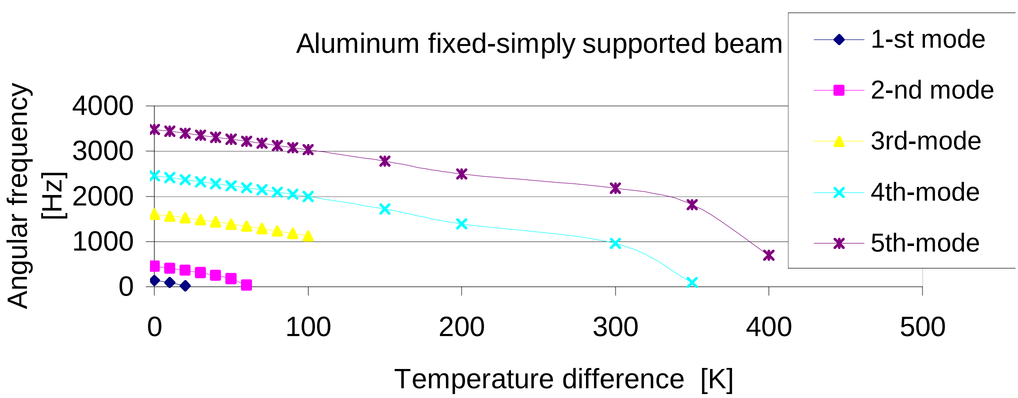

Fig. 14. Aluminium clamped-simply supported beam

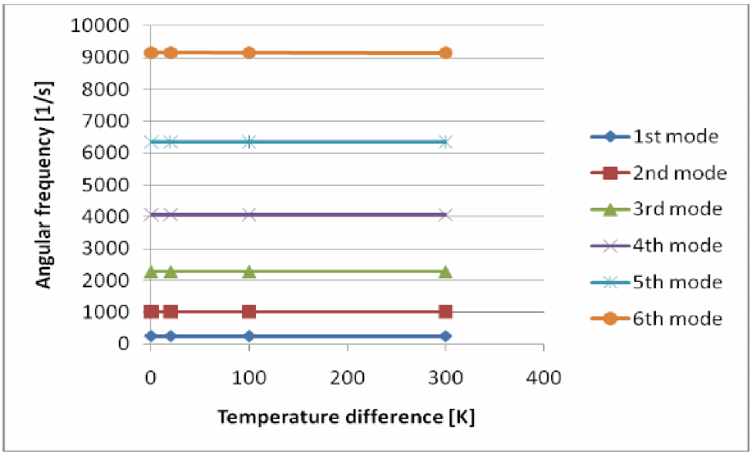

Fig. 15. Supported-simply supported rotor (Ω=0)

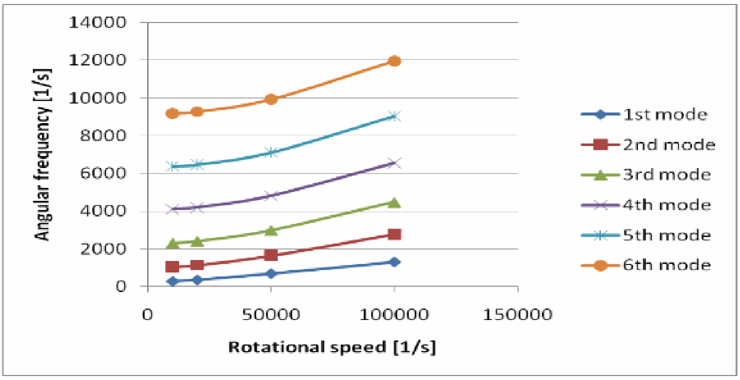

Fig. 16. Vibration of rotor in dependence of rotor speed (Ω =100 K)

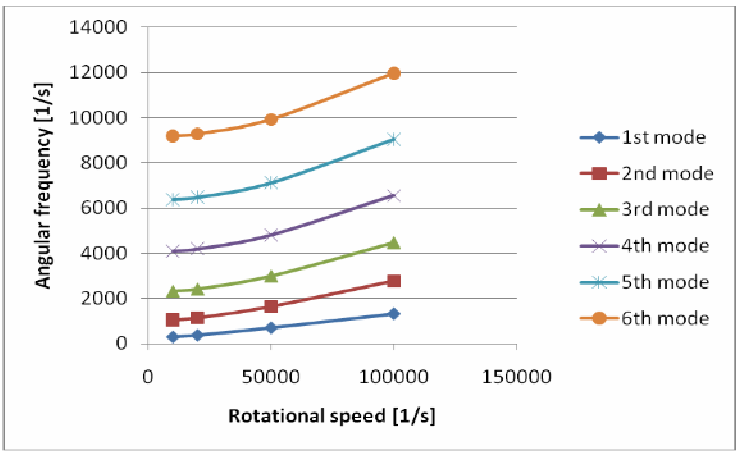

Fig. 17. Vibration of rotor in dependence of rotor speed and gyroscopic effect (Ω =100 K)

Conclusion

The paper demonstrates for the first time in scientific literature the combination of temperature, gyroscopic and rotor speed effect on shaft and beam vibrations. In the presented paper we have concentrated on the analysis of the influence of temperature on angular frequencies in modes of vibration. The thermo physical properties of state, such as modulus of elasticity and linear expansion coefficient, are regarded as constants in this paper. The analysis indicates that a minor change in temperature results in a considerable alteration in vibrations.

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