439.Modeling of a mechanical system using output data of the hammer blow sequence response

V. Slivinskas[1,2] **and V. Šimonytė**[1]
[1]Vilnius Pedagogical University, Studentų 39, Vilnius, Lithuania
[2]Vilnius Gediminas Technical University, Saulėtekio al. 11, Vilnius, Lithuania
e-mail: *astera@astera.lt* *simonyte@vpu.lt*

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Abstract. A linear stationary dynamical system that is excited by a sequence of unit impulses is considered. The method is based on the system impulse response formant decomposition. A new system parameter estimation method based on deconvolution of the system output process and explicit Levenberg optimization method is presented. The method efficiency is verified using simulated and real data.

Keywords: linear dynamical system, impulse response, convolution, deconvolution, nonlinear Levenberg optimization method.

Introduction

The system is driven by the input variable $u(t)$ at time moment t. The output of the system is the convolution of the input $u(t)$ and the impulse response $h(t)$, and can be calculated as the convolution integral:

$$y(t) = \int_0^t h(\tau) \cdot u(t-\tau)\, d\tau. \qquad (1)$$

It is well known that system response $y(t)$ to Dirac delta function $\delta(t)$ is equal to the impulse response $y(t) = h(t)$. In the case when the system is excited by a sequence of $\delta(t)$ functions $\delta(t)$, $\delta(t-T)$, $\delta(t-2T), \ldots, \delta(t-K \cdot T)$ where T is the time period and K is equal to $\mathrm{int}(t/K)$, the output of the system is periodic and is expressed as a sum of shifted impulse responses:

$$y(t+k \cdot T) = \sum_{n=0}^{K} h(t+nT), \ 0 \le t < T, \qquad (2)$$

for some large enough integer k.

In order to obtain the impulse response of a mechanical system, the system is driven by a hammer blow. To get the impulse response of an electronic filter, the filter is driven by a very narrow impulse. Using a single blow or impulse is not reliable for obtaining impulse response. It seems that it is better to excite the system by a sequence of blows or impulses with period T.

But as we see from equation (2), in this case the output of the system is not the impulse response but the convolution of shifted impulse responses. One can simplify the situation by assuming that the impulse response of the system is damping rapidly and assuming that, for example, $h(t+2\cdot T)\cong 0$ for all $t\geq 0$. Then the output of the system $y(t)$ in the interval $[k\cdot T, k\cdot T+T), k>0$ is expressed as follows:

$$y(t+k\cdot T)\ = h(t)+h(t+T). \tag{3}$$

The impulse response $h(t)$ can be expressed as a sum of finite number m of formants $f_i(t)$:

$$h(t)\ = \sum_{i=1}^{m} f_i(t). \tag{4}$$

In general case, the formant $f_i(t)$ is a mixture of a damped sinusoid and an L_i-th degree polynomial with frequencies ω_i, damping factors λ_i, amplitudes $a_{i1,}a_{i2},\dots,a_{iL_i}$ and phases $\varphi_{i1},\varphi_{i2},\dots,\varphi_{iL_i}$[4]:

$$f_i(t) = e^{\lambda_i t}(a_{i1}\sin(\omega_i+\varphi_{i1})+a_{i2}t\sin(\omega_i t+\varphi_{i2})+ \cdots + a_{iL}t^{L-1}\sin(\omega_i t+\varphi_{iL})) \tag{5}$$

From (3)-(4) we can obtain the following expression:

$$y(t+i\cdot T)\ = \sum_{i=1}^{m}\big(f_i(t)+f_i(t+T)\big), 0\leq t<T\,. \tag{6}$$

Our task is to develop an algorithm for estimating parameters $(\lambda_i,\omega_i,a_{ij},\varphi_{ij})$ of the formants $f_i(t)$ of the impulse response $h(t)$ using a given sequence $y(0), y(\Delta t), y(2\Delta t), \dots, y((N-1)\Delta t)$ of the measured equidistant samples of two periods $[k\cdot T,(k+2)\cdot T)$ of $y(t)$. Denote by $\boldsymbol{\theta}$ a vector that consists of damping factors and frequencies and by $\boldsymbol{\alpha}$ a vector that consists of amplitudes and phases:

$$\boldsymbol{\theta} = [\lambda_1,\omega_1,\dots\lambda_m,\omega_m]^T,$$
$$\boldsymbol{\alpha} = [a_{11},\varphi_{11},\dots,a_{K_1},\varphi_{K_1},\dots,a_{m1},\varphi_{m1},\dots,a_{mK_m},\varphi_{mK_m}]^T \tag{7}$$

Denote by $\boldsymbol{y_1},\boldsymbol{y_2}$ vectors whose coordinates are measured samples:

$$\boldsymbol{y_1} = [y(0), y(\Delta t),\dots, y((N/2-1)\Delta t)]^T,$$
$$\boldsymbol{y_2} = [y\left(\left(\tfrac{N}{2}\right)\Delta t\right),\dots,y((N-1)\Delta t)]^T. \tag{8}$$

Levenberg optimization method

Levenberg optimization method was applied for estimation of parameters of model (4)-(5) in the case when the data was without convolution and sampled from $h(t)$ [2-3]. The method was based on the basis signal space model. This model is represented by a matrix $\boldsymbol{S}(\boldsymbol{\theta})$ of dimension $N\times k$ whose columns are products of damped cosinusoidal and sinusoidal functions and t powers. Denote by $\boldsymbol{y}$ a vector of measured equidistant samples of $h(t)$, by $\boldsymbol{x}(\boldsymbol{\alpha})$ a column-vector of amplitudes and by $\boldsymbol{e}$ a column-vector of errors. Then the model can be presented as follows:

$$\boldsymbol{y} = \boldsymbol{S}(\boldsymbol{\theta})\cdot\boldsymbol{x}(\boldsymbol{\alpha})+\boldsymbol{e}\,. \tag{9}$$

The model (3-6) differs a little from (9). Let $\boldsymbol{S}_1(\boldsymbol{\theta})$ be a matrix that is equal to the upper half of the matrix $\boldsymbol{S}(\boldsymbol{\theta})$, and $\boldsymbol{S}_2(\boldsymbol{\theta})$ be a matrix that is equal to the lower half of the matrix $\boldsymbol{S}(\boldsymbol{\theta})$. In a vector form our model is as follows:

$$\begin{bmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{S}_1(\boldsymbol{\theta}) + \boldsymbol{S}_2(\boldsymbol{\theta}) \\ \boldsymbol{S}_1(\boldsymbol{\theta}) + \boldsymbol{S}_2(\boldsymbol{\theta}) \end{bmatrix} \cdot \boldsymbol{x}(\boldsymbol{\alpha}) + \begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_2 \end{bmatrix}. \quad (10)$$

Denote

$$\boldsymbol{\Phi}(\boldsymbol{\theta}) = \begin{bmatrix} \boldsymbol{S}_1(\boldsymbol{\theta}) + \boldsymbol{S}_2(\boldsymbol{\theta}) \\ \boldsymbol{S}_1(\boldsymbol{\theta}) + \boldsymbol{S}_2(\boldsymbol{\theta}) \end{bmatrix}, \quad (11)$$

and

$$\boldsymbol{y} = \begin{bmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{bmatrix}, \boldsymbol{e} = \begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_2 \end{bmatrix}. \quad (12)$$

Then our model is of the same form as (9):

$$\boldsymbol{y} = \boldsymbol{\Phi}(\boldsymbol{\theta}) \cdot \boldsymbol{x}(\boldsymbol{\alpha}) + \boldsymbol{e}. \quad (13)$$

Denote by $\boldsymbol{I}_N$ the identity matrix of size $N \times N, \boldsymbol{P}_{\boldsymbol{\Phi}(\boldsymbol{\theta})}$ the orthogonal projector onto the column space of $\boldsymbol{\Phi}(\boldsymbol{\theta})$:

$$\boldsymbol{P}_{\boldsymbol{\Phi}(\boldsymbol{\theta})} = \boldsymbol{\Phi}(\boldsymbol{\theta})(\boldsymbol{\Phi}(\boldsymbol{\theta})^{\mathrm{T}}\boldsymbol{\Phi}(\boldsymbol{\theta}))^{-1}\boldsymbol{\Phi}(\boldsymbol{\theta})^{\mathrm{T}}, \quad (14)$$

and by $\boldsymbol{P}^{\perp}_{\boldsymbol{\Phi}(\boldsymbol{\theta})}$ the orthogonal projector onto the orthogonal complement of the column space of $\boldsymbol{\Phi}(\boldsymbol{\theta})$:

$$\boldsymbol{P}^{\perp}_{\boldsymbol{\Phi}(\boldsymbol{\theta})} = \boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{\Phi}(\boldsymbol{\theta})}. \quad (15)$$

Then the problem reduces to determination of estimates of parameters $\boldsymbol{\theta}$ that minimize the functional [5]:

$$r_2(\boldsymbol{\theta}) = \left\|\boldsymbol{P}^{\perp}_{\boldsymbol{\Phi}(\boldsymbol{\theta})} \cdot \boldsymbol{y}\right\|^2. \quad (16)$$

In other words, we try to choose the damping factors λ_i and frequencies ω_i that the error would be as small as possible. After estimating the parameter $\boldsymbol{\theta}$, the amplitudes and phases are estimated by a simple least squares method. Levenberg method for minimization of the functional $r_2(\cdot)$ is an iterative procedure of updating an initial estimate of $\boldsymbol{\theta}$. Let $\boldsymbol{D}$ stands for the Frechet derivative of a mapping. Then $\boldsymbol{V}(\boldsymbol{\theta}) = \boldsymbol{D}\boldsymbol{P}^{\perp}_{\boldsymbol{\Phi}(\boldsymbol{\theta})}\boldsymbol{y}$ is an $N \times k$ matrix, and $\boldsymbol{b}(\boldsymbol{\theta}) = \boldsymbol{P}^{\perp}_{\boldsymbol{\Phi}(\boldsymbol{\theta})} \cdot \boldsymbol{y}$ is an $N \times 1$ vector. Denote by $\boldsymbol{I}_k$ the identity matrix of size $k \times k$, and by c_l the constant of Levenberg algorithm in the l-th iteration (usually $c_0 = 0.001$). An iterative procedure for updating estimates is as follows:

$$\boldsymbol{\theta}^{l+1} = \boldsymbol{\theta}^l - (\boldsymbol{V}^T(\boldsymbol{\theta}^l)\boldsymbol{V}(\boldsymbol{\theta}^l) + c_l\boldsymbol{I}_k)^{-1}\boldsymbol{V}^T(\boldsymbol{\theta}^l)\boldsymbol{b}(\boldsymbol{\theta}^l). \quad (17)$$

It is not difficult to modify the explicit algorithm to solve (17) that is presented in [2-3] and applied for data without convolution to our case (3-5). We implemented Levenberg algorithm in Visual Studio 2005 VB. Net language using BlueBit matrix library for matrix operations.

An algorithm to calculate the matrix $\boldsymbol{V}(\boldsymbol{\theta})$

Denote by $\boldsymbol{B}$ a special generalized inverse of the basic signal matrix $\boldsymbol{\Phi}$ satisfying the equations:

$$\begin{aligned} &\boldsymbol{\Phi B \Phi} = \boldsymbol{\Phi}, \\ &(\boldsymbol{\Phi B})^{\mathrm{T}} = \boldsymbol{\Phi}, \\ &\boldsymbol{B \Phi B} = \boldsymbol{B}. \end{aligned} \tag{18}$$

$\boldsymbol{B}$ has the following additional properties:

$$\begin{aligned} &\boldsymbol{\Phi B} = \mathrm{P}_{\boldsymbol{\Phi}}, \\ &\boldsymbol{B \Phi} = \boldsymbol{I}_N. \end{aligned} \tag{19}$$

It is shown in [4] that:

$$\boldsymbol{D P}_{\boldsymbol{\Phi}}^{\perp} = -\boldsymbol{P}_{\boldsymbol{\Phi}}^{\perp}\boldsymbol{D \Phi B} - (\boldsymbol{P}_{\boldsymbol{\Phi}}^{\perp}\boldsymbol{D \Phi B})^{\mathrm{T}}. \tag{20}$$

The matrix $\boldsymbol{B}$ is obtained using a standard QR- decomposition of the matrix $\boldsymbol{\Phi}$. Denote by $\boldsymbol{S}$ a $k \times k$ permutation matrix, by $\boldsymbol{T}_1$ a $k \times k$ upper triangular matrix with decreasing diagonal elements, by $\boldsymbol{Q}$ a $N \times N$ orthogonal matrix, and by $T = \begin{bmatrix} T_1 \\ 0_{(N-k)\times k} \end{bmatrix}$ a matrix obtained from $\boldsymbol{T}_1$ by adding $N-k$ zero rows. Then the matrix $\boldsymbol{B}$ is calculated as follows:

$$\boldsymbol{B} = \boldsymbol{S}[\boldsymbol{T}_1^{-1}\, \mathbf{0}_{k\times(N-k)}]\boldsymbol{Q}^{\mathrm{T}}. \tag{21}$$

Since a differential operation $\frac{\partial}{\partial \theta_i}$ is linear, then one can use the following expression for calculation of $\boldsymbol{D\Phi}$:

$$\frac{\partial}{\partial \theta_i}\boldsymbol{\Phi}(\boldsymbol{\theta}) = \begin{bmatrix} \frac{\partial}{\partial \theta_i}\boldsymbol{S}_1(\boldsymbol{\theta}) + \frac{\partial}{\partial \theta_i}\boldsymbol{S}_2(\boldsymbol{\theta}) \\ \frac{\partial}{\partial \theta_i}\boldsymbol{S}_1(\boldsymbol{\theta}) + \frac{\partial}{\partial \theta_i}\boldsymbol{S}_2(\boldsymbol{\theta}) \end{bmatrix}. \tag{22}$$

The elements s_{mn} of the matrices $\boldsymbol{S}_1(\boldsymbol{\theta})$ and $\boldsymbol{S}_2(\boldsymbol{\theta})$ are analytic functions of the coordinates θ_{i_1} and θ_{i_2} of the parameter vector $\boldsymbol{\theta}$ that correspond to the damping factor λ_m and frequency ω_m. Thus it is not difficult to calculate $\frac{\partial}{\partial \theta_i}\boldsymbol{S}_1(\boldsymbol{\theta})$ or $\frac{\partial}{\partial \theta_i}\boldsymbol{S}_2(\boldsymbol{\theta})$. For example, if $s_{ij} = (i-1)^l e^{\lambda_m \cdot (i-1)} \cdot \cos(\omega_m \cdot (i-1))$ then the derivative:

$$\frac{\partial}{\partial \lambda_m}s_{ij} = -(i-1)^{l+1} e^{\lambda_m \cdot (i-1)} \cdot \sin(\omega_m \cdot (i-1)).$$

To conclude, the matrix $\boldsymbol{V}(\boldsymbol{\theta}) = \boldsymbol{D P}_{\boldsymbol{\Phi}(\boldsymbol{\theta})}^{\perp}\boldsymbol{y}$ is calculated in four steps:

1. Calculate the matrix $\boldsymbol{B}$ using QR- decomposition of the matrix $\boldsymbol{\Phi}$ (21);
2. Calculate the Frechet derivative $\boldsymbol{D\Phi}$ of the basic signal matrix $\boldsymbol{\Phi}$ (22);
3. Obtain the projector $\boldsymbol{P}_{\boldsymbol{\Phi}}^{\perp}$ by (14,15);
4. Finally, calculate $\boldsymbol{V}(\boldsymbol{\theta})$ using (20).

Simulation results

In order to check the algorithm of deconvolution of an output process using Levenberg method (17) we did the calculations with the simulated data. Let $\Delta t = {}^{1}/_{4800}\, s$; $T = \Delta t \cdot 800;\ N = 1600.$ The impulse response of our system was as follows:

$$h(\Delta t \cdot i) = 10^5 (\Delta t)^2 i^2 e^{-300\Delta t \cdot i} \sin(2\pi 600 \cdot \Delta t \cdot i + \pi/2),\ \ i = 0,1,\dots,1600. \quad (23)$$

The output process of our system was produced by formula (3) and adding white Gaussian noise (Fig. 1).

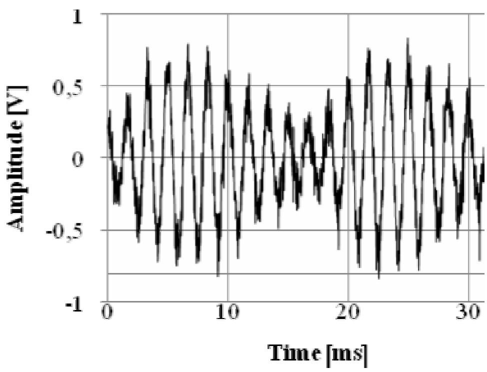

Fig. 1. The simulated system output process.

The initial data for Levenberg algorithm is presented in table 1.

Table 1. Initial data for Levenberg algorithm

	Size	Dimension
Initial frequency	500	[Hz]
Initial damping	-160	[1/s]
Number of iterations	31	[units]

The output process deconvolution was realized by the method, presented in this paper. The estimated impulse response is presented in Fig.2.

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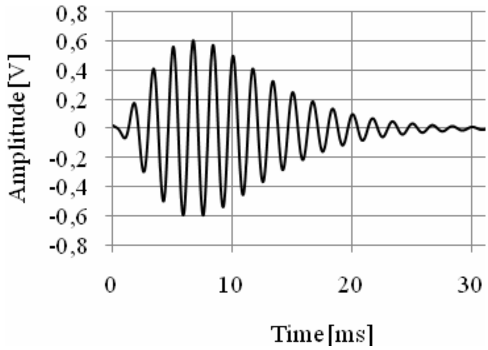


Fig. 2. The estimated impulse response of the simulated system

In order to investigate the influence of additive noise the calculations of the deconvolution were made with different noise value. The summary of results is presented in Fig. 3, where relative signal estimation error is put on the vertical axis and signal-to-noise ratio (SNR) is put on the horizontal axis. These values are calculated along standard signal processing formulas [4] depending on generated noise dispersion and signal estimate obtained.

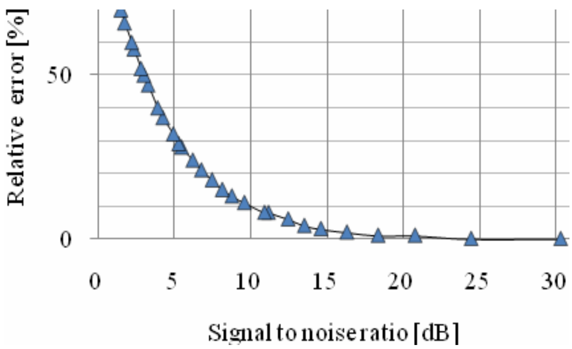


Fig. 3. Relative error versus SNR

The investigation demonstrated that when SNR is larger than 10 dB the precision of estimation of impulse response is more than 10%.

Modeling of a mechanical system

In order to check the effectiveness of the method for real mechanical systems a laboratory experiment was carried out by registering the vibration in one point of an elastic band fastened to a massive stand excited by a hammer blow into the stand. Experimental equipment is illustrated in Fig. 4.

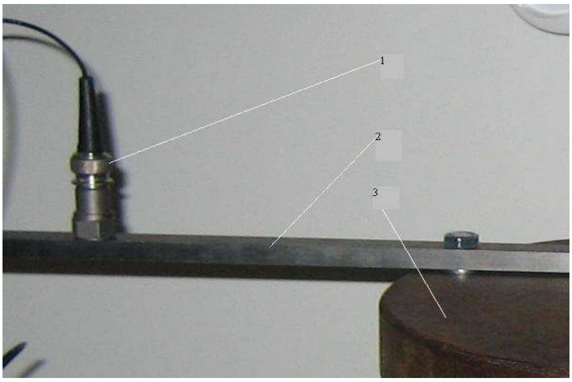


Fig. 4. Equipment for investigation of oscillations of the band:
1 – piezoelectric accelerometer; 2 – band, 3 – stand

The oscillations from the accelerometer were transferred into the computer by means of the microphone input of the sound card and sound recording program. The data was registered in PCM (pulse code modulation) format with attributes 48,000 kHz and 16 bit Stereo. Since sensing element measures the motion acceleration, therefore in order to get the way of the motion the data was digitally integrated two times. Besides the oscillations with frequency larger than 5000 Hz were filtered. To model the impulse response of the band in the fixed point the method presented above was used. Two fragments of the hammer blow response are provided in Fig. 5.

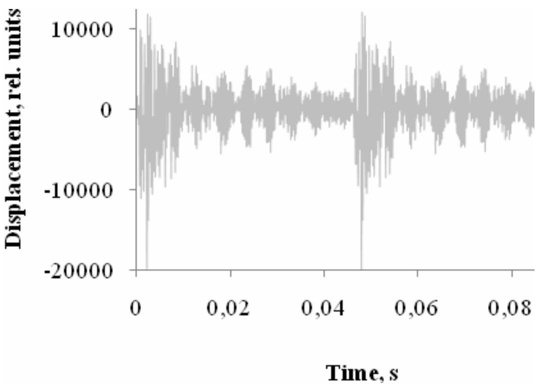


Fig. 5. Two fragments of the measured data

The oscillation amplitudes were measured in relative units from the interval (-20000, 20000). That is related to the format of the registered data. If one wants to get the amplitudes of oscillations in real units, for example, in microns, calibration of the data must be carried out.

In order to get the entire model of the impulse response, the data was filtered as in [1] and used as the initial data for Levenberg algorithm. The following frequency ranges were investigated: 1 - 200 Hz, 200 – 1000 Hz, 1000 – 2000 Hz, 2000 – 2960 Hz, 2960 – 3420 Hz, 3200 – 3320 Hz, 3320 – 3420 Hz, 3420 – 4000 Hz, 4000 – 5000 Hz. The range of 3000 – 4000 Hz was divided into more bands since Fourier analysis shows that the signal energy is concentrated mostly in this region. We calculated as many formants of the model in a specific band as there were peaks of the data Fourier transform in this band. Taking into account the model accuracy and calculation results, we have chosen the third order formants.

An i-th formant $g_i(t)$ is expressed by the following formula:

$$g_i(t) = a_{i1}e^{\lambda_i t}\sin(2\pi f_i t + \varphi_{i2}) + a_{i2}f_d t e^{\lambda_i t}\sin(2\pi f_i t + \varphi_{i2}) + a_{i3}{f_d}^2 t^2 e^{\lambda_i t}\sin(2\pi f_i t + \varphi_{i3}) \quad (24)$$

here a_{i1} is the first amplitude measured in units; a_{i2} – the second amplitude measured in $units \cdot Hz$; a_{i3} – the third amplitude measured in $units \cdot Hz^2$; $\varphi_{i1}, \varphi_{i2}, \varphi_{i3}$ - the phases measured in radians; λ_i - the damping factor measured in 1/s; f_i - the frequency measured in Hz; f_d. – the sampling frequency measured in Hz. In total we have calculated 15 formants.

The values of the formant model parameters are presented in Table 2.

TABLE 2: The values of frequency f and damping factor λ of the formants : a) formant No 1 – 6; b) formant No 7-12; c) formant No 13 – 15.

a)

No	1	2	3	4	5
f	88	403	595	808	920
λ	-113	-155	-177	-99	-299
a_1	3598	2535	3960	3056	2947
a_2	38,6	34,6	107,8	26,3	134,7
a_3	0,0845	0,117	0,458	0,0497	0,955
φ_1	-3,139	1,526	-2,43	-0,9706	1,254
φ_2	0,1745	-1,74	0,606	1,9538	-1,32
φ_3	-2,854	1,335	-2,86	-1,3511	1,964

b)

No	6	7	8	9	10
f	1050	1612	2198	2774	3284
λ	-407	-707	-384	-382	-136
a_1	1502	1908	3052	3339	5987
a_2	106,5	268,7	245,6	484,3	117,6
a_3	0,8962	4,7812	2,811	5,899	0,81730
φ_1	1,1846	-2,429	2,0574	-0,85	-3,0892
φ_2	-2,901	0,3985	-1,604	2,673	2,5077
φ_3	0,2733	-2,738	1,3893	-0,42	-0,7305

c)

No	11	12	13	14	15
f	3497	3702	4647	4187	4858
λ	-131	-119	-97	-1307	-231
a_1	3415	1061	1303	3065	1918
a_2	98,6	27,9	19,7	652,1	90,7
a_3	0,7906	0,1948	0,0877	25,09	0,431
φ_1	-0,330	1,547	-1,608	2,794	0,117
φ_2	2,8006	2,2623	0,5666	-1,25	3,021
φ_3	-0,418	-0,3436	-2,494	1,867	-0,11

In Fig. 6 the impulse response of the modeled system is presented.

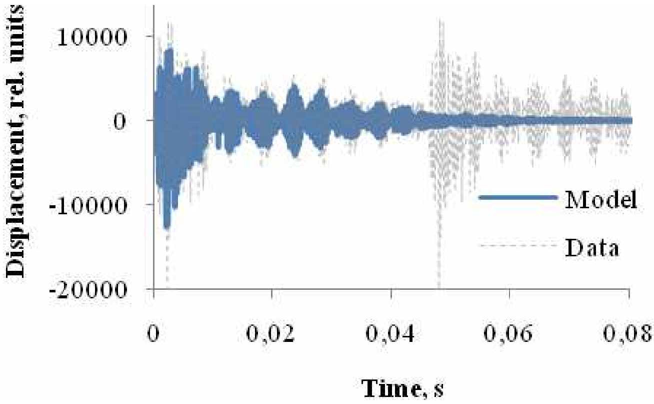
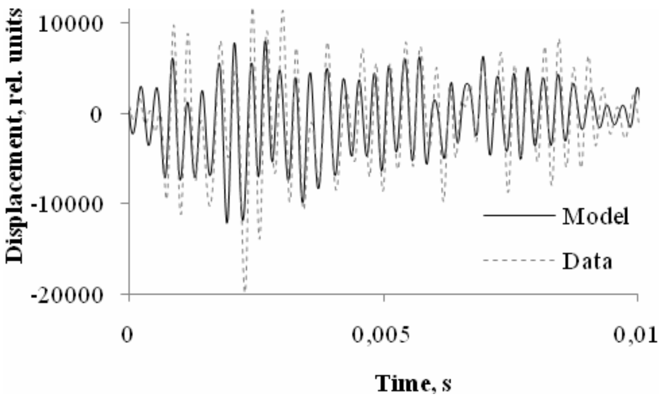
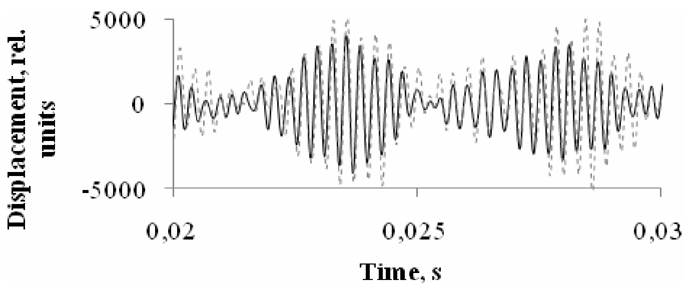


Fig. 6. The impulse response of the band in the fixed point

Fig. 7 presents more detailed comparison of the modeled impulse response and data.



a)



b)

Fig. 7. Comparison of the data and modeled impulse response:
a) in the time interval (0, 0.01); b) in the interval (0.02, 0.03)

In Fig. 9 we present the magnitude responses of the data from the time interval (0, 0.046) and the modeled impulse response.

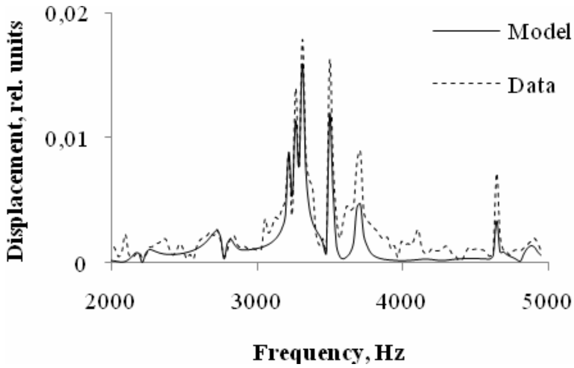


Fig. 8. The magnitude responses of the initial fragment of the data and the model

Fig. 9 demonstrates that the model sufficiently exactly matches real oscillations, although in order to fit the data exactly, one needs to add some three formants (see the frequency intervals 2000-2050 Hz, 3000-3050 Hz, 4000-4050 Hz.

6. Conclusions

A new oscillation modeling method based on signal modeling by quasi-polynomials using an iterative Levenberg optimization procedure has been presented when the data is the system response to a sequence of delta impulses. Mathematical formulas have been derived and an explicit parameter estimation method has been presented. Provided modeling examples demonstrate efficiency of the method. The paper presented the results of the elastic band impulse response modeling using hammer blow response data registered in a computer via the microphone input of the sound card.

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