438. Investigation of rotary system containing a flexible centrifugal coupling with controlled damping

A. Jakštas, B. Spruogis, V. Turla and E. Jurkonis
Vilnius Gediminas Technical University
J. Basanaviciaus 28, LT-03224, Vilnius, Lithuania
e-mail: *arunasj@me.vgtu.lt* *bsp@ti.vgtu.lt*
Vytautas.Turla@me.vgtu.lt *eugjurk@yahoo.com*

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Abstract. The possibility of isolation of the driving shaft of the vehicle from vibrations of the engine shaft using a coupling with a controlled damping element based on magnetorheological liquid and a supplemental disc connected to the driving or driven half–coupling is discussed herein. The impact of the parameters of the coupling and the feedback upon the amplitude–frequency response of the system is analyzed. It is demonstrated that effective damping of vibrations in the resonance zone is possible.

Keywords: rotary system, coupling, controlled damping

Introduction

One of the key factors that limit the accuracy of a precise mechanism is the impact vibrations that appear in it. Various elastic elements are used for their reduction. In case of rotating systems, flexible couplings are applied. Vibration-isolating features of usual couplings with constant parameters are thoroughly studied to the sufficient extent [1]. Couplings with controlled parameters provide new opportunities. Such coupling is adapted to changing excitation of the vibrations. The controlled damping forces can be generated using magnetorheological liquid [2]. It is used in shock–absorbers of vehicles [3] as well as in drives of mechatronic systems [4]. A controlled damping force is generated by the magnetorheological liquid that is crossed by magnetic field generated by the electromagnet. The liquid fills the gap between the parts of the coupling and thus a force resisting to movement of the surfaces with respect to each other that form the gap is generated.

When using this controlled damper, its imperfection is obvious: its vibration-isolating features change abruptly on transition from one mode to another, i.e. depending on presence or absence of current in the electromagnet. Although in the constructions discussed upon in the above–mentioned references the current is controlled by feedback, certain indetermination in analytical examination and designing of vibration–isolating systems of this type is unavoidable upon certain parameters of excitation, in particular, in transitional processes.

In the presented paper, a flexible coupling that allows eliminating the above-mentioned imperfection is considered. It differs from the known designs by the damping element that is connected to the remained part of the system via a supplemental elastic element.

The structure of the coupling under discussion

The half–couplings 1 and 2 of the coupling under discussion are connected to the shafts of the engine and the working machine. The half-couplings are connected to each other using freely deformable ring 3 that contacts with the pins 4 and 5 fixed to both half–couplings. This connection is analogous to the one used in the construction described in the reference [5].

In the coupling, the role of the supplemental mass is played by the disc 6 connected to the half–coupling 2 using elastic arc–type elements 7 that are fixed to the disc and the half–coupling by the pins 8 and 9. In the half–coupling 1, the electromagnet 10 is mounted; the lines of its magnetic field cross the gap 11 between the half–coupling 1 and the supplemental disc 6. The part of the gap 11 between the sealants 12 and 13 is filled with the magnetorheological liquid. Namely this liquid generates damping forces in the coupling. The link between the half–couplings is realized via the elastic elements 7 and the magnetorheological liquid; it is parallel to the elastic link formed by the freely deformable ring 3.

The electric current flowing in the coils of the electromagnet 10 is regulated depending on the vibrations of the engine shaft. The force resisting to movement of the surfaces that form the gap 11 with respect to each other depends of the current flowing in the coils. The current is controlled by the feedback impacted by the signals from the sensor of vibrations.

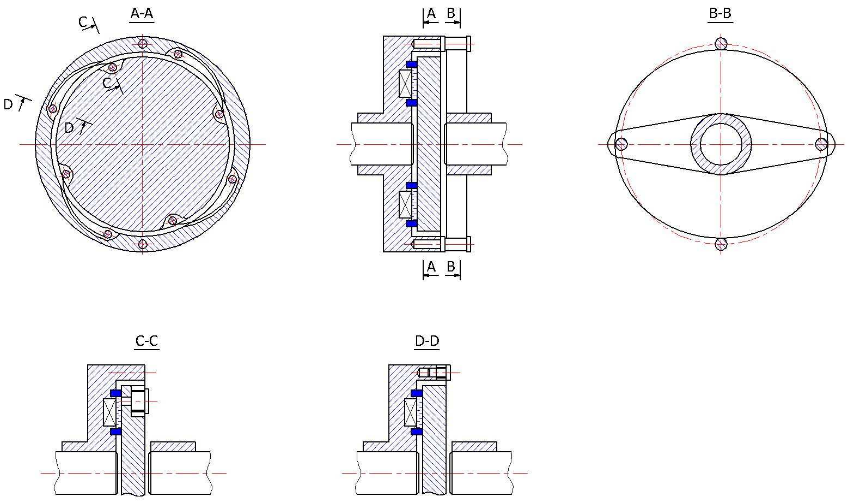

Fig. 1. Scheme of a coupling with an elastic disc

In the paper, the possibilities of the coupling to protect the sensitive mechanism against the rotational vibrations of the engine shaft are considered. In most cases, mounting of sensors of vibrations in a driven mechanism is more complicated than on the engine shaft, thus in our case the possibilities of vibration–isolation by controlling the current flowing in the coils of the electromagnet depending on the amplitude of the rotational vibrations of the engine shaft are discussed. For this purpose, an accelerometer oriented towards the directions of the tangent of the engine shaft can be used. Earlier research showed that the vibration–isolating systems of the type under discussion are more effective when the current flowing in the coils of the electromagnet is proportional to the amplitude of the vibrations [5]. So, the signal of the accelerometer should be integrated twice in the system of the feedback.

Two versions of using the coupling that are predetermined by the half–coupling chosen for connecting to the engine shaft are possible. If the half–coupling 1 is fixed to the engine shaft, the supplemental elastic link appears between the engine (the source of the vibrations) and the damping element. If the half–coupling 2 is fixed to the engine shaft, the supplemental elastic link appears between the isolated object (the working machine) and the damping element.

Mathematical model of the vibration–isolating system

The force resisting to the movement of the surfaces that form the gap filled with the ferromagnetic liquid with respect to each other practically does not depend on the speed of such movement, so it is described as a dry friction in analytical consideration. The vibration–isolating system examined in such a way can be considered an active vibration–isolating system with controlled dry friction [6]. The value of this friction force depends on the induction of the magnetic field in the gap, thus on the current flowing through the coils of the electromagnet. In the case under discussion, this current will depend on the amplitude of the rotational vibrations of the engine shaft. The moment of the friction forces that impacts the supplemental disc:

$$T_f = \tau Q \frac{dm}{2} = A\xi_0 Q \frac{dm}{2} , \qquad (1)$$

where τ – relative resistance to the movement of the surfaces with respect to each other (N/m^2) proportional to magnetic induction in the gap, Q – area of the gap filled with the magnetorheological liquid, A – coefficient of transfer of the feedback between the meter of vibrations and the current flowing in the electromagnet, ζ_0 – amplitude of rotational vibrations of the engine shaft.

In the case when the elastic link is formed between the engine shaft and the supplemental disc, i.e. the half–coupling 1 is fixed on the engine shaft (Fig. 1):

$$\begin{aligned} & I_1\ddot{\varphi}_1 + C_1\varphi + T_f sign\dot{\psi} = C_1\xi_0 \sin\dot{\psi} , \\ & I_2\left(\ddot{\varphi} - \ddot{\psi}\right) + C_2\left(\varphi - \psi\right) - T_f sign\dot{\psi} = \\ & = C_2\xi_0 \sin\omega t , \end{aligned} \qquad (2)$$

where I_1 – moment of inertia of the driven shaft, C_1 – stiffness of rotation of the principal elastic element (the freely deformable ring), ψ – angle of turning of the supplemental disc with respect to the half–coupling 1, I_2 – moment of inertia of the supplemental disc, C_2 – stiffness of rotation of the supplemental elastic element (arc–shaped elastic elements), ω – frequency of vibrations. The point over the variables means differentiation with respect to time.

In this paper, an approximate solution obtained on applying the method of harmonic linearization is used [7]. Then a member of the nonlinear system (2):

$$\frac{4T_f}{\pi\omega\psi_0}\psi , \qquad (3)$$

where ψ_0 – amplitude of vibrations of the supplemental disc with respect to the half–coupling.

In accordance with [8], the stiffness of rotation of the principal elastic link on the base of the freely deformable ring is:

$$C_1 = 96\frac{B}{C^2}\frac{EI}{R}\delta^2 , \qquad (4)$$

where B and C – coefficients that assess the initial deformation of the free ring, E – modulus of elasticity of the material of the ring, I – moment of inertia of the cross–section of the ring, δ – torsion angle of the coupling, R – initial radius of the ring.

The stiffness of rotation of the supplemental elastic element formed by arc–shaped elements is as follows:

$$C_2 = \frac{nEIR}{R_0^3 K} , \qquad (5)$$

where n – number of the elastic elements (in the Fig. 1 $n = 4$), R – radius of location of the pins in the half–coupling, R_0 – radius of the curvature of the arc, K – coefficient that takes into account ratios of the geometrical parameters of the elastic element.

If the supplemental arc–shaped elements are situated between the supplemental disc and the driven shaft, i.e. the half–coupling 2 is fixed on the engine shaft, the rotational vibrations of the systems are described by the following system of equations:

$$\begin{aligned} & I_1\ddot{\varphi} + C_1\varphi - C_2\left(\varphi - \chi\right) = \left(C_1 + C_2\right)\xi_0 \sin \omega t \ , \\ & I_2\ddot{\varphi} - C_1\left(\varphi - \chi\right) + T_f \operatorname{sign} \dot{\chi} = \\ & = \left(I_2\xi_0\omega^2 - C_2\xi_0\right)\sin \omega t \ , \end{aligned} \tag{6}$$

where χ – angle of turning of the supplemental disc with respect to the driven shaft. The other marking is the same as in (2).

If the method of harmonic linearization is applied, a nonlinear member:

$$\frac{4T_f}{\pi\chi_0\omega}\dot{\chi} \ , \tag{7}$$

where χ_0 – amplitude of the vibrations of the supplemental disc with respect to the driven shaft.

Research results and their analysis

In the vibration–isolating system under consideration, two modes are possible. In one case, the supplemental disc turns with respect to the half–coupling 1 and the moment of its forces of inertia will have vibration–isolating features of the coupling. In the other case, upon a strong magnetic induction, the cohesion forces appearing in the magnetorheological liquid steadily connect the supplemental disc and the half–coupling 1, i.e. the coupling turns into an ordinary flexible coupling and its stiffness of rotation is equal to $C_1 + C_2$.

Vibration–isolating features of the coupling are defined by the ratio of the amplitudes of vibrations of the driven shaft and the engine shaft:

$$\alpha = \frac{\varphi_0}{\xi_0} . \tag{8}$$

In the case of the first connection, i.e. when the supplemental arc–shaped elastic elements are between the engine shaft and the supplemental disc:

$$\alpha = \sqrt{\frac{\left(N - mz^2\right)^2\left(N-\mu\right)z^4 + \dfrac{8\delta^2}{\pi^2}\left\{\begin{bmatrix}\left(2\mu + \mu N + \mu^2\right)z^4 - \\ -\begin{pmatrix}2N + 3\mu N + \\ 2\mu + N^2\end{pmatrix}z^2 + \\ +N\left(1+2N\right)\end{bmatrix} \times \begin{bmatrix}\mu\left(N-\mu\right)z^4 - \\ N\left(N-\mu\right)z^2 + N\end{bmatrix}\right\}}{\left[\mu z^4 - \left(N+\mu\right)z^2 + N\right]^2\left(N-\mu\right)^2 z^4}} , \tag{9}$$

where $N = C_2/C_1$, $\mu = I_2/I_1$, $\delta = A/C_2$, $z = \sqrt{\dfrac{I_1\omega^2}{C_1}}$.

This solution will be valid when the supplemental disc can turn with respect to the half–coupling 1. Such mode exists, when:

$$\beta^2 = \frac{\left[\mu(1+N) - \left(N - \mu z^2\right)\right]^2 z^4 - \dfrac{8\delta^2}{\pi^2}\left[1 + N - (1+\mu) z^2\right]^2}{\left[\mu z^4 - (N-\mu) z^2 + N\right]^2}, \tag{10}$$

where

$$\beta = \frac{\psi_0}{\xi_0} \tag{11}$$

is positive.

Otherwise, the surfaces that form the gap shall not move against each other and the ratio of the amplitudes of vibrations of the driven shaft and the engine shaft:

$$\alpha = \sqrt{\frac{1+N}{\left[1 + N - (1+\mu) z^2\right]^2}}. \tag{12}$$

Vibrations of the supplemental disc with respect to the engine shaft will exist in one or two frequency ranges. They do not appear in the low frequency range because of low moment of the forces of inertia of the supplemental disk, i.e. the damping element does not operate. Damping, i.e. relative movement of the surfaces forming the gap, will appear when the certain frequency referred to as the opening frequency (z_{op}), is reached. The cohesion force in the range of higher frequencies is sufficient to ensure a steady link between the supplemental disc and the half–coupling 1. Such a mode is set when the so-called closing frequency (z_{ce}) is exceeded.

Namely, in the range between z_{op} and z_{ce} damping effect is manifested. If the parameters of the coupling are chosen properly, resonance vibrations may be damped efficiently.

In the range of higher frequencies, existence of a second opening frequency (z_{sop}) is possible: if this frequency is exceeded, the supplemental disc will move with respect to the half–coupling 1 again. The dependence of these frequencies on the parameters of the coupling is presented in Fig. 2.

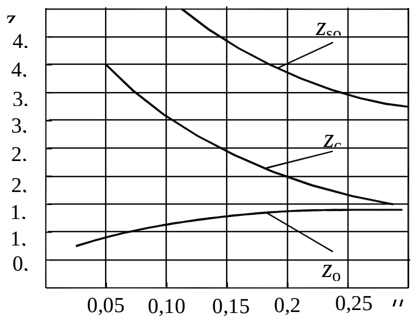

Fig. 2. Dependence of the frequencies z_{op}, z_{ce} and z_{zop} on the ratio of the moments of inertia (μ), when $N = 2$ and $\delta = \sqrt{2}$

In the Fig. 3, the dependence of α on frequency z according to (9), when $z_{op} < z < z_{ce}$ and $z > z_{sop}$ as well as according to (12) when $z < z_{op}$ and $z_{ce} < z < z_{sop}$, is provided.

It may be seen from the provided amplitude-frequency responses that significant increase of the amplitude of the vibrations in the resonance zone is avoided. In the case of a larger moment of inertia of the supplemental disc, an increase of vibrations in the range of high frequencies is possible because of the increased moment of the forces of inertia.

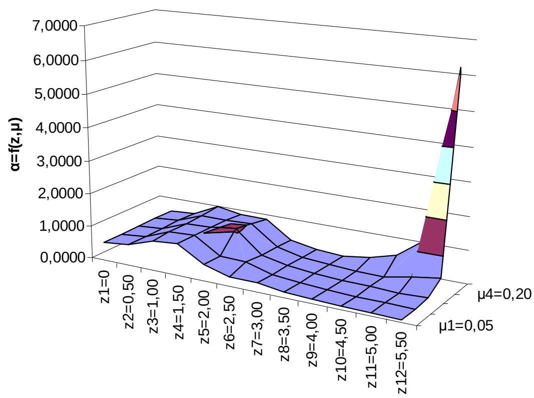

Fig. 3. Amplitude–frequency responses when the supplemental elastic element is between the engine shaft and the disc and $N = 2$, $\delta = \sqrt{2}$ upon various values of $\mu = I_2/I_1$

In the second case, when the half–coupling 1 is fixed to the driven shaft, the dependence of the ratio of the amplitudes of the driven shaft and the engine shaft on the frequency is determined by solution of the system (6) using the method of harmonic linearization:

$$\alpha = \sqrt{\frac{\left[\mu(1+N)+\left(\mu z^2-N\right)\right]^2 z^4\left(N-\mu z^2\right)^2 - \frac{8\delta^2}{\pi^2}\left\{\left[\begin{array}{l}2(1+N)\mu z^2 - \\ -\left(\begin{array}{l}4N\mu+2N+\\+\mu N^2+\mu+N\end{array}\right)z^2 + \\ +2N(1+N)\end{array}\right]\times \\ \times\left[-N\mu z^4+\left(\mu+\mu N^2+N^2\right)z^2\right]\right\}}{\left[\mu z^4-(N+\mu+\mu N)z^2+N\right]^2\times \\ \times\left[\mu(1+N)-\left(N-\mu z^2\right)\right]^2 z^4}}. \tag{13}$$

Here, the notations are analogous to those used in (9). The dependence (14) will exist, when:

$$\gamma^2 = \frac{\left[\mu(1+N)-\left(N-\mu z^2\right)\right]^2 z^4 - \frac{8\delta^2}{\pi^2}\left(1+N-z^2\right)^2}{\left[\mu z^4-(N+\mu+\mu N)z^2+N\right]^2} \rangle\, 0, \tag{14}$$

where

$$\gamma = \frac{\chi_0}{\xi_0}. \tag{15}$$

In the contrary case:

$$\alpha = \sqrt{\frac{1+N}{\left(1+N-z^2\right)^2}}. \qquad (16)$$

The analysis of the solutions revealed that in this case the range between the frequencies z_{op} and z_{ce} narrows and an increase of vibrations of the driven shaft in the resonance zone is inevitable. However, no increase of vibrations in the range of high frequencies was observed.

In the Fig. 4, amplitude–frequency response obtained according to (13) and (16) is provided.

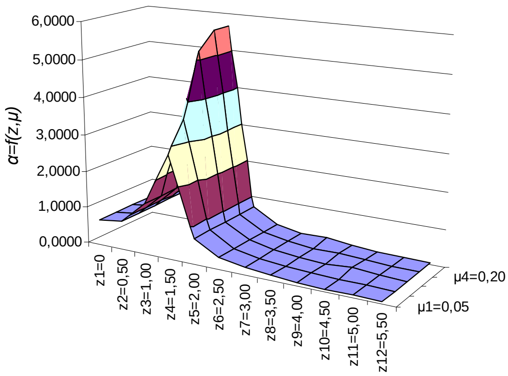

Fig. 4. Amplitude–frequency responses when the supplemental elastic element is between the driven shaft and the disc upon various values of $\mu = I_2/I_1$, $N = 2$, $\delta = \sqrt{2}$

Conclusions

1. A coupling with a damping element based on magnetorheological liquid and a supplemental disc having a feedback between the engine shaft and the damping element suppresses the vibrations of the driven shaft efficiently, if a proper combination of the parameters of the feedback and the coupling is chosen.
2. In the resonance zone the coupling is more efficient if the supplemental elastic element is inserted between the driving half–coupling and the supplemental disc.
3. If a supplemental elastic element is inserted between the supplemental disc and the driven half–coupling, the amplitude of the driven shaft does not increase in the range of high frequencies.

References

[1] **Spruogis B.** The devices of transmission and stabilization of rotary motion. Monograph. Vilnius: Technika, 1997. 476 p.
[2] **Bica, L.** Advances in magnetorheological suspension: Production and properties. Journal of Industrial and Engineering Chemistry. V. 12, Issue 4, July 2006, p. 501–515.
[3] **Kum-Gil Sung, Young-Min Han, Jae-Wan Cho and Seung-Bok-Choi.** Vibration control of vehicle ER suspension system using furry moving sliding controller. Journal of Sound and Vibration. V. 311, Issues 3-5, 8 April 2008, p. 1004–1019.
[4] **Spruogis B., Turla V., Jakštas A.** Universal coupling for shaft of actuators in mechatronic systems. Solid State Phenomena: Mechatronic Systems and Materials: a collection of papers from the 1st international conference (MSM 2005), Vilnius, Lithuania, 20–23 October 2005.
[5] **Kavolėlis A.-P. and others.** Centrifugal Elastic Coupling. USA Patent No. 4118953. Washington, 1978.
[6] **Spruogis B., Jakštas A.** Research of the Efficiency of Use of Shock Absorber with Controlled Dissipative Force in Motor Cars. Transportas – Transport Engineering. V. XVI, No. 4, 2001.
[7] **Kelly, S. Graham.** Advanced Vibration Analysis / S. Graham Kelly Mechanical Engineering; 204. Boca Raton (Fla.): CRC/Taylor & Francis, 2007.