435. Stability control of a hyperredundant arm for a grasping operation

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Abstract: In this paper a problem of a class of hyperredundant arms with continuum elements that perform the grasping function by coiling is discussed. This function is often met in the animal world as in the case of elephant trunk or octopus tentacle. First, the dynamic model in 3D-space is developed. The equations that describe the motion of the arm that carries a load by coiling are inferred. The stability of the motion is discussed. Numerical simulations of the motion towards an imposed target are presented.

Keywords: distributed parameter systems, force control, grasping, hyperredundant robots.

Introduction

Hyperredundant arms are a class of arms that can achieve any position and orientation in 3D-space. The control of these systems is very complex and a great number of researchers have tried to offer solutions. In [2], Gravagne analyzed the kinematical model of "hyper-redundant" robots, known as "continuum" robots. Important results were obtained by Chirikjian and Burdick [3]-[5] which laid the foundations for the kinematical theory of hyper-redundant robots. Their results are based on a "backbone curve" that captures the macroscopic geometric features of a robot. The inverse kinematical problem is reduced to determining the time varying backbone curve behavior. New methods for determining "optimal" hyper-redundant manipulator configurations based on a continuous formulation of kinematics are developed. Mochiyama has also investigated the problem of controlling the shape of an HDOF rigid-link robot with two-degree-of-freedom joints using spatial curves [6], [7]. [8, 9] presents the state of the art of the continuum robots, outline their areas of application and introduce some control issues.

In other papers [10, 11] several technological solutions for actuators used in hyper-redundant structures are presented and conventional control systems are introduced.

In this paper, the problem of a class of hyperredundant arms with continuum elements that performs the grasping function by coiling is discussed. This function is often met in the animal world as in the case of the elephant trunk (Fig. 1), octopus tentacle or constrictor snakes. First, the dynamical model presented in [2] is developed for 3D-space. The equations that describe the motion of the arm that carries a load by coiling are inferred.

The paper is organized as follows: section II presents the hyperredundant structure with continuum elements; section III studies the dynamic model in 3D-space; section IV presents the control algorithm; section V verifies the control laws by means of computer simulations.

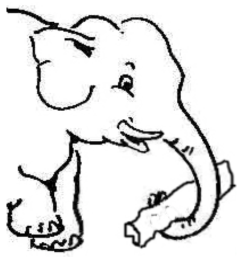

Fig. 1. Elephant trunk

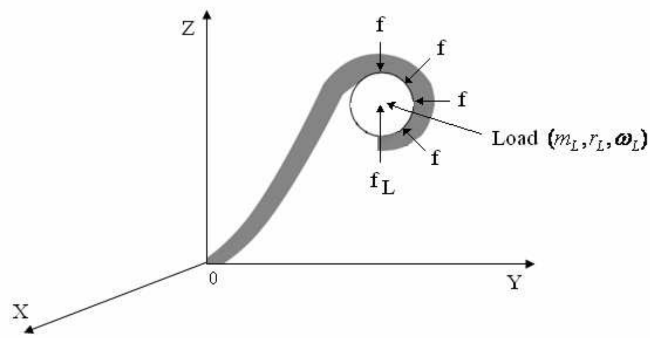

Fig. 2. Distribution of forces around the object-load

Technological model

The paper studies a class of hyperredundant arms, that can achieve any position and orientation in 3D space, and that can perform a coil function for the grasping (Fig. 2). The arm is a high degree of freedom structure or a continuum structure.

Technologically, these arms are based on the use of flexible composite materials in conjunction with active controllable electro-rheological (ER) fluids that can change their mechanical characteristics in the presence of electrical fields.

The general form of the arm is presented in Fig. 3. It consists of a number (N) of elements, cylinders mode of fiber-reinforced rubber. There are four internal chambers in the cylinder, each of them containing the ER fluid with an individual control circuit. The deformation in each cylinder is controlled by an independent electrohydraulic pressure control system combined with the distributed viscosity control of the ER fluid. The chambers of the segment have reinforced rubber walls with fiber in a circular direction. Thus, it is easy to deform it in axial direction while it resists deformation in the radial direction. The cylinder can be bent in any direction by appropriately controlling the pressure in the four chambers. The electrical control of the ER fluid viscosity is obtained by an electrode network distributed on the length of the cylinder.

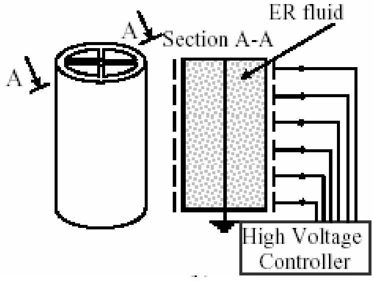

Fig. 3. The cylinder structure

The technological model can be considered as one with a control, highly flexible and elastic backbone. We shall assume that the backbone never bends beyond the “small-strain region” where an applied stress produces a strain that is recoverable and observes approximately linear stress-strain relationship. Similarly, the system is frictionless and any other damping and friction are neglected.

The last m elements (m<N) represent the grasping terminal. These elements contain a number of force sensors distributed on the surface on the cylinders. These sensors measure the contact with the load ensure the distributed force control during the grasping. The sensor network constitutes a number of impedance devices [12] (see Fig. 3) that define the dynamic relationship between the grasping element displacement and the contact force.

Theoretical model

The essence of the hyperredundant model is a 3-dimensional backbone curve *C*. The independent parameter *s* is related to the arc-length from the origin of the curve *C*, $s \in [0, \quad L]$, where: $L = \sum_{i=1}^{N} l_i$ and l_i represent the length of the elements *i* of the arm in the initial position. The position of the point *s* on curve *C* is defined by the position vector $r = r(s)$, $s \in [0, \quad L]$ and the orientation is given by two continuum angles $\theta(s)$ and $q(s)$. It is assumed that the bending of the element is produced by the fluid pressure control in the θ-plane chamber and then *q*-plane chamber (Fig. 4).

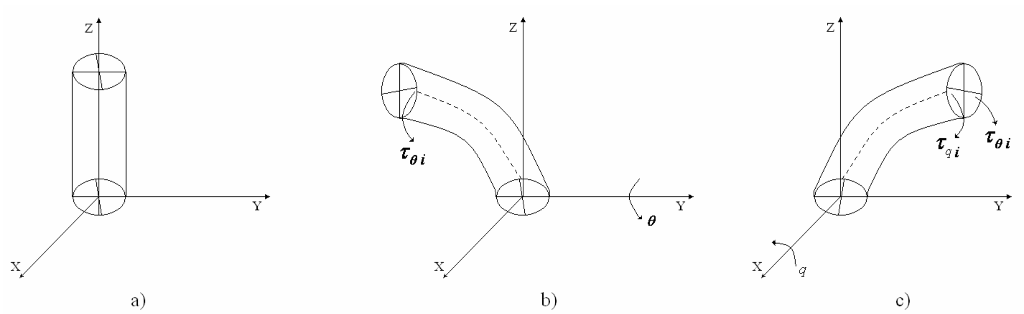

Fig. 4. The tentacle movements: a) initial position; b) step 1: θ – plane bending; c) step 2: *q* – plane bending

The position vector on curve *C* is given by:

$$r(s) = [x(s) \quad y(s) \quad z(s)]^T, \tag{1}$$

where $x(s) = \int_0^s \sin\theta(s')ds'$,

$$y(s) = -\int_0^s \sin q(s')\cos\theta(s')ds', \tag{2}$$

$$z(s) = \int_0^s \cos q(s')\cos\theta(s')ds', \; s' \in [0, \quad s]$$

For a dynamic motion, the time variable will be introduced, $r = r(s,t)$. The arm has equivalent bending stiffness *EI* with linear mass density ρ and rotational inertial density *I*.

a) No load arm

The kinetic and potential energies are:

$$T = \frac{1}{2}\int_0^L I\left(\dot{\theta}^2 + \dot{q}^2 + \rho\left\|\dot{r}^2\right\|\right)ds\,, \tag{3}$$

$$V = \frac{1}{2}\int_0^L EI\left[\left(\frac{\partial\theta}{\partial s}\right)^2 + \left(\frac{\partial q}{\partial s}\right)^2\right]ds\,, \tag{4}$$

where $\dot{r}$, $\dot{\theta}$, $\dot{q}$ denote $\frac{\partial r(t,s)}{\partial t}$, $\frac{\partial \theta(t,s)}{\partial t}$, $\frac{\partial q(t,s)}{\partial t}$, respectively.

The bending of the arm is determined by the distributed torques $\tau_{\theta i}$, τ_{qi}, $i = 1,2,\ldots,N$ in each chamber of the cylinder:

$$\tau_{\theta i} = \left(p_{\theta i1} - p_{\theta i2}\right)\cdot\frac{\pi d^2}{16} \tag{5}$$

$$\tau_{qi} = \left(p_{qi1} - p_{q\theta i2}\right)\cdot\frac{\pi d^2}{16} \tag{6}$$

where $p_{\theta i1}$, $p_{\theta i2}$, p_{qi1}, p_{qi2} are the fluid pressures in each θ, q – pair of chambers and *d* is the diameter of the cylinder.

The distributed moments can be defined as:

$$\mathfrak{M}_\theta\left(s,t\right) = \sum_{i=1}^{N-1}\tau_{\theta i}\left(t\right)\delta\left(s - s_i\right) \tag{7}$$

$$\mathfrak{M}_q\left(s,t\right) = \sum_{i=1}^{N-1}\tau_{qi}\left(t\right)\delta\left(s - s_i\right) \tag{8}$$

with $s_i = \sum_{k=1}^{i} l_k$, $i = 1,2,\ldots,(N-1)$.

The arm can be also considered as a linear viscoelastic damping mechanism with damping coefficient *b*.

The grasping force $f(s)$ is a distributed force along the last elements of the arm (Fig. 2). We denote by ω the generalized coordinate vector:

$$\omega = \begin{bmatrix}\theta \\ q\end{bmatrix} \tag{9}$$

and the moment vector as:

$$\mathfrak{M} = \begin{bmatrix} m_\theta \\ m_q \end{bmatrix} \tag{10}$$

Using the same procedure as in [2], the dynamic model of the arm can be derived as:

$$I\ddot{\omega} + b\dot{\omega} - EI\frac{\partial^2\omega}{\partial s^2} = A^T f + \mathfrak{M} \tag{11}$$

$$\rho\ddot{r} = \frac{\partial f}{\partial s} \tag{12}$$

with the boundary conditions:

$$EI\,\frac{\partial \theta(t,L)}{\partial s} = \tau_{\theta L};\;\; EI\,\frac{\partial q(t,L)}{\partial s} = \tau_{qL} \qquad (13)$$

where $$A = \begin{bmatrix} \cos\theta & 0 \\ \sin q \sin\theta & -\cos q \cos\theta \\ -\cos q \sin\theta & -\sin q \cos\theta \end{bmatrix} \qquad (14)$$

b) Load arm

The load of the arm is represented by a circular body with diameter d_L and mass m_L. The position of the load can be approximated by the vector $r_L(t) = r(L,t)$. We denote by ω_L, f_L, $\tau_{\theta L}$, τ_{qL} the positions, force and the torques at the end point of the arm:

$$\begin{cases} \omega_L(t) = \omega_L(L,t) = \begin{bmatrix} \theta(L,t) \\ q(L,t) \end{bmatrix} \\ f_L(t) = f(L,t) \\ \tau_{\theta L}(t) = \tau_\theta(L,t) \\ \tau_{qL}(t) = \tau_q(L,t) \end{cases} \qquad (15)$$

The kinetic energy will be:

$$T = \frac{1}{2}\int_0^1 \left[I\left(\dot\theta^2 + \dot q^2\right) + \rho\|\dot r\|^2 \right] ds + \frac{1}{2} m_L \|\dot r_L\|^2 + + \frac{1}{2} I_{\theta L}\dot\theta_L^2 + \frac{1}{2} I_{qL}\dot q_L^2 \qquad (16)$$

where $I_{\theta L}$ and I_{qL} represent the inertia moments of the load with respect to θ and q rotations, respectively.

We will discuss a light-weight arm in which the gravitational component of the arm is neglected with respect to the load. The potential gravitational energy will be:

$$V_G = \int_0^L m_L g \cos q \cos\theta ds \qquad (17)$$

From (4), (16), (17), the dynamic model can be easily derived,

$$I\ddot\omega + b\dot\omega - EI\,\frac{\partial^2 \omega}{\partial s^2} + m_L g C = A^T f + \mathfrak{M} \qquad (18)$$

$$\rho \ddot r = \frac{\partial f}{\partial s} \qquad (19)$$

$$I_L \ddot\omega_L + EI\,\frac{\partial \dot\omega_L}{\partial s} = \tau_L \qquad (20)$$

$$m_L \ddot r_L + f_L = 0 \qquad (21)$$

where $$C = \begin{bmatrix} -\sin q \cos\theta & 0 \\ 0 & -\cos q \sin\theta \end{bmatrix} \tag{22}$$

$$I_L = diag\left(I_{\theta L}, I_{qL}\right) \tag{23}$$

Control algorithm

We consider that the initial state of the system is given by:

$$\omega_0 = \omega(0,s) = \left[\theta_0, \quad q_0\right]^T \tag{24}$$

$$\dot{\omega}(0,s) = \left[\dot{\theta}_0, \quad \dot{q}_0\right]^T = \left[0, \quad 0\right]^T \tag{25}$$

where $$\begin{cases} \theta_0 = \theta(0,s) \\ q_0 = q(0,s) \end{cases} \tag{26}$$

corresponding to the initial position of the arm defined by the curve C_0:

$$C_0 : \left(\theta_0(s), \quad q_0(s)\right),\ s \in [0, \quad L] \tag{27}$$

The desired position is represented by the curve C_d:

$$C_d : \left(\theta_d(s), \quad q_d(s)\right),\ s \in [0, \quad L] \tag{28}$$

with $$\dot{\omega}_d = [0, \quad 0]^T \tag{29}$$

We define by $e_\theta(t,s)$, $e_q(t,s)$ and $e(t)$ the position errors:

$$e_\theta(t,s) = \theta(s,t) - \theta_d(s) \tag{30}$$
$$e_q(t,s) = q(s,t) - q_d(s), \text{ with } s \in [0, \quad L] \tag{31}$$

or the global error:

$$e(t) = \int_0^L \left[\left(\theta(s,t) - \theta_d(s)\right) + \left(q(s,t) - q_d(s)\right)\right] ds \tag{32}$$

Theorem. The control system of the position for a load arm in a grasping function by coiling is stable if the distributed torqueses have the form:

$$\tau_{\theta i} = \tau_{d\theta i} - k_{d\theta i}\dot{e}_{\theta i}(s_i, t) - k_{p\theta i} e_{\theta i}(s_i, t) \tag{33}$$

$$\tau_{qi} = \tau_{dqi} - k_{dqi}\dot{e}_{qi}(s_i, t) - k_{pqi} e_{qi}(s_i, t) \tag{34}$$

where $\tau_{d\theta i}$ and τ_{dqi} are the desired static holding torques [3, 4] and $k_{d\theta i}$, $k_{p\theta i}$, k_{dqi}, k_{pqi} are positive control coefficients.

Proof. See Appendix.

Simulation

A hyperredundant manipulator with 6 elements is considered. The position control problem is first analyzed for the initial position $r_0 = (4.5, \quad 0, \quad -0.5)$ and the final position

$r_F = (1,\ \ 3.5,\ \ 3.5)$. A discretization for each element with an increment $\Delta = \frac{l}{6}$ is introduced and a MATLAB code is applied. The result is presented in Fig. 5.

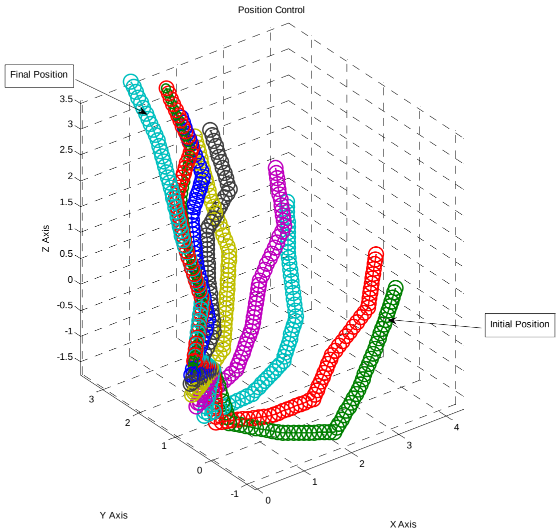

Fig. 5. No load position control

A control algorithm for an arm with 11 elements that carries a circular load by coiling in θ-plane are simulated in Fig. 6. The initial position is given by $r_0 = (5,\ \ 0,\ \ 0)$ and the final position is $r_F = (1.5,\ \ 0,\ \ 5)$.

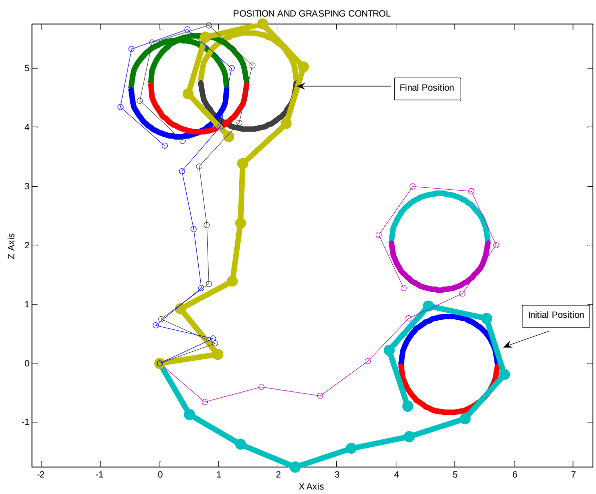

Fig. 6. 2D control of position and grasping with load

Then, the same algorithm is applied for a 11-element arm in 3D space, with a circular load between initial position $r_0 = (4,\ \ 2,\ \ 1)$ and final position $r_F = (1,\ \ 3,\ \ 4)$. The result of the simulation is presented in Fig. 7.

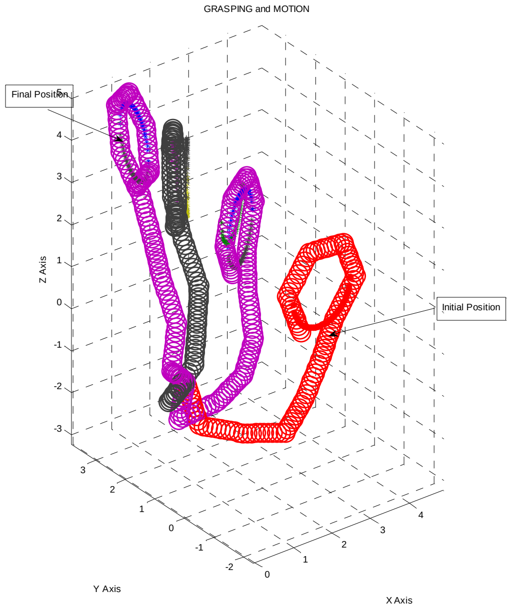

Fig. 7. Grasping the load and motion with the load

Conclusion

The paper considers the control problem of a hyperredundant robot with continuum elements that perform the coil function for grasping. The structure of the arm consists of flexible composite materials in conjunction with active – controllable electro-rheological fluids. The dynamic model in 3D space is developed. The equations that describe the motion of the arm that carries a load by coiling are inferred. The stability of the motion is discussed. Numerical simulations of the motion towards an imposed target are presented.

Appendix

We consider the following Lyapunov functional [2]:

$$V(t)=\frac{1}{2}\int_0^l\left(\rho\dot{r}^T\dot{r}+\dot{\omega}^T I\dot{\omega}+EI\left(\frac{\partial e}{\partial s}\right)^2\right)ds+ \\ +\frac{1}{2}m_L\dot{r}_L^T\dot{r}_L+\frac{1}{2}\dot{\omega}_L^T I_L\dot{\omega}_L+\frac{1}{2}\sum_{i=1}^{n}k_{pi}e_i^2(s_i,t) \tag{A.1}$$

The time derivative will be as follows:

$$\dot{V}(t)=\int_0^l\left(\rho\dot{r}^T\ddot{r}+\dot{\omega}^T I\ddot{\omega}+EI\cdot\frac{\partial e}{\partial s}\cdot\frac{\partial \dot{e}}{\partial s}\right)ds+ \\ +m_L\dot{r}_L^T\ddot{r}_L+\dot{\omega}_L^T I_L\ddot{\omega}_L+\sum_{i=1}^{n}k_{pi}e_i(s_i,t)\dot{e}_i(s_i,t) \tag{A.2}$$

For a desired position:

$$\omega_d=[\theta_d,\quad q_d]^T=const.$$

and using the dynamic model (18), (19) and the boundary condition conditions (20), (21), the relation (A.2) can be rewritten as:

$$\dot{V}(t)=\int_0^l \begin{pmatrix} -\dot{\omega}^T A^T f - EI\dot{e}^T \cdot \dfrac{\partial^2 e}{\partial s^2} + \\ + \dot{\omega}^T \begin{pmatrix} -b\dot{\omega} + EI\,\dfrac{\partial^2 \omega}{\partial s^2} - \\ - m_L gC + A^T f + \eta \end{pmatrix} \end{pmatrix} ds +$$
$$+\dot{\omega}_L^T\left(\tau_L - I_L\ddot{\omega}_L\right)+ \tag{A.3}$$
$$+\sum_{i=1}^{N} k_{pi} e_i\left(s_i,t\right)\dot{e}_i\left(s_i,t\right)+\dot{\omega}_L^T I_L \dot{\omega}_L + m_L \dot{r}_L^T \ddot{r}_L$$

Substituting the control (…) in (A.3), we obtain:

$$\dot{V}(t) = -\sum_{i=1}^{N} k_{vi}\dot{\omega}_i^T \omega_i \le 0 \tag{A.4}$$

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