432. Vibratory positioning and search of automatically assembled parts on a horizontally vibrating plane

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Abstract. Part motion on a horizontal plane, which is excited in perpendicular directions according to harmonic law, is analyzed in this paper, considering an unconstrained part on the plane and a part having elastic and damping constraints. Motion trajectories of the part were investigated, when the plane vibrates along a circular, elliptical or complex trajectory, which is obtained by subjecting the plane to excitation of different frequency. Based on the character of motion trajectories and taking into account the parameters of the dynamic system, 5 regimes of body motion were defined. Zones of existence of these regimes were established. Characteristics of body motion were determined as functions of coefficient of friction between the part and the plane, excitation amplitude, frequency and phase shift of the excitation signals.

Keywords: vibrations, assembly automation, positioning, search.

Introduction

For assembly automation it is necessary to accomplish operations of parts feeding into assembly position, orienting and positioning, matching, joining and removing assembled units from the working area. Matching of connective surfaces is the main stage of automated assembly during which the parts are matched the way they can be assembled without difficulties. In recent years the manipulation of the parts and assembly automation are performed using vibrations. The method of vibratory search can be used for matching of parts connective surfaces [1]. During the search one part must move along certain trajectories in respect to the other part by the plane, which is perpendicular to the connection axis. The centre of the connective part must fall into the zone of allowable error, which is defined by the clearance between the parts under assembly, the size of the chamfers and the axial tilt angle. For one of the parts being assembled, the search motion can be provided by a robot or a manipulator. A less expensive and simpler is a method of vibratory search, which is based on the body motion on the horizontally vibrating plane. When the plane vibrates, due to the friction forces between the surfaces of the part and the plane, the body on the plane can move into the predetermined position along different trajectories and to perform the search motion in this position by a circular, elliptic or more complex trajectory.

The transportation and orientation of the parts on the horizontal plane have been investigated by numerous researchers. D.S. Reznik and J.F. Canny [2,3] were experimentally investigating the manipulation of a part on a horizontal plane which is excited by four linear motors. When the plane is moving in complex trajectories the friction force fields are created. Because of this the parts located on the plane at different places may move in different trajectories. Winncy Y. Du has proposed a device for the vibratory transportation of the parts [4], which main component is a horizontally-vibrating thin plate. The plate is excited by modified pulse-width modulation signals. Thereby the part put on the plane can be transferred

to the required position. This device can transport different parts and it has a simple construction but it is impossible to change the direction of transportation and the motion trajectory. Horizontally vibrating planes are widely applied not only for the vibratory search but also for the group transportation of parts. Vibration lines are usually used for the transportation of powdery products or uniform parts. Frei P. investigates a line that transports multiple objects along different trajectories [5,6]. The plane of the conveyor consists of separate segments that vibrate both horizontally and vertically (in two perpendicular directions). By applying the mentioned principle it is possible to orient, sort out and transport the parts at different velocities along different trajectories. Bohringer K.-F. has proposed the mechanism for parts orientation, which systematically performs orientation and localization of the parts [7]. The elastic vibrations of the plane are excited for that purpose. Changing the frequencies of the vibrations, different modes of the plane vibrations are obtained. The orientation of the parts and the order depend on the vibration nodes zone. Fedaravičius A. and Tarasevičius K. investigated motion of a body on a vibrating platform under periodically controlled effective coefficient of friction [8]. Circular motion was provided to the platform. Controlling dry friction makes it possible to change the direction and velocity of the body being transported. The moving disc-shaped part displacement and the turn-off angle dependences on excitation parameters have been determined experimentally. In this paper, transient regimes of body motion and dependence of body motion trajectories on the system parameters have not been analyzed.

In the aforementioned works the manipulation of parts on vibrating plane is analyzed by averaging moment friction forces when any instantaneous planar motion of the plane corresponds to a rotation about a point. When the part is moving on the plane inertia forces appear because of relative and translational acceleration. Those inertia forces determine the character of motion. Various aspects of manipulation on a vibrating plane are considered in many works taking into account inertia forces. Therefore the problems that are still unstudied are related to the regimes of part motion on vibrating plane, the influence of excitation parameters on a character of part motion trajectory and displacement direction. In addition, research efforts are needed in field of manipulation of the elastically constrained and damped part on the vibrating plane. This case is typical for automatic assembly when one part is movably based on a plane and performs search motion with respect to other part.

The aim of this paper is by taking into account dynamic processes investigate theoretically the motion of cylindrical part on a horizontally vibrating plane and to establish motion regimes that are most suitable for manipulation of the parts being assembled automatically. Two cases are under consideration:

1. the part is unconstrained;
2. the part is elastically constrained and damped.

The plane is excited in two perpendicular directions. By changing the excitation amplitude, frequency, the phase of the excitation signals and the coefficient of friction between the part and the plane, the part can be easily and quickly redirected and provided with search motion of different trajectories, and this way the matching of connective surfaces is possible.

1. Equations of motion

For matching of the connective surfaces of the parts the vibratory search method, based on the movement of the part on the vibrating plane, is proposed. The plane is provided with vibratory excitation along two perpendicular directions. By changing the amplitudes of the perpendicular components of the vibrations, frequencies and phase shift between the components of vibrations, it is possible to obtain different kinds of plane motion law. The character of the part motion on the plane depends on the plane motion law.

The motion of the cylindrical body on the vibrating plane is investigated. The assumption is made that the diameter of the body is not large and its mass is concentrated in the mass center

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of the body. In this case the body may be assumed as a flat part. On the horizontal plane the immovable system of coordinates' $\xi O \eta$ is situated. The coordinate system $x O_1 y$ is related to the vibrating plane. The axes x and y are parallel to the ξ and η axes accordingly (Fig.1, a). It is assumed that the plane is moving uniformly in the $\xi O \eta$ coordinate system in a way that each point of it draws a circle of a radius R_e. Thus, the movement of any point of the plane is determined by such equations:

$$\begin{cases} \xi = \xi_0 + R_e \cos \omega t, \\ \eta = \eta_0 + R_e \sin \omega t, \end{cases} \tag{1}$$

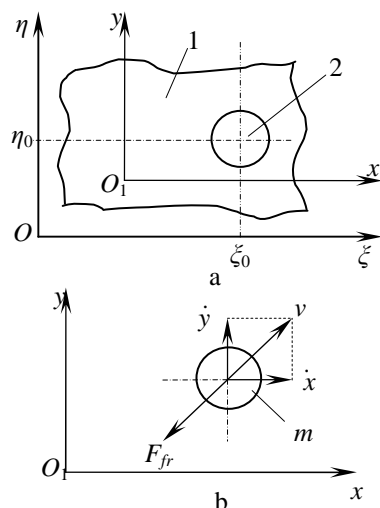where ω is the frequency of harmonic excitation; t is time.

Fig. 1. Dynamic scheme of the vibratory search: a - arrangement of the axes, *1* – plane, *2* – cylindrical part; b – direction of friction force acting on the body

On the plane there is a body of mass m, which mass centre coordinates in the movable system are denoted by x and y. When the body is sliding in respect to the plane, $x=x(t)$ and $y=y(t)$ and projections of mass centre acceleration, are expressed by two components:

$$\begin{cases} a_\xi = \ddot{x} + \ddot{\xi} = \ddot{x} - R_e \omega^2 \cos \omega t, \\ a_\eta = \ddot{y} + \ddot{\eta} = \ddot{y} - R_e \omega^2 \sin \omega t, \end{cases} \tag{2}$$

The first component determines the relative acceleration of the body, and the second component determines the acceleration of translation.

The body moving on a plane is influenced by resistance dry friction force F_{fr}, which has the direction opposite to the relative velocity. It may be assumed that friction force is hardly related to the speed of the body motion. As the body performs non-stop movement on a vibrating plane the rest friction may not be taken into account and it may be considered that the friction coefficient between the contacting surfaces is equal to the sliding friction coefficient. The projections of the relative velocity of the body are denoted as $\dot{x}$ and $\dot{y}$. Thereby the projections of the friction force onto the axes ξ and η (as well as onto the axes x and y) are as follows:

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$$\begin{cases} F_\xi = -\mu mg \, cos(v, x) = -\mu mg \, \dfrac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}}, \\[3mm] F_\eta = -\mu mg \, cos(v, y) = -\mu mg \, \dfrac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}}, \end{cases} \qquad (3)$$

where μ is the coefficient of sliding friction between the surfaces of the part and the plane.

Using the (3) and (4) expressions differential equations of the body motion on the plane are obtained:

$$\begin{cases} \ddot{x} + \mu g \, \dfrac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = R_e \omega^2 \, cos \, \omega t, \\[3mm] \ddot{y} + \mu g \, \dfrac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = R_e \omega^2 \, sin \, \omega t, \end{cases} \qquad (4)$$

These equations are valid when the body is sliding on the plane, i.e. when $\dot{x}^2 + \dot{y}^2 \neq 0$.

The body on the vibrating plane has two characteristic stages of motion – transient and steady state. During the transient stage the body slides on the plane following the particular trajectory. Then steady circular, elliptical or other motion trajectory is taking place. Equation (10) defines the radius of the steady circular trajectory of the body motion. Both stages of the body motion on the vibrating plane have been specifically investigated by numerical simulation of dynamic equations.

2. Unconstrained part motion on a plane

Motion of the part being assembled automatically on a vibrating plane is under consideration. The force of friction acts against the moving part and has the direction opposite to the velocity vector direction. Plane excitation by harmonic vibrations enables preliminary positioning of the part on the plane with respect to the other connective part and because of search motion of the part along the circular, elliptic or other trajectory assures matching of connective surfaces of the parts being assembled.

To solve equations of motion, MATLAB software was applied and the calculation code was developed. Results of mathematical simulation demonstrated that the part moving from the initial point towards the positioning point has characteristic transient and steady regimes of motion (Fig. 2). The character of the trajectory for transient regime depends both on excitation amplitude and frequency as well as on the coefficient of friction and initial velocity. The part can move to a predetermined position along a circular looping trajectory (Fig. 2), or along a curvilinear or linear trajectory. As the plane vibrates by a circular trajectory, the trajectory of steady motion of the part is also circular (Fig. 2). Relative to the plane motion, the part repeats the trajectory of the plane, though the part motion is slightly delayed in respect to the plane motion. It is possible to control the direction of motion by changing the initial phase of excitation signals (Fig. 3).

For accurate and quick matching of connective surfaces of the parts, it is necessary to identify what parameters have influence on the radius of circular steady motion, because this radius predetermines the search zone of the connective surfaces. As the amplitude of excitation increases the radius of the part motion trajectory also increases (Fig. 4, a). When the coefficient of friction increases the radius decreases and when excitation frequency increases radius of part motion trajectory increases as well (Fig. 4, b).
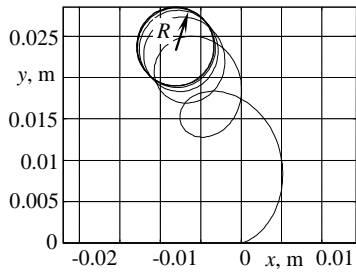
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Fig. 2. Trajectory of the part on a vibrating plane during transient and steady motion regimes
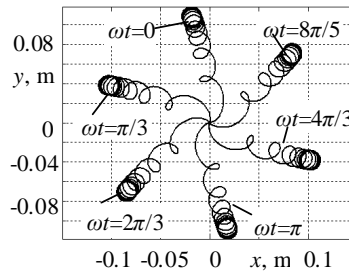

Fig. 3. Part motion trajectory versus initial phase of excitation signals, as $\mu=0.2$, $R_e=0.01$ m, $\omega=35$ s^{-1}
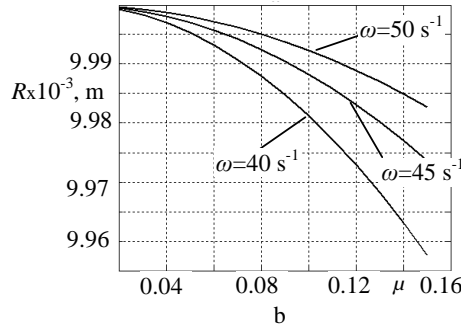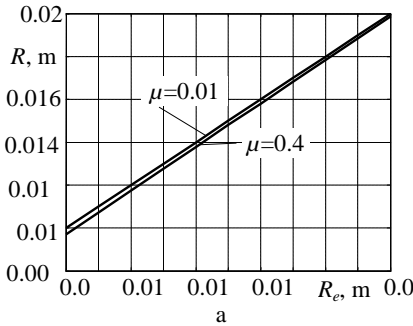

Fig. 4. Dependence of circular trajectory radius R: a - on excitation amplitude R_e and friction coefficient μ, as $\omega=40$ s^{-1}; b - on friction coefficient μ and excitation frequency ω, as $R_e=0.01$ m

The character of the part motion trajectory depends on the coefficient of friction (Fig. 5). As friction coefficient μ increases, displacement of the part from the initial position to position of steady motion diminishes (Fig. 5 a and b). When $\mu=0.08$, the part moves along an unwinding helix trajectory (Fig. 5, c), and further increasing friction coefficient part no longer moves but only rotates circularly (Fig. 5, d). The character of the trajectory is influenced not only by friction coefficient μ, but also by excitation amplitude R_e and frequency ω. Under higher values of excitation frequency or/and amplitude, the part can again move following the circular trajectory.
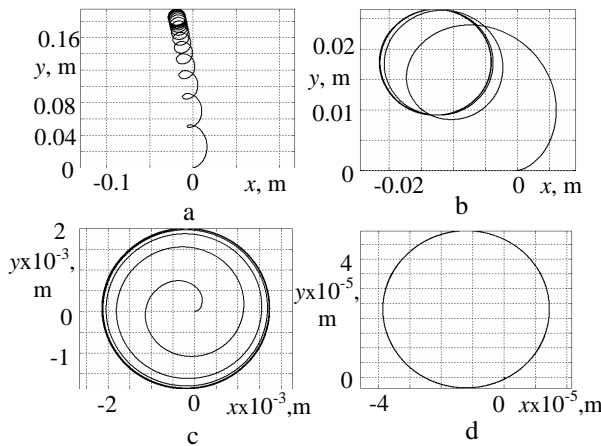

Fig. 5. Character of the trajectories, as $R_e=0.01$ m, $\omega=10$ s^{-1}: a - $\mu=0.01$, b - $\mu=0.05$, c - $\mu=0.1$, d - $\mu=0.15$

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Based on motion trajectories and taking into account sets of parameters R_e, and ω it is possible to determine 5 regimes of body motion. The zones of the existence of these regimes were established (Fig. 6). With the first zone parameters the part initially moves along a loop trajectory, which later changes into a circular motion (Fig. 5, a). Within the second zone the part moves rotating along an arch of a circle, and the distance from the initial position to the centre of steady circular trajectory is larger than the radius of the mentioned circle (Fig. 5, b). In the third zone the trajectory of part motion is unwinding helix (Fig. 5, c). The fourth zone characterizes circular motion of the body (Fig. 5, d) without the transient stage of motion. When parameters R_e and ω are within the fifth zone, the part performs chaotic motion, because both the excitation amplitude and frequency are too small for the body to be able to move along a circular trajectory.

In assembly automation, the main criterion used to define the efficiency of the assembly process is the duration of the parts assembly. Commonly, the assembly duration is highly affected by the positioning duration, i.e. the time while the motion trajectory settles into a steady state. It was determined that when increasing both the excitation amplitude R_e, and frequency ω, the positioning duration increases (Fig. 7, a). The increase in friction coefficient μ results in the reduction of positioning duration (Fig. 7, b).
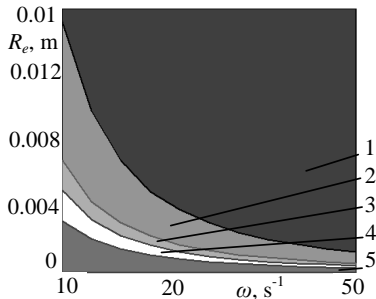

Fig. 6. Zones of motion regimes in coordinates R_e and ω, as μ=0.05
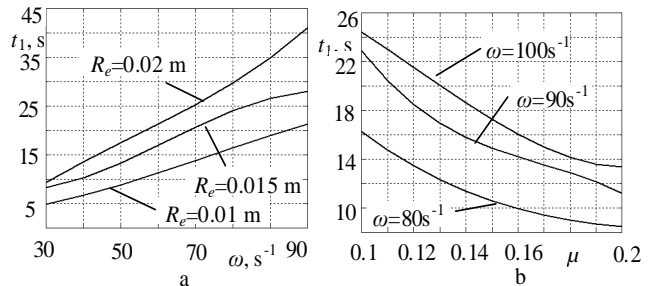
Fig. 7. Dependencies of the time positioning duration a - on excitation frequency ω and amplitude R_e, as μ=0.1; b - on friction coefficient μ and excitation frequency ω, as R_e=0.01 m

It is possible to obtain different trajectories of the plane motion by providing phase-shifted harmonic excitation to the plane. Then equations of the part motion are as follows:

$$\begin{cases} \ddot{x} + \mu g \dfrac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = A_e \omega^2 \sin(\omega t + \alpha_1), \\ \ddot{y} + \mu g \dfrac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = A_e \omega^2 \sin(\omega t + \alpha_2), \end{cases} \quad (11)$$

where α_1, α_2 are the phase angle of the excitation signals.

In this case, the character of the motion trajectory and the direction of the body motion do not depend on the values of phases α_1 and α_2, but depend on phase difference $\theta = \alpha_1 - \alpha_2$. Variation of phase difference enables generation of linear, circular or elliptic trajectories of body motion (Fig. 8).

Motion trajectories of the part may be complex when excitation along the axes ξ and η is of different frequencies. Then equations of the body motion are:

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$$\begin{cases} \ddot{x} + \mu g \dfrac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = R_e \omega_1{}^2 \cos(\omega_1 t); \\[4mm] \ddot{y} + \mu g \dfrac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = R_e \omega_2{}^2 \sin(\omega_2 t). \end{cases} \qquad (12)$$

During the simulation it was determined, that as difference between the frequencies is equal to 1 ($|\omega_1 - \omega_2| = 1$), then search trajectory of the part is located within the square, having boundaries depending on excitation amplitude of the plane (Fig. 9), i. e. the magnitude of the square search area is $2R_e$ and essentially does not depend neither on excitation frequency of the plane, nor on friction coefficient. The higher is the excitation frequency of the plane, the denser is the search area that the part goes through. To ensure more distant positioning of the part, it is necessary to increase the excitation frequency or reduce the coefficient of friction.
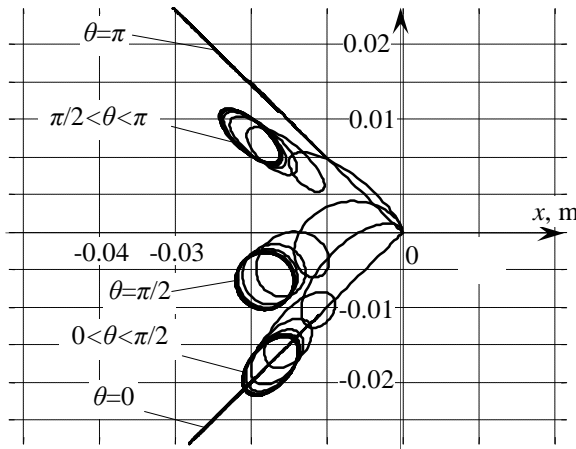

Fig. 8. Dependencies of part motion trajectories on phase difference as $\mu = 0.2$; $A_e = 0.01$ m; $\omega = 20$ s^{-1}
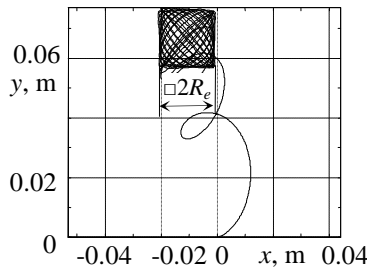

Fig. 9. Complex trajectories of the part motion, as $\mu = 0.1$; $\omega_1 = 20\omega_2$; $\omega_2 = 21$ s^{-1}; $A_e = 0.01$ m

In order to match connective surfaces and join cylindrical parts, the axis of part performing search must fall into the zone of allowable error. The area of the mentioned zone is predetermined both by joining clearance and chamfers of the parts. The probability that the axis of the bushing, which moves along the circular or elliptic search trajectory, would fall into the zone of allowable error depends on the position of the centre point of the trajectory in respect of the mentioned zone, and also on the magnitudes of the circle radius or ellipse axes. If the minor

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axis of an ellipse is shorter than the diameter of the allowable error zone, then the axis of the part will fall into the mentioned zone twice per search cycle. Therefore, at the elliptic trajectory of the search, the probability to match connective surfaces would be higher. When the plane is provided with excitation of different frequencies along the mutually perpendicular directions, the conditions for parts matching get better. The part moves repeatedly making loops in different positions and a higher probability exist that the bushing axis would fall into the zone of allowable error and unhindered joining of the parts occurs.

3. Elastically constrained and damped part motion on a vibrating plane

The analysis was carried out when the mating part being assembled (peg) was fed into assembly position by a manipulator or robot and the other part (bushing) was mobile based on the plane. Due to the positioning error of the robot the part had interdependent misalignment in the assembly position. In order to eliminate the errors of positioning, horizontal search motion by means of vibratory excitation was provided to the bushing, witch had elastic and damping constraints in all directions (Fig. 10).
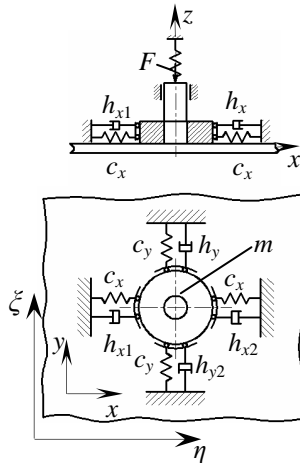

Fig. 10. Scheme for vibratory search of the constrained part

Motion of constrained bushing on the horizontal plane is described by equations:

$$\begin{cases} m\ddot{x} + \mu F \dfrac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} + h_x \dot{x} + c_x x = mR_e \omega^2 \cos \omega t, \\[4mm] m\ddot{y} + \mu F \dfrac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} + h_y \dot{y} + c_y y = mR_e \omega^2 \sin \omega t, \end{cases} \qquad (13)$$

$F = mg + F_1.$

where F_1 is the force of the peg pressing on the bushing; $h_x = h_{x1} + h_{x2}$, $h_y = h_{y1} + h_{y2}$ are damping coefficients along x and y directions; $c_x = c_{x1} + c_{x2}$, $c_y = c_{y1} + c_{y2}$ are spring stiffness along x and y directions.

By means of numerical simulation it was determined that at the beginning the part moves on the plane along a complex trajectory from the starting position to a particular point predetermined by the system parameters (Fig. 11). Then the part returns and undergoes a steady

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circular trajectory of motion. The part displacement in the transient movement regime and radius of steady circular motion depend on the stiffness of resilient elements and initial velocity, and also on the friction coefficient and excitation frequency. Before the steady circular search trajectory is reached, the part moves over the plane in loops. Thus the probability that the centre of the moving bushing will fall into the zone of allowable error increases.
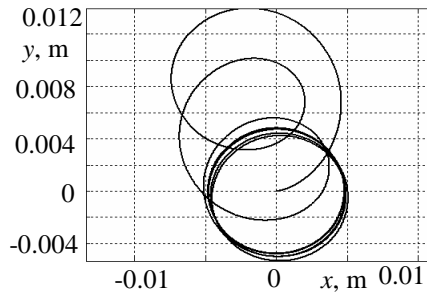

Fig. 11. Search trajectory of the part

The motion trajectory of the part depends on friction coefficient μ, excitation amplitude and frequency (Fig. 12). When friction coefficient μ increases, the part displacement from the initial position towards the farthest point of the trajectory shortens (Fig. 12 a and b). When μ=0.1, the part moves along the unwinding helix trajectory (Fig. 12, c). When friction coefficient exceeds 0.6, the part no longer moves and only rotates circularly (Fig. 12, d). But at higher values of the excitation frequency or/and amplitude or if elastic elements of smaller stiffness are chosen, the part can move again and follow the steady circular trajectory.
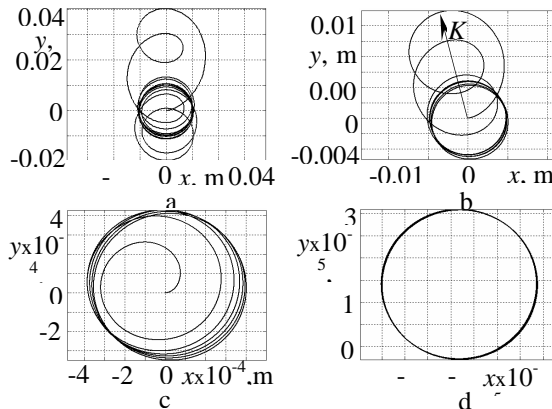

Fig. 12. Character of motion trajectories xy plane, as R_e=0.01 m, m=0.01 m, F=5 N, h_x=h_y=0.2 Ns/m, c_x=c_y=10 N/m, ω=180 s^{-1}: a – μ=0.01, b – μ=0.1, c – μ=0.45, d – μ=0.63

Considering the character of motion trajectories, it is possible to identify 5 regimes of the part motion, taking into account the sets of parameters F, R_e, and c. Zones of the existence of these regimes were established (Fig. 13). With the given parameters from the first zone, the part initially moves backwards and forwards in respect to the initial point by a looping trajectory, which later changes into a steady circular motion (Fig. 12, a). Within the second zone the part has characteristic sliding motion along a complex trajectory and steady circular motion, but without the forward-backward sliding motion (Fig. 12, b). In the third zone the part has

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characteristic helical unwinding motion (Fig. 12, c). The fourth zone defines circular motion of the part (Fig. 12, d). When the parameters are within the fifth zone, the part motion is chaotic and of small amplitude.

In Fig. 13, the largest zones are the first and the second, as the regime of the part motion on the plane is transient, later changing into a steady circular search motion. These two trajectories are most suitable for the search.

When mutually dependent connective surface location error in the assembly position is more significant, it is important to know what maximum displacement K from the initial point the part is able to perform (Fig. 12, b). All the parameters of the system have influence on the distance of the part, but the most significant are amplitude R_e and frequency ω. It was determined that increase in excitation amplitude results in rapid increase in part displacement K (Fig. 14, a). As friction coefficient increases displacement decreases and as excitation frequency increases displacement increases as well (Fig. 14, b).
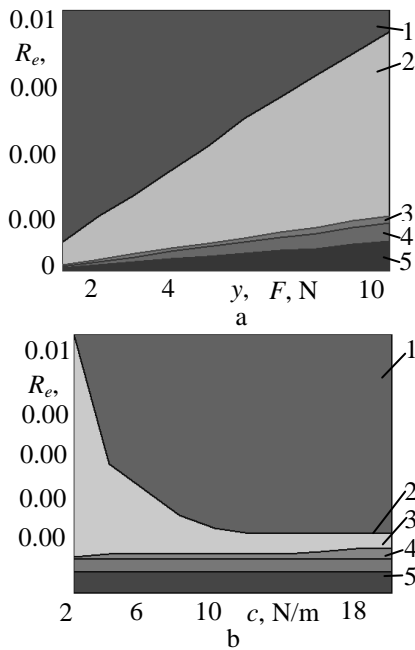

Fig. 13. Zones of motion regimes in coordinates:
a - F and R_e, as $\omega=150$ s^{-1}, $\mu=0.05$, $h_x=h_y=0.2$ Ns/m,
b - c and R_e, as $\omega=150$ s^{-1}, $\mu=0.1$, $m=0.01$ kg,
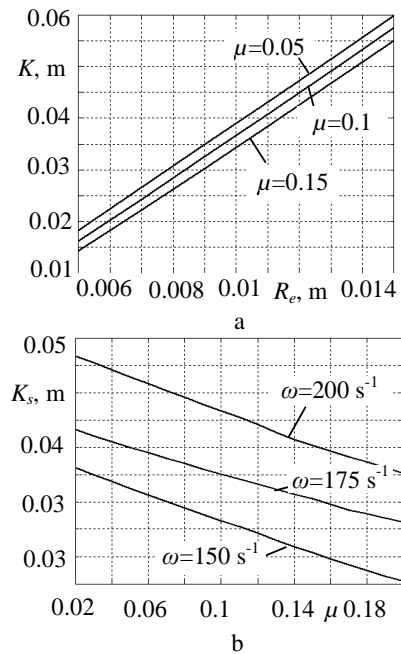$F=3$ N; $c=c_x=c_y$, $h_x=h_y=0.2$ Ns/m

Fig. 14. Dependence of part displacement on:
a - excitation amplitude R_e and friction coefficient μ, as $h_x=h_y=0.2$ Ns/m, $\omega=150$ s^{-1},
$c_x=c_y=10$ N/m, $m=0.01$ kg, $F=2$ N; b - friction coefficient μ and excitation frequency ω, as $R_e=0.01$ m, $h_x=h_y=0.2$ Ns/m, $c_x=c_y=10$ N/m, $m=0.01$ kg, $F=2$ N

The character of the part motion on the elliptically vibrating plane is similar to the motion of the part as the trajectory of the plane is circular. It is possible to obtain a helix trajectory of steady motion by subjecting the plane to the excitation of different or equal amplitudes along the mutually perpendicular directions and by using different stiffness resilient elements along x and y directions.

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In previously investigated cases, during the search connective parts were not contacting or they were pressed towards each other by constant force. Then a steady circular or elliptic search trajectory is obtained. It is possible to increase the pressing force during the search and thereby obtain the interwinding helix trajectory. As the pressing force increases, friction force between the connective parts and between the search-performing part and the vibrating plane also increases. Thus the probability that the centre of the moving bushing will fall into the zone of allowable error increases.

The equations of the constrained part motion on a horizontal plane under varying pressing force are written as follows:

$$\begin{cases} m\ddot{x} + \mu F_1 \dfrac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} + h_x \dot{x} + c_x x = mR_e \omega^2 \cos \omega t, \\[4mm] m\ddot{y} + \mu F_1 \dfrac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} + h_y \dot{y} + c_y y = mR_e \omega^2 \sin \omega t, \end{cases} \qquad (14)$$

where $F_1 = mg + c_1 \cdot v \cdot t$, c_1 is spring stiffness; v is pressing velocity;

An increase in the pressing force causes a decrease in the amplitude of search motion over the plane and the trajectory is similar to the interwinding helix (Fig. 15). Some time after search the motion of the part stops.

The duration till the relative velocity of the part motion with respect to the plane reaches zero depends on the parameters of the dynamic system. If friction coefficient and amplitude of excitation are increased, the stop time reduces and if excitation amplitude and frequency increased, the stop time is increased (Figs. 16 and 17).

If the size of the allowable error zone is known, it is possible to choose system parameters so that the distance between adjacent loops of interwinding helix (helix pitch Δ) is not exceeding the allowable error zone (Fig. 15). Then the trajectory of the bushing axis certainly falls into the allowable error zone and matching of the parts connective surfaces takes place. It was determined that the pitch of the interwinding helix increases over time.
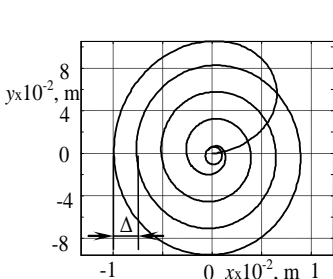
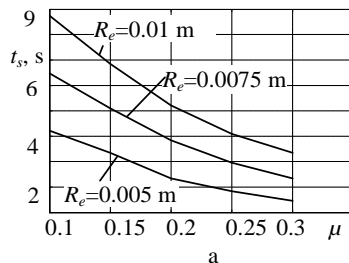**Fig. 15.** Search trajectory of the part under varying pressing force

Fig. 16. Dependencies of the part stop time on friction coefficient μ and excitation amplitude R_e, as m=0.005 kg, v=0.01 m/s, h_x=h_x=0.1 Ns/m, c_x=c_y=5 N/m, c_1=15 N/m, ω=50 s^{-1}
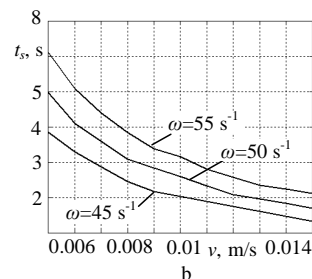
Fig. 17. Dependencies of the part stop time on the peg pressing velocity v and excitation frequency ω, as m=0.005 kg, c_1=15 N/m, h_x=h_x=0.2 Ns/m, c_x=c_y=10 N/m, μ=0.3, R_e=0.01 m

4. Conclusions

The character of the part motion on a vibrating plane was analyzed as the plane moves along circular, elliptical and complex trajectories, for the part resting freely on a plane and for a movably-based part. It was determined that the part on the plane is able to follow transient

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motion regime and can move along a certain steady trajectory. The performed analysis revealed that the direction of part motion depends on the initial phase of the excitation signal.

Depending on the character of motion trajectory, 5 regimes of body motion have been defined and, taking into account different sets of parameters, the zones of the existence of these regimes have been established. The best conditions for interdependent search of the parts are when the motion character of the part on the vibrating plane is described as falling into the first and second zone. It was determined that the motion of the part is delayed in comparison to the motion of the plane.

This study demonstrated that the phase shifted harmonic excitation of the plane in two perpendicular directions provides a possibility to obtain circular, elliptic and linear trajectories of motion.

Excitation of the plane in mutually perpendicular directions by harmonic signals of different frequency results in part motion along complex trajectories.

Obtained results indicate that movably based part, under the constant peg pressing force, initially moves in a transient search motion regime, which later changes into the regime of steady motion along a circular or elliptic trajectory. Under varying pressing force, the trajectory of the part search on a vibrating plane is similar to the interwinding helix of decreasing amplitude and the part stops after a while. Dependencies of stop time on spring stiffness, amplitude and frequency of excitation as well as on pressing velocity have been determined for the part.

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