411. Bifurcation analysis by method of complete bifurcation groups of the driven system with two degrees of freedom with three equilibrium positions

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Abstract. This paper devoted to application of the new method of complete bifurcation groups (MCBG), which shows very good results in single-degree-of-freedom tasks, for global bifurcation analysis of systems with two degrees-of-freedom on example of two-mass chain system with symmetrical elastic characteristic with two potential wells between masses. It is shown, that using of the MCBG allows to implement global bifurcation analysis of nonlinear oscillators with 2 DOF, and to find new nonlinear effects, bifurcation groups, and unknown before periodic and chaotic regimes.

Keywords: two-degrees-of-freedom, driven system, bifurcation analysis, method of complete bifurcation groups, rare chaotic and regular attractors, fully unstable subharmonical isles.

Introduction

Behaviour of the nonlinear driven oscillators may be very complex and unexpected phenomena occur rather often in these systems. The authors of this paper for many years have been studying nonlinear phenomena in one-mass driven oscillators with symmetrical elastic characteristics with two potential wells. Usual phenomena, as periodic and chaotic motion, and unusual, as stable forced oscillations near unstable equilibrium position [1-5], rare attractors (RA) [6-10] were found in such kind of systems. For those investigations algorithms and approaches later named as method of complete bifurcation groups [11-14] were used.

In this paper authors apply the method of complete bifurcation groups to global bifurcation analysis of two-mass chain driven system with symmetrical elastic characteristic with two potential wells between masses. It is shown, that all phenomena typical for one-mass system remain and there are new bifurcation groups with unusual structure – fully unstable subharmonical isles [5, 11, 12]. This phenomenon is concerned with Andronov-Hopf bifurcation. Changing system parameters leads to appearance of rare stable periodic and chaotic regimes on these fully unstable isles.

Dynamical model and methods

A system under consideration is a chain driven system consisting of two bodies coupled by nonlinear elastic constrain with three equilibrium positions (two potential wells) and linear damping and attached to base by linear elastic constrain and linear damping. External periodic force is applied to the body, attached to the base (Fig. 1).

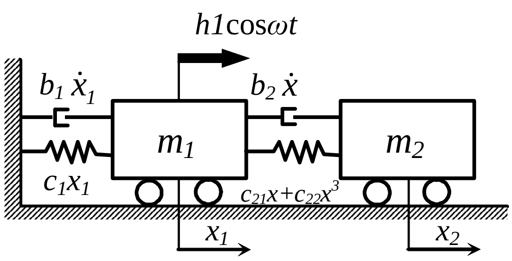

Fig. 1. Physical model of the system

Elastic and damping forces are represented in Fig. 2a. Also several additional characteristics, describing the system, are represented in this figure - potential wells of both elastic constrains (Fig. 2a) and free damped oscillations of both masses (Fig. 2b).

Corresponding equations of motion are:

$$\begin{cases} m_1\ddot{x}_1 + b_1\dot{x}_1 + c_1x_1 - b_2\dot{x} - c_{21}x - c_{22}x^3 = h1cos(\omega t + \varphi_0) \\ m_2\ddot{x}_2 + b_2\dot{x} + c_{21}x + c_{22}x = 0 \end{cases}$$

where x_1, x_2 – generalized coordinates ($x = x_2 - x_1$); m_1, m_2 – mass of oscillating bodies; b_1, b_2 – linear dissipation coefficients; c_1 – stiffness coefficient of the first linear elastic constrain; c_{21}, c_{22} - stiffness coefficients of the second nonlinear elastic constrain; $h1$, ω, φ_0 – amplitude, frequency and phase of excitation.

Method of complete bifurcation groups was used for investigations. This method consists in direct numerical modeling of originate existing nonlinear model, that is, without its simplification. Under complete bifurcation groups method we understand complex of approaches to analysis of dynamic systems, which involves the following procedures: at fixed system parameters – search of all periodic stable and unstable regimes and bifurcation subgroups with unstable periodic infinitiums (UPI) on plane of states, constructing of regimes' basins of attraction on plane of states; at varying system parameters – constructing of bifurcation diagrams and bifurcation maps. Special importance of the method is continuation of parameter solution (in one-parameter task) along solution branch of definite regime (not along parameter), and that allows to find new unknown before stable regimes in broadly used dynamic models of strongly nonlinear oscillation systems.

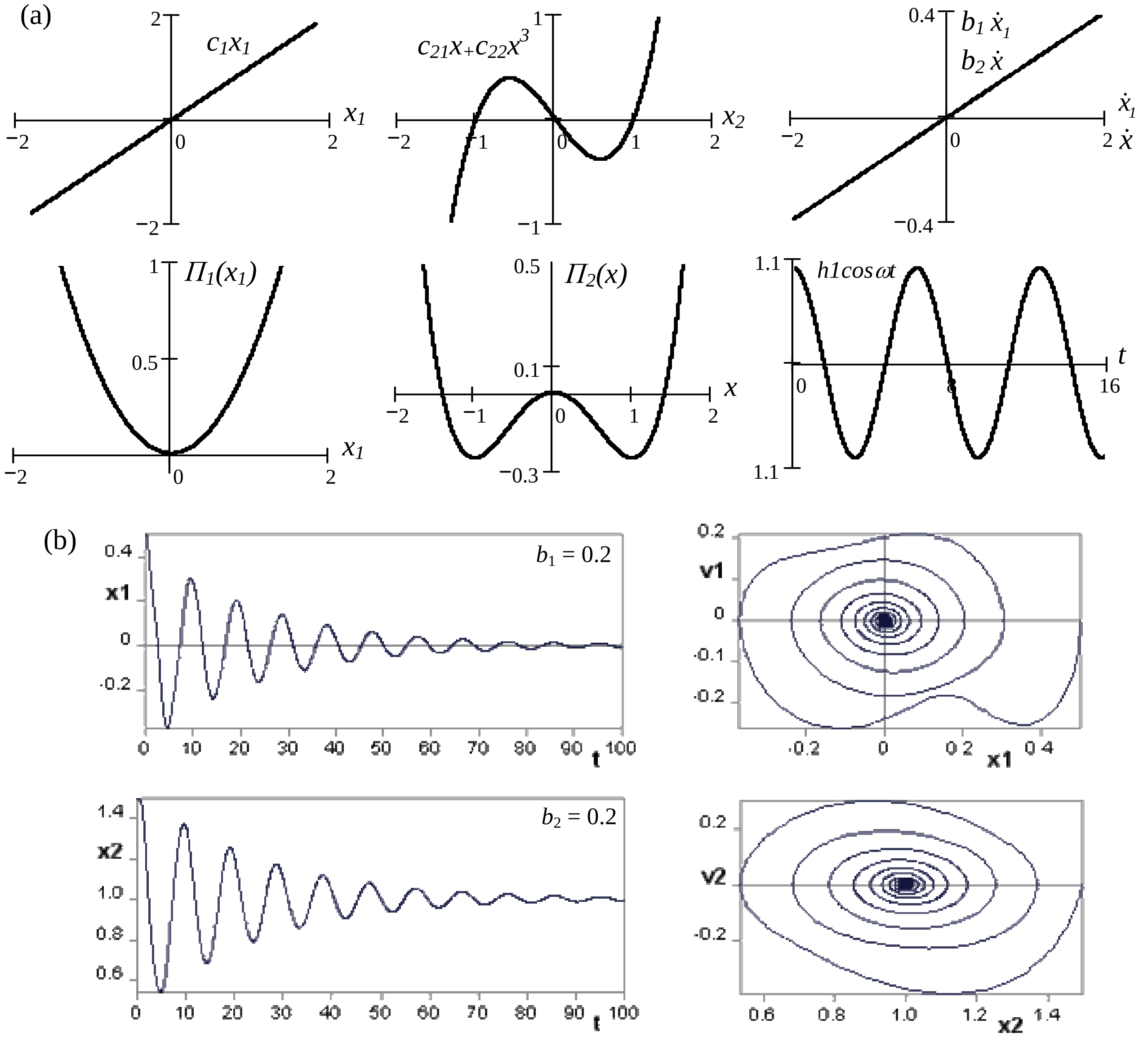

Fig. 2. Characteristics of chain system with two degrees-of-freedom with three equilibrium positions of second mass $m2$ and linear dissipation at harmonic excitation. (a) characteristics of elastic and dissipative forces, acting in a model, characteristics of potential energies, corresponding to elastic forces and diagram of external harmonic excitation; (b) free oscillations of the first and second masses. Parameters: $m_1 = m_2 = 1$, $b_1 = b_2 = 0.2$, $c_1 = 1$, $c_{21} = -1$, $c_{22} = 1$

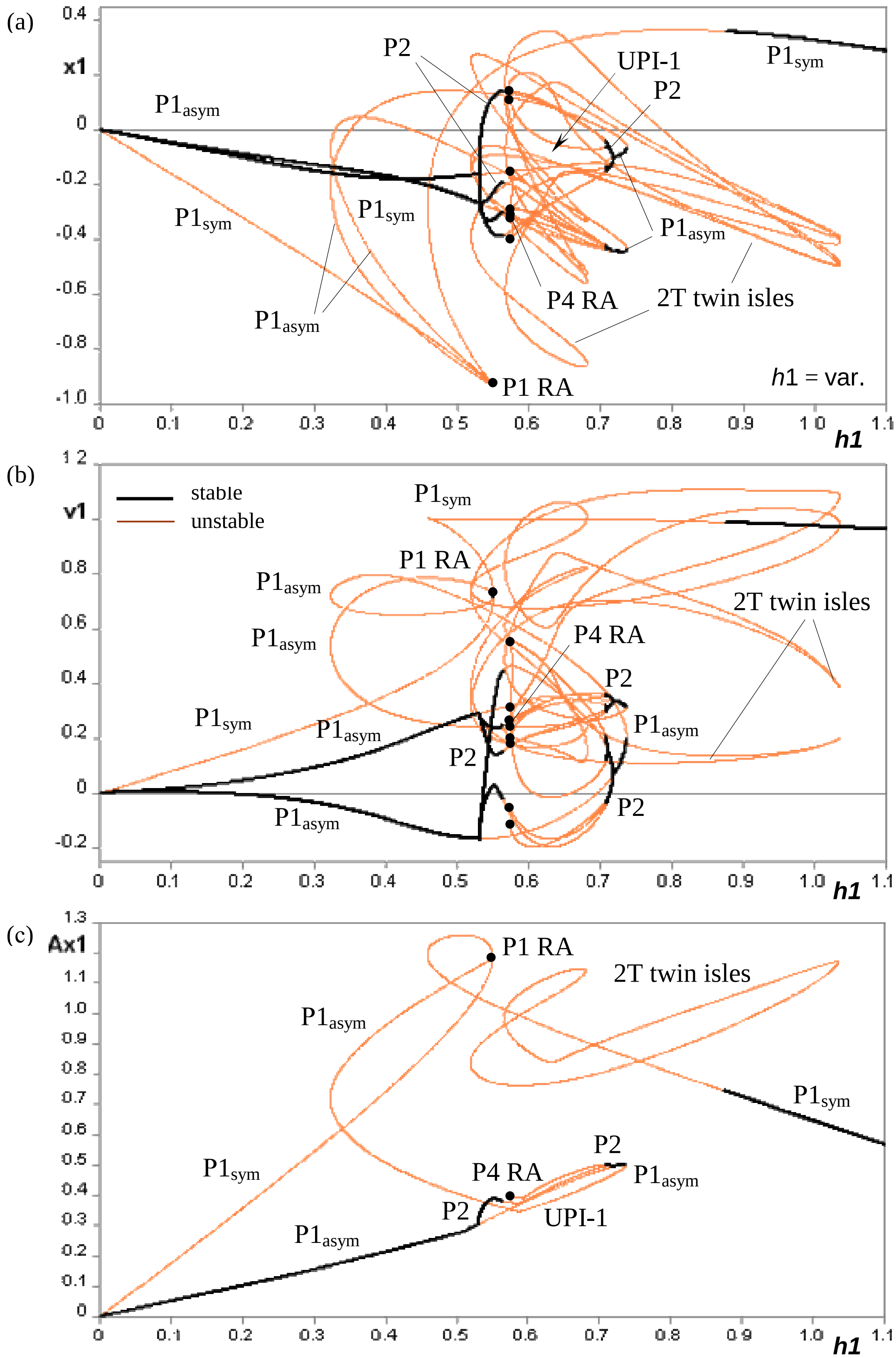

Fig. 3. Two bifurcation groups, group 1T with rare attractor, UPI region and almost periodic oscillations and group of completely unstable subharmonic isles 2T, are shown. Bifurcation diagrams for driven chain system with two degrees-of-freedom with three equilibrium positions of mass m_2 and linear dissipation at harmonic excitation (eq.). (a), (b) coordinates x_1, v_1 of fixed point of the first mass vs excitation amplitude $h1$; (c) oscillation amplitude Ax1 of the first mass vs $h1$. Parameters: $m_1 = m_2 = 1$, $b_1 = b_2 = 0.2$, $c_1 = 1$, $c_{21} = -1$, $c_{22} = 1$, $\omega = 1$, $\varphi_0 = 0$, $k = 7$, $h1$ = var.

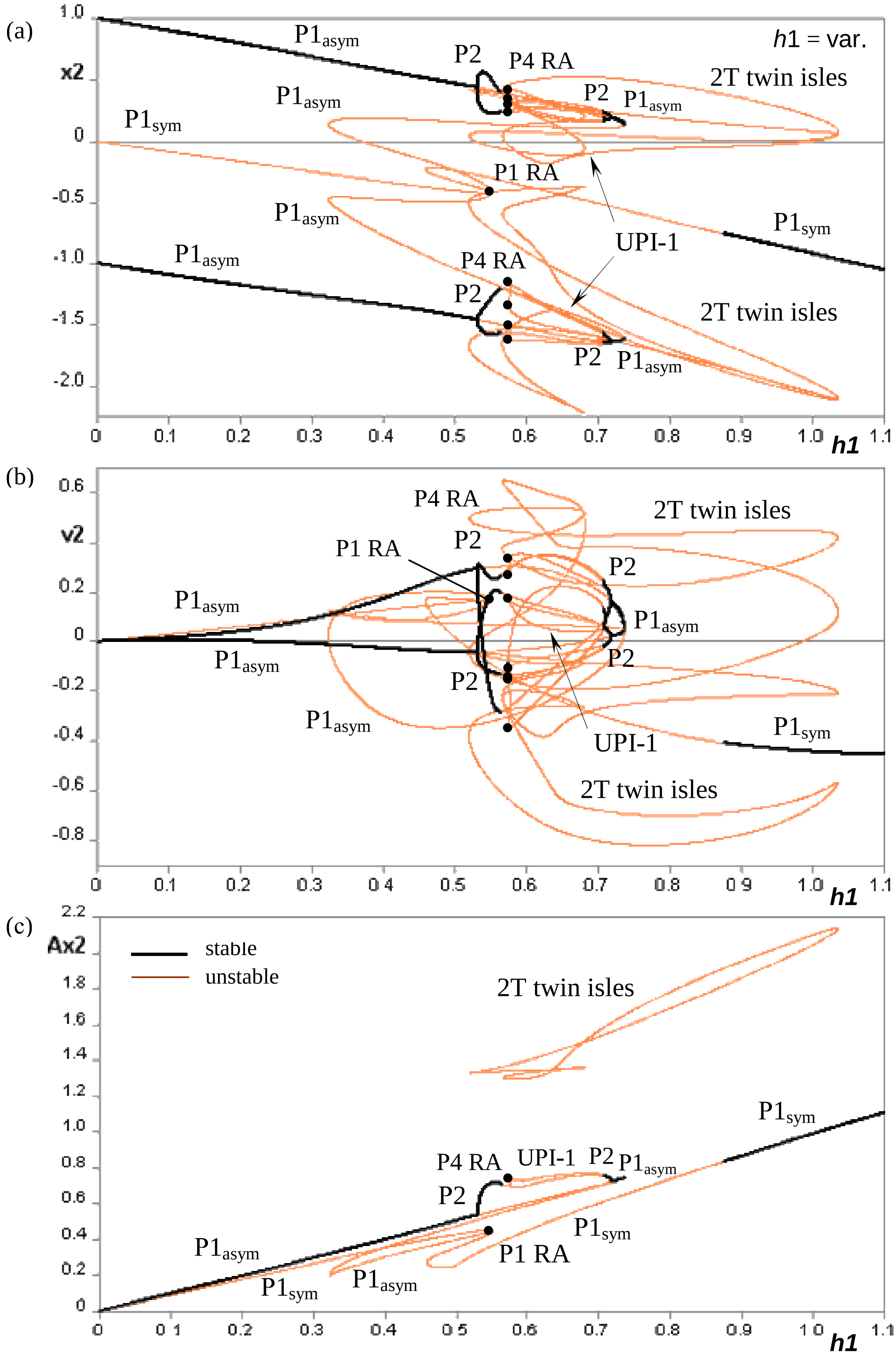

Fig. 4. Two bifurcation groups, group 1T with rare attractor, UPI region and almost periodic oscillations and group of completely unstable subharmonic isles 2T, are shown. Bifurcation diagrams for driven chain system with two degrees-of-freedom with three equilibrium positions of mass m_2 and linear dissipation at harmonic excitation (eq.). (a), (b) coordinates x_2, v_2 of fixed point of the first mass of excitation amplitude $h1$; (c) oscillation amplitude Ax2 of the second mass vs $h1$. Parameters: $m_1 = m_2 = 1$, $b_1 = b_2 = 0.2$, $c_1 = 1$, $c_{21} = -1$, $c_{22} = 1$, $\omega = 1$, $\varphi_0 = 0$, $k = 7$, $h1$ = var.

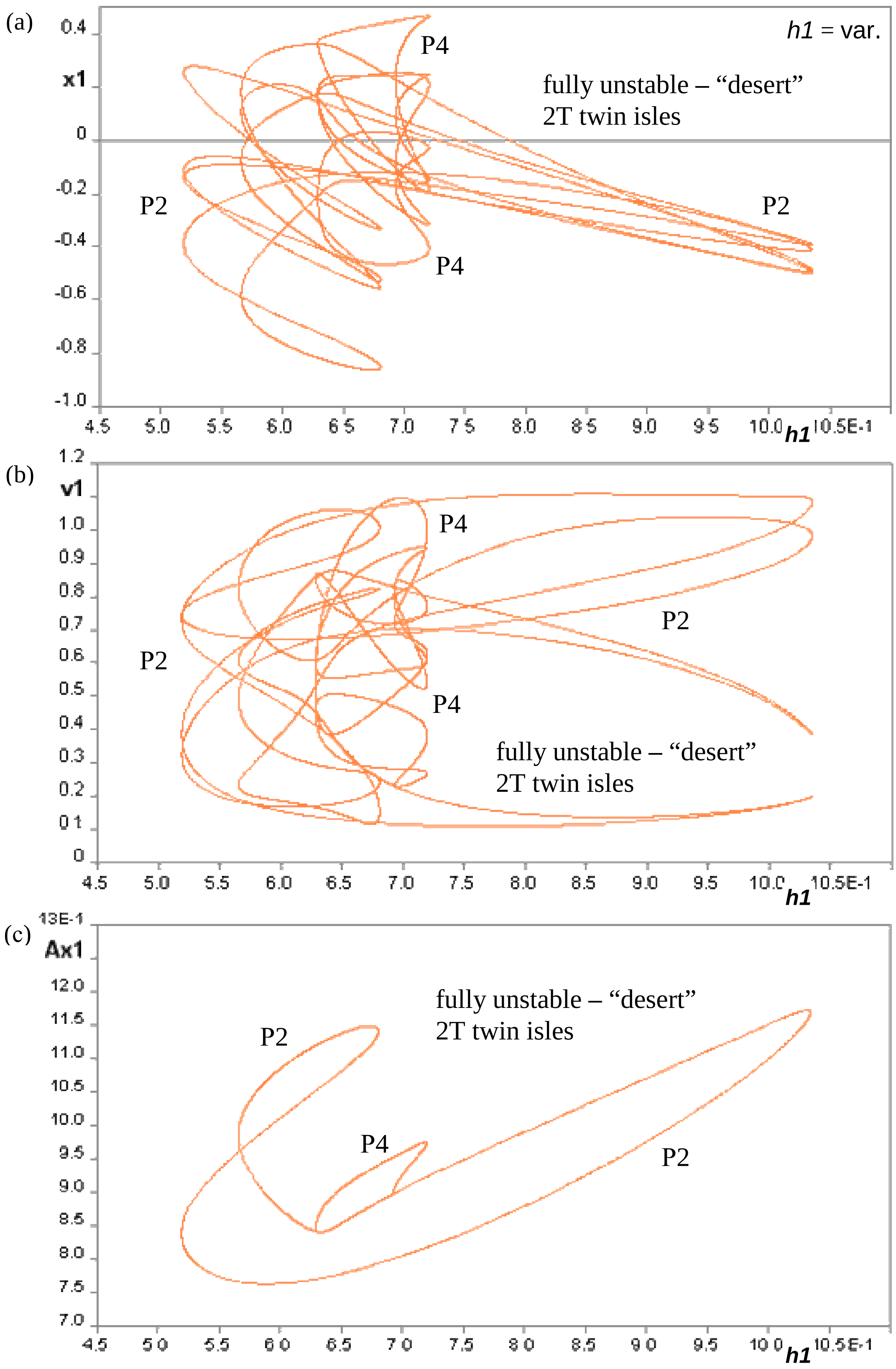

Fig. 5. New bifurcation group of completely unstable subharmonic twin isles 2T. Bifurcation diagrams for driven chain system with two degrees-of-freedom with three equilibrium positions of the second mass m_2 and linear dissipation at harmonic excitation (eq.). (a), (b) coordinates x_1, v_1 of fixed point of the first mass of excitation amplitude $h1$; (c) oscillation amplitude Ax1 of the first mass vs $h1$. Parameters: $m_1 = m_2 = 1$, $b_1 = b_2 = 0.2$, $c_1 = 1$, $c_{21} = -1$, $c_{22} = 1$, $\omega = 1$, $\varphi_0 = 0$, $k = 7$, $h1$ = var.

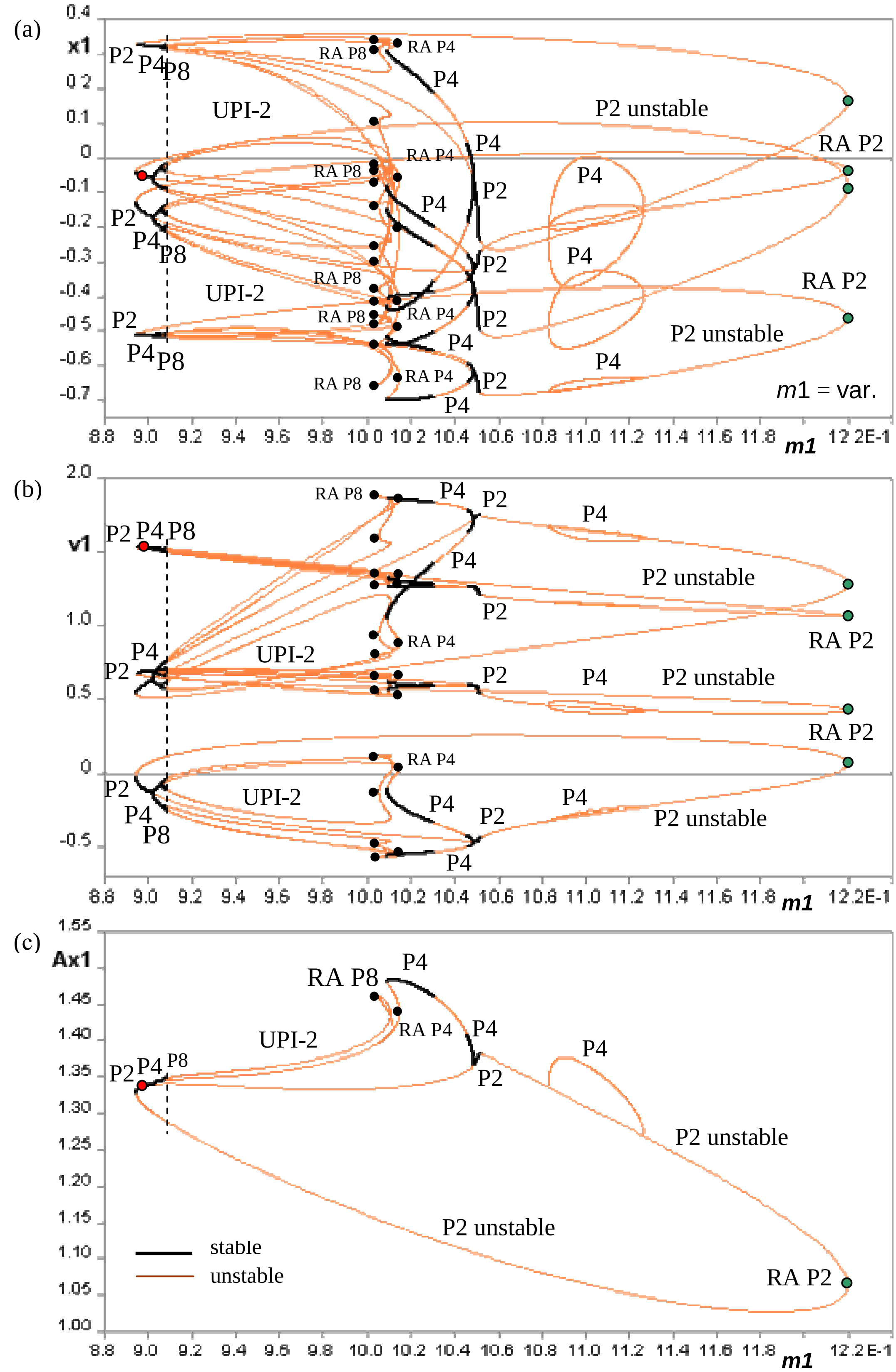

Fig. 6. Appearance of stable regimes – rare attractors – on the fully unstable subharmonic isles 2T (Fig. 5) at varying the first mass m_1. Bifurcation diagrams for driven chained system with two degrees-of-freedom with three equilibrium positions of the second mass m_2 and linear dissipation at harmonic excitation (eq.). (a), (b) coordinates x_1, v_1 of fixed point of the first mass vs m_1; (c) oscillation amplitude Ax1 of the first mass vs m_1. Parameters: $m_2 = 1$, $b_1 = b_2 = 0.2$, $c_1 = 1$, $c_{21} = -1$, $c_{22} = 1$, $h1 = 1$, $\omega = 1$, $\varphi_0 = 0$, $k = 7$, $m_1 =$ var.

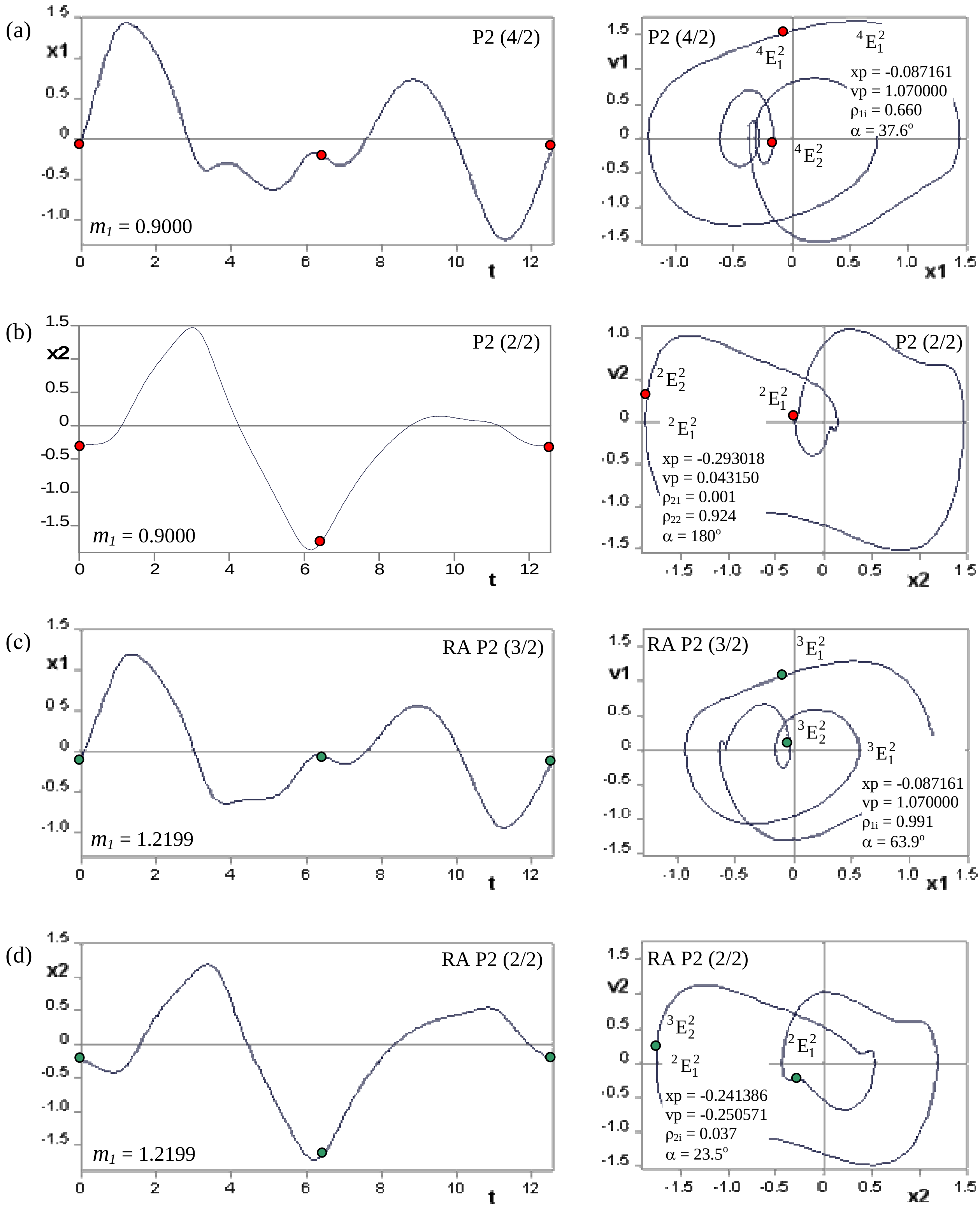

Fig. 7. Time histories and phase trajectories projections for driven chained system with two degrees-of-freedom with three equilibrium positions of the second mass m_2 and linear dissipation at harmonic excitation (eq.): (a), (b) for stable periodic regime P2, which belongs to bifurcation group of subharmonic isle 2T (Fig. 6 at m_1 = 0.9000); (c), (d) for rare attractor RA P2 – stable periodic regime P2, which belongs to bifurcation group of subharmonic isle 2T (Fig. 6 at m_1 = 1.2199). Red and green dots mark fixed point of P2 and RA P2 regimes correspondingly. Parameters: $m_2 = 1$, $b_1 = b_2 = 0.2$, $c_1 = 1$, $c_{21} = -1$, $c_{22} = 1$, $h1 = 1$, $\omega = 1$, $\varphi_0 = 0$, $k = 7$

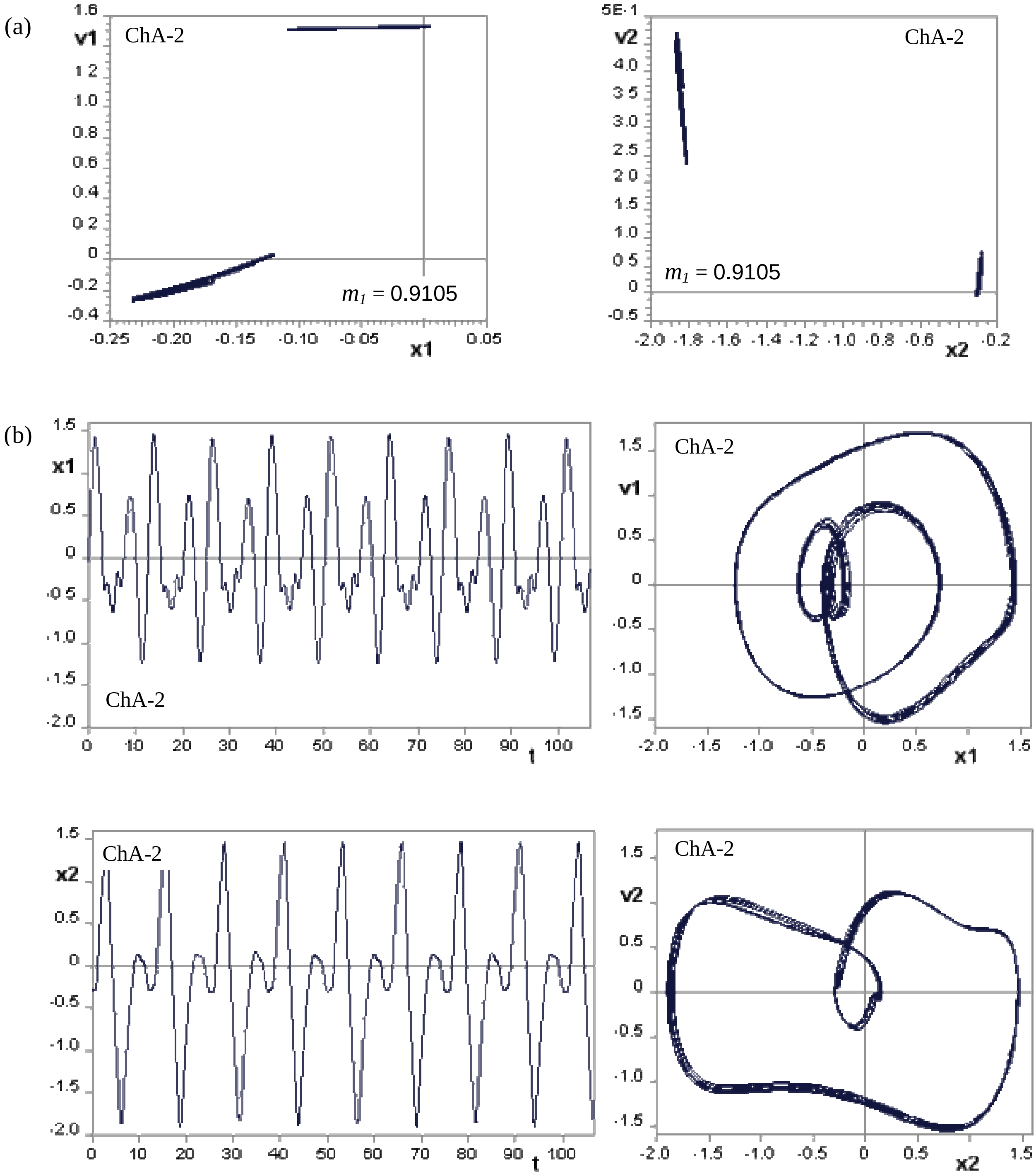

Fig. 8. Chaotic oscillations, belonging to bifurcation group of subharmonic isle 2T (Fig. 6 at m_1 = 0.9105, dashed line) for driven chained system with two degrees-of-freedom with three equilibrium positions of the second mass m_2 and linear dissipation at harmonic excitation (eq.). (a) Poincaré plain for first and second masses, 15000 periods are shown from initial conditions x_{10} = -0.042325, v_{10} = 1.517680, x_{20} = -0.289490, v_{20} = 0.031809; (b) time histories and phase trajectories projections for first and second masses correspondingly, 16 periods are shown. Parameters: m_1 = 0.9105, m_2 = 1, b_1 = b_2 = 0.2, c_1 = 1, c_{21} = -1, c_{22} = 1, $h1$ = 1, ω = 1, φ_0 = 0, k = 7

Results

Results of bifurcation analysis, at varying initial conditions and system parameter – amplitude *h1* of excitation, are represented in Figs. 3, 4. The fixed system parameters are $m_1 = m_2 = 1$, $b_1 = b_2 = 0.2$, $c_1 = 1$, $c_{21} = -1$, $c_{22} = 1$, $\omega = 1$, $\varphi_0 = 0$.

In Figs. 3 and 4 bifurcation diagrams of two bifurcations groups, 1T group of main regime P1 and 2T group of subharmonic regime P2, are shown for the first and second mass correspondingly. 1T group has: rare attractors P1 RA, which corresponds to stable periodic oscillations near unstable equilibrium position – hilltop oscillations, and P4 RA; region with unstable periodic infinitium UPI-1 [10, 15]; and Andronov-Hopf bifurcation at $h1 \approx 0.88$. 2T group is the twin subharmonic isles, which consist only of unstable solutions. "Twin isles" means that regimes of these isles are mutually symmetric.

On example of 1T bifurcation group we can conclude that all phenomena typical for 1dof systems, like stable periodic oscillations near unstable equilibrium position, rare attractors of tip type, UPI regions and chaotic behaviour, remain also in 2dof systems. Also we can observe phenomena usual only for systems with several degrees of freedom – Andronov-Hopf bifurcation, which leads to almost periodic oscillations, and fold bifurcation with both brunches unstable. And, as a result of such kind of bifurcation, new bifurcation group with unusual structure (topology) was found. It is fully unstable "desert" subharmonical isle, e.g. at least one of stability markers is outside of unity circle. Separately bifurcation diagrams of this new bifurcation group, only for first mass, are shown in Fig. 5.

Varying system parameters such as mass of oscillating bodies, linear dissipation coefficients, particular stiffness coefficients or excitation frequency, leads to appearance of rare attractors on unstable branches of "desert" isles. As example, in Fig. 6, bifurcation diagrams for the unstable subharmonic isles now with fixed parameter of amplitude of excitation $h1 = 1$ and varying parameter of first mass m_1 are shown. In this Fig. we can observe stable periodic regimes, several rare attractors of tip type, regions with UPI, chaotic attractors and almost periodic oscillations.

It is known, that in systems with one degree of freedom tip rare attractors has the following structure: from one side it is bounded by fold bifurcation and from the other side by unstable periodic infinitium through cascade of bifurcations of period doubling. Let's compare this structure with rare attractors' structure in system with two degrees of freedom shown in Fig.6.

Structure of rare attractor near $m1 \approx 0.9$ (Fig. 6 and Fig. 7a,b) is similar to one in systems with one degree of freedom – fold bifurcation from the left and UPI-2 from the right. And as it is known, in systems with one degree of freedom transition to unstable periodic infinitium leads to chaotic attractor. In case of rare attractor near $m1 \approx 0.9$ situation is the same, corresponding chaotic attractor is represented in Fig. 8.

Rare attractor RA P2 near $m1 \approx 1.2$ (Fig. 6 and Fig. 7c,d) has different structure. From the right there is fold bifurcation and from the left there is Andronov-Hopf bifurcation, which leads to almost periodic oscillations.

Conclusion

Bifurcation analysis of driven chain system with two-degrees-of-freedom with three equilibrium positions of second body is done using method of complete bifurcation groups. It is shown, that all phenomena typical for one-degree-of-freedom system with three equilibrium positions remain and there are new bifurcation groups with unusual structure (topology) – fully unstable "desert" subharmonical isles, which consist only of unstable periodic regimes. This phenomenon is concerned with Andronov-Hopf bifurcation. Changing system parameters leads to appearance of rare stable periodic and chaotic regimes on these fully unstable isles.

Acknowledgements

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References

[1] **Zakrzhevsky M., Frolov V.** Bifurcation Analysis of the Stable Forced Oscillations near Unstable Equilibrium Position // In: 2nd ENOC, Proceedings, Vol.2 - Prague, 1996 - p. 265-267.

[2] **Zakrzhevsky M.** Global Stable Oscillations near Unstable Equilibrium Positions: The Hilltop Effect // In: IUTAM Symposium on New Applications of Nonlinear and Chaotic Dynamics in Mechanics, Proceedings, F.C. Moon (ed.) - Ithaca, USA, Kluwer Academic Publishers, 1998 - p. 117-124.

[3] **Zakrzhevsky M.** Stable Forced Hilltop Oscillations // In: IUTAM / IFToMM Symposium "Synthesis of Nonlinear Dynamical systems", Proceedings, E.Lavendelis, M.Zakrzhevsky (eds.) - Riga, 1998 - p.79-80.

[4] **Zakrzhevsky M.V. and Zevin A.A.,** The Forced and Parametric Oscillations Near Unstable Equilibrium Positions, XIX Scientific School on Nonlinear Oscillations, Kaunas, 1989.

[5] **Zakrzhevsky M., Schukin I., Yevstignejev V.** Rare attractors in Driven Nonlinear Systems with Several Degrees of Freedom, Scientific Proceedings of Riga Technical University – Transport and Engineering, sērija 6, sējums 24, Riga, 2007, pp. 79-93.

[6] **Thompson J.M.T. and Stewart H.B.** Nonlinear Dynamics and Chaos, 2nd Edition, Wiley, 2002.

[7] **Vakakis A.F., Manevitch L.I., Mikhlin Y.V., Pilipchuk V.N., Zevin A.A.** Normal Modes and Localization in Nonlinear Systems, Wiley, 1996.

[8] **Zakrzhevsky M.** Nonlinear Oscillatory and Vibro-Impact Systems: Rare Attractors. In: Proc. XIII Symposium on the Dynamics of Vibroimpact (Strongly Nonlinear) Systems, (Eds.: Astashev V.K. and Krupenin V.L.), Russia, Moscow-Zvenigorod, 2001, 156-162.

[9] **Zakrzhevsky M.** Rare Attractors and Protuberances in Nonlinear Dynamics, in the Proceedings of the XIV Symposium “The Dynamics of Vibroimpact (Strongly Nonlinear) Systems”, Moscow-Zvenigorod, 2003, pp. 114-115.

[10] **Zakrzhevsky M.** Typical Bifurcation Groups and Their Interaction in Nonlinear Dynamic Systems. Rare Attractors, Protuberances, Unstable Periodic Infinitiums, and Chaotic Behaviour, in Book of Abstracts of 1st International Conference on Vibro-Impact Systems, Loughborough, 2006, pp. 53-55.

[11] **Schukin I.T.** Development of the methods and algorithms of simulation of nonlinear dynamics problems. Bifurcations, chaos and rare attractors, PhD Thesis, Riga – Daugavpils, 2005, 205 p. (in Russian)

[12] **Yevstignejev V.Yu.** Application of the Complete Bifurcation Groups Method for Analysis of Strongly Nonlinear Oscillators and Vibro-Impact Systems, PhD thesis, Riga, 2008, 146 p. (in Russian)

[13] **Zakrzhevsky M.V., Schukin I.T., Smirnova R.S., Yevstignejev V.Yu.** Method of Complete Bifurcation Groups and its Applications for Prediction of Unknown Chaotic and Regular Attractors in Nonlinear Dynamical Systems. In the Book of Abstracts, Chaotic Modeling and Simulation International Conference CHAOS 2008, Chania, Crete, Greece, pp. 97-99, 2008.

[14] **Zakrzhevsky M.** Method of Complete Bifurcation Groups (MCBG) and its Application for Prediction of Rare Dangerous Dynamical Phenomena. In the Book of Abstracts of XXXV Summer School – Conference “Advanced problems in Mechanics”, St.Petersburg(Repino), Russia, pp. 116-117, 2007.

[15] **Zakrzhevsky M.V.** Nonlinear and Chaotic Dynamics: New about Unstable Periodic Infinitiums, Proceedings of the 1st Baltic-Bulgarian Conference on Mechanics and Biomechanics, Varna, 2000, pp. 61-66.