407. Nonlinear optimal synthesis of the vibrator with flow excitation

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Abstract. In the daily life and techniques people all time have interaction with continue media like air or water. In report motion of vibrator with constant air or water flow excitation is observed. In first part of report motion of a vibrator with constant air or water flow velocity excitation is investigated. The main idea is to find out optimal control law for variation of additional area of vibrating object within limits. For solution of the high-speed problem the maximum principle is used. It is shown that optimal control action is on bounds of area limits. Examples of synthesis real mechatronic systems are given.

Keywords: vibrator, criterion of optimization, fluid excitation.

1. Introduction

Motion of a vibrator with two degree of freedom and constant fluid flow $\bar{V}_0$ excitation is analyzed (Fig. 1.). System consists of masses m1, m2 with springs c1, c12 and dampers b1, b12.The main idea is to find out optimal control interaction law with fluid flow for variation of additional area S(t) of vibrating mass m2 within limits :

$$S_1 \le S(t) \le S_2,$$

where S_1 - lower level of additional area of mass m2; S_2 - upper level of additional area of mass m2, t - time.

The criterion of optimization is time required to move object from initial position to end position. First of all to understand process of fluid excitation and optimal solution of control problem observe system with one degree of freedom when mass m1 is very large (massive fundament).

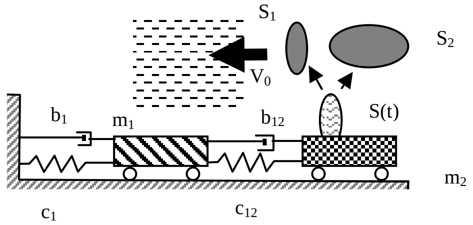

Fig. 1. Scheme of model with area S(t) control

To simplify the equation it is easy (for system with one degree of freedom) to miss indexes of motion description. Then the differential equation is :

$$m\,\ddot{x} = -c\,x - b\,\dot{x} - u(t)\cdot(V_0 + \dot{x})^2$$

where $u(t) = S(t)\cdot k$, m = m2 – mass, $\ddot{x} = \ddot{x}_2$ - acceleration, x = x2 – displacement of object, $\dot{x} = \dot{x}_2$ - velocity of object, c = c12 – stiffness of spring, b = b12 – damping coefficient, V_0 - constant velocity of wind, S(t) – area variation, $u(t)$ - control action (3), k – constant. It is required to determine the control action u = u(t) for displacement of a system (2) from initial position x(t0) to end position x(t1) in minimal time T (criterion K) K= T, if area S(t) has limit (1).

2. Solution of optimal control problem for system with one degree pf freedom

For system excitation any time must be solved the high-speed problem [1 - 9]:

$$K = \int_{t_0}^{t_1} 1\cdot dt$$

.

To assume $t_0 = 0;\quad t_1 = T,$ we have $K = T$.
System, transforms to:

$x_1 = x; \quad \dot{x}_1 = x_2$ or

$\dot{x}_1 = x_2; \quad m\,\dot{x}_2 = -c\,x - b\,\dot{x} - u(t)\cdot(V_0 + \dot{x})^2,$

and Hamiltonian is [1 – 3]:

$$H = \psi_0 + \psi_1 x_2 + \psi_2 (\frac{1}{m}\cdot\left(-cx_1 - bx_2 - u(t)\cdot(V_0 + x_2)^2\right),$$

here $H = \psi \cdot X$, where

$$\psi = \begin{Bmatrix}\psi_0\\ \psi_1\\ \psi_2\end{Bmatrix}; \quad X = \begin{Bmatrix} 0 \\ x_2 \\ \frac{1}{m}\cdot[-cx_1 - bx_2 - u(t)\cdot(V_0 + x_2)^2] \end{Bmatrix}$$

Scalar multiplication of two last vector functions ψ and X in any time (Hamiltonian H [3]) must be maximal [2 – 9]. To have such maximum, control action u(t) must be within limits $u(t) = u_1; \quad u(t) = u_2$, depending only from the sign of function ψ_2 (see, for example, [3 – 6]):

$$H = \max H,$$
$$if \quad \psi_2 \cdot (-u(t)\cdot(V_0 + x_2)^2) = \max$$

Therefore if $\psi_2 > 0,_1$ the $u(t) = u_1$ and if $\psi_2 < 0,$ the $u(t) = u_2$, where $u_1 = S_1 \cdot k$ and $u_2 = S_2 \cdot k$, see (1). Examples of very simple control action (with one and three switch points) are shown in Fig. 2, 3.

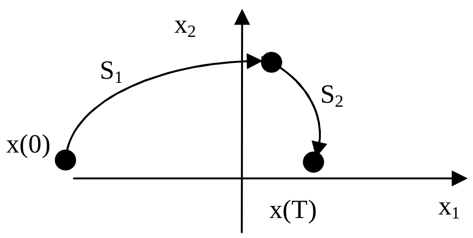

Fig. 2. Example of optimal control with one switches point

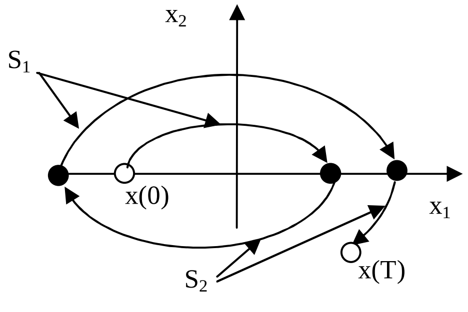

Fig. 3. Example of optimal control with three switch points when x2 = 0

How to find switches points (e.g., $\psi_2 > 0,$ or $\psi_2 < 0,$) here do not be observed [3 – 9]. But the main conclusion of optimal control law is that value of area any time must be on bounds S(t) = S1 or S(t) = S2 (5).

In real systems it allows to synthesizing quasi optimal control actions (see, for example, [10 - 13]). Additionally must be mentioned that optimal control in time domain u(t) (like programming control) in real nonlinear systems without feed back often are unstable. Therefore in this case task of synthesis new real control systems include step of forming control like mixed function of phase coordinates and time u(t) = u(x1, x2,t) (see, for example, [10 – 13]).

3. Solution of optimal control problem for system with two degree pf freedom

The equation of motion may be described as :

$$m_1\ddot{y} = -c_1 y - c_{12}(y - z) - b_1\dot{y} - b_{12}(\dot{y} - \dot{z});$$
$$m_2\ddot{z} = c_{12}(y - z) + b_{12}(\dot{y} - \dot{z}) - u(t)\cdot(V_0 + \dot{z})^2,$$

where $y, \dot{y}, \ddot{y} -$ displacement, velocity and acceleration of mass m1; $z, \dot{z}, \ddot{z} -$ displacement, velocity and acceleration of mass m2. To use new variables (phase coordinates) $x_1 = y, \quad x_2 = \dot{x}_1 = \dot{y},$ $x_3 = z, \quad x_4 = \dot{x}_3 = \dot{z}$ the system may be written in first order differential equation form

$$\dot{x}_1 = x_2;$$
$$\dot{x}_2 = \frac{1}{m_1}[-c_1x_1 - c_{12}(x_1 - x_3) - b_1x_2 - b_{12}(x_2 - x_4)];$$
$$\dot{x}_3 = x_4;$$
$$\dot{x}_4 = \frac{1}{m_2}[c_{12}(x_1 - x_3) + b_{12}(x_2 - x_4) - u(t)\cdot(V_0 + x_4)]$$

In this system with two degree of freedom Hamiltonian is

$$H = \psi_0 + \psi_1 x_2 + \psi_2 (\frac{1}{m_1}\cdot\begin{pmatrix} -c_1x_1 - c_{12}(x_1 - x_3) \\ -b_1x_2 - b_{12}(x_2 - x_4)\end{pmatrix} +$$
$$+ \psi_3 x_4 + \psi_4 (\frac{1}{m_2}(c_{12}(x_1 - x_3) + b_{12}(x_2 - x_4) -$$
$$- u(t)\cdot(V_0 + x_4)).$$

Optimal control law is the same structure then solution:

$$H = \max H,$$
$$if \quad \psi_4 \cdot (-u(t)\cdot(V_0 + x_4)^2) = \max.$$

The main conclusion of optimal control law (9) for system with two degree of freedom is the same like for system with one degree of freedom: value of area any time must be on the bounds (1), i.e. S(t) = S1 or S(t) = S2.

4. Synthesis of real control action

For realizing optimal control actions (in general case) system of one degree of freedom needs a feedback system with two adapters: one for displacement measurement and another - for velocity measurement. There is a simple case of control existing with only one adapter when motion changes directions, as shown in Fig. 3. [12]. It means that control action is similar to negative dry friction and switch points are along zero velocity line. In that case equation of motion for large velocity $|V0| \geq |\dot{x}|$ and dry friction is :

$$m \cdot \ddot{x} = -c \cdot x - b \cdot \dot{x} - F \cdot sign(\dot{x}) + U(\dot{x}),$$

where

$$U(\dot{x}) = -\left[k \cdot (V0+\dot{x})^2 \cdot S1 \cdot \frac{1+sign(\dot{x})}{2}\right] - \left[k \cdot (V0+\dot{x})^2 \cdot S2 \cdot \frac{1-sign(\dot{x})}{2}\right],$$

and m – mass; c, b, F, k, V0 – constants. Examples of modeling are shown in Fig. 4. – Fig. 7.

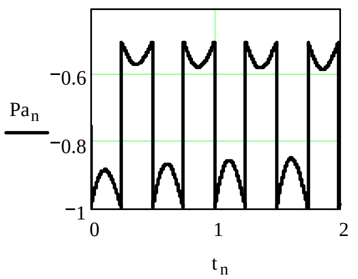

Fig. 4. Full control action (10) $Pa_n = U(\dot{x})$ in time tn domain (SI system)

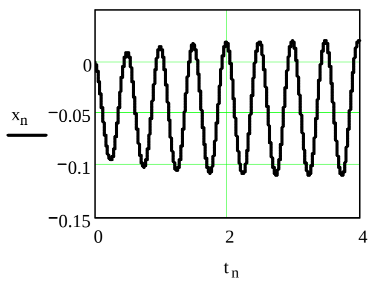

Fig. 5. Displacement x_n in time t_n domain (SI system)

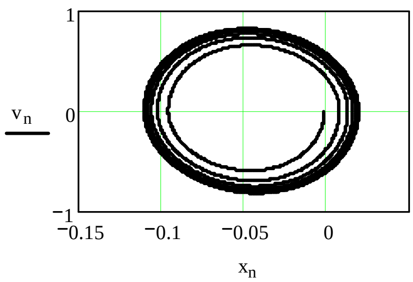

Fig. 6. Motion in phase plane ($x = x_n$; $\dot{x} = v_n$) with initial conditions inside of limit cycle

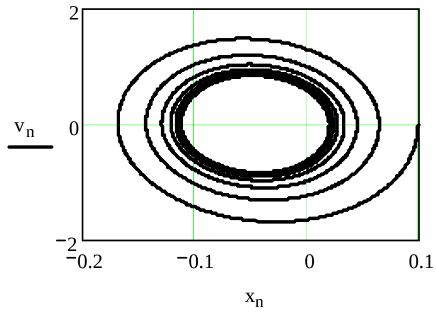

Fig. 7. Motion with initial conditions outside of limit cycle

An attempt to find more, than one limit cycle was investigated complicated system with cubic resistance force and dry friction (11). Answer is positive: for a system with non-periodical excitation (e.g. constant velocity V0 of air or water flow) there can be more, than one limit cycles. Both cycles are separated by different initial conditions (Fig. 8., 9.).

$$m \cdot \ddot{x} = -c \cdot x^3 - b \cdot \dot{x} - F \cdot sign(\dot{x}) - \\ - \left[k \cdot (V0+\dot{x})^2 \cdot S2 \cdot \frac{1-sign(\dot{x})}{2}\right] - \\ - \left[k \cdot (V0+\dot{x})^2 \cdot S1 \cdot \frac{1+sign(\dot{x})}{2}\right].$$

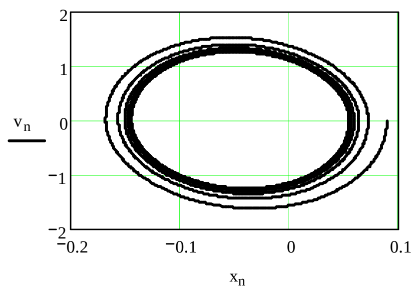

Fig. 8. Motion in phase plane for small limit cycle

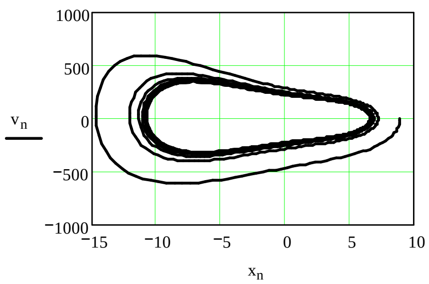

Fig. 9. Motion for large limit cycle

For system with two degree of freedom (6) was investigated the same synthesizing controls action U (see (10 - 12)):

$$U(\dot{z}) = -\left[k \cdot (V0 + \dot{z})^2 \cdot S1 \cdot \frac{1 + sign(\dot{z})}{2}\right] - \\ -\left[k \cdot (V0 + \dot{z})^2 \cdot S2 \cdot \frac{1 - sign(\dot{z})}{2}\right],$$

Results of modeling are shown in Fig. 10. – 13.

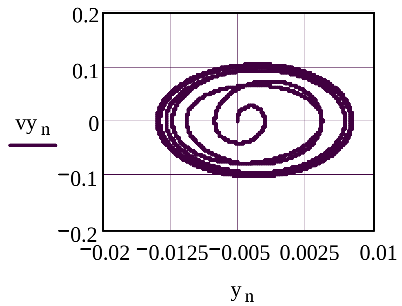

Fig. 10. Motion of mass m1 in phase plane from small initial conditions

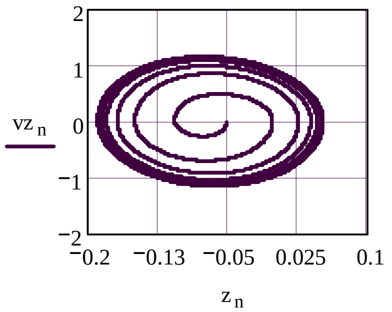

Fig. 11. Motion of mass m2 in phase plane from small initial conditions

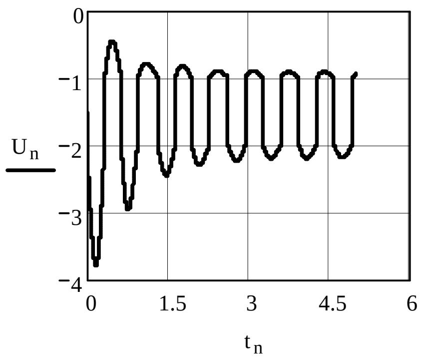

Fig. 12. Control action in (12) time domain

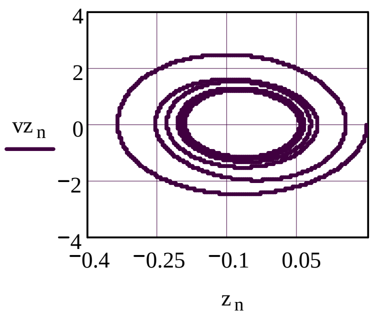

Fig. 13. Motion of mass m2 in phase plane from large initial conditions

5. Conclusion

Air or water flow may be used for excitation objects motion in vibration technique. Control of object area allows finding very efficient innovative mechatronic systems. Algorithm synthesis of strongly non – linear mechanical systems includes tasks of optimization to get principally new inventions. For realization such systems adapters, controllers and actuators must be used. Very simple control actions have solutions with use of sign functions. Description how in real system does additional area change is out of this report (for modern mechatronic systems now problem to do it).

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