406. Influence of spatial resonant oscillations of the vibratory machine working organ on the technological load behaviour

V. Zviadauri[a], G. Tumanishvili[b], T. Nadiradze
R. Dvali institute of machine mechanics, 10, Mindeli str.,
Tbilisi, 0186, Georgia
e-mail: [a] *v_zviadauri@yahoo.com*, [b] *gtumanishvili@yahoo.com*

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Abstract. For raising the quality of operation of the vibratory technologic machines a precision of the treatment and assemblage of their constructions separate details and units has a particular importance. Specificity of elastic elements (especially of springs) – to oscillate not only in direction of the acting force but also in spatial (non-working) directions, is of no small importance.
An attempt is made in the presented work to elaborate a common generalized dynamical model of the vibratory technologic machine, moving in the space due to above mentioned reasons.
The following possible deviations are envisaged in the model: of treatment and assemblage of the machine; of interlocation of elastic elements with supporting surfaces; of transfer of the exciting force.
Dynamical and mathematical models of spatial movement of the loaded vibratory technologic machine, considering above mentioned possible deviations, are elaborated.
Numerical experiments of generation of non-working spatial vibrations and their influence on behaviour of the friable technologic load, are carried out. A part of the research results is presented in this work.

Keywords: spatial oscillations, technological load, resonance, vibrating transportation, spatial dynamical model.

Introduction

Oscillation systems suspended on elastic elements (springs) together with working oscillations are subjected to spatial oscillations caused by their spatial rigidity. Because of possible errors and deviations of manufacturing and installing of machine constructional elements and assemblies arise initial deviations that should be taken into account to correct the oscillation system calculating model; among them: 1) deviations of the coordinate axes, rigidly connected to masses (parts of the system), and 2) deviation of exciting power transmission.

Numerous works [3, 4] are devoted to questions of vibratory displacement of separate particles and mass weights. However the problem of complex research of influence of all the possible kinematic and dynamic parameters of the vibratory machine on behaviour of technological loading remains still insufficiently solved.

Finding out of the mechanism of arising of non-working (parasitic) fluctuations, their development and a role in the course of vibratory technological process is required.

With this objective a development of the specified unified pattern and the mathematical model of spatial movement of the loaded vibratory technological machine, identical to actual processes, is appropriate.

Spatial dynamical model

Let's consider spatial motion of the loaded vibratory machine in this aspect (Fig. 1).

The spatial motion equations we can obtain by introduction of speeds of free points A_i and B_i in kinetic energies of masses m_1 and m_2 accomplishing accordingly translational and relative motions:

$$V_{A_i} = V_{O_1} + \omega_{O_1} \times r_{1i}$$
$$V_{B_i} = V_{O_1} + \omega_{O_1} \times R_{2i} + V_{O_2} + \omega_{O_2} \times r_{2i}. \quad (1)$$

Position of the ***n*** mass spatial interconnected

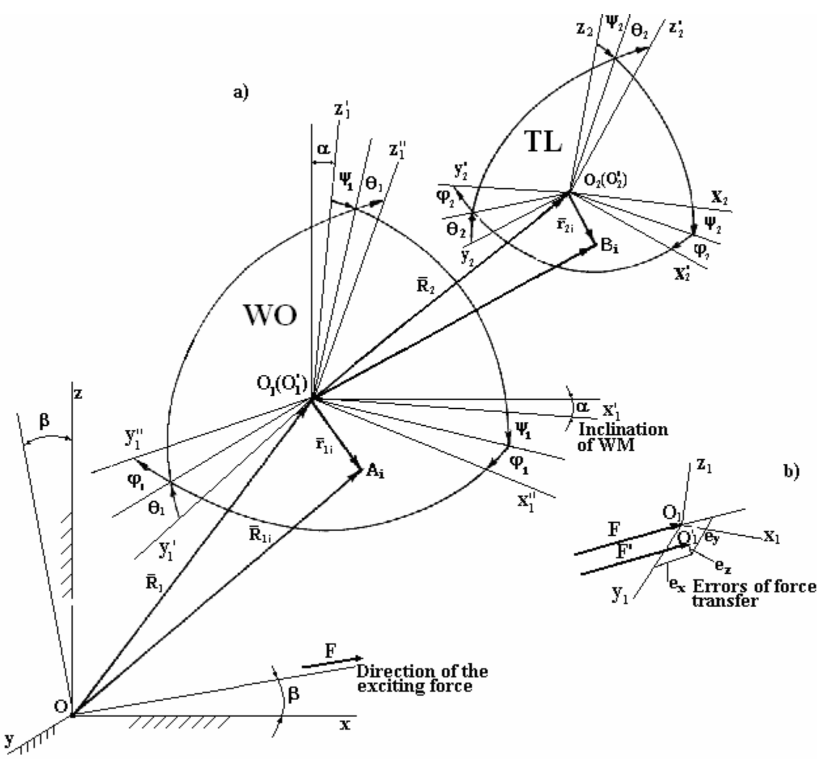

Fig. 1. Motion of the WO (O_1) and TL (O_2) in space

oscillatory system is defined by coordinates of centroids of masses, by points of fastening of elastic elements and by their reduction to a certain coordinate system using the directing cosines by table 1:

Table 1

	x_i	y_i	z_i
x	$(\alpha_i)_{11}$	$(\alpha_i)_{12}$	$(\alpha_i)_{13}$
y	$(\alpha_i)_{21}$	$(\alpha_i)_{22}$	$(\alpha_i)_{23}$
z	$(\alpha_i)_{31}$	$(\alpha_i)_{32}$	$(\alpha_i)_{33}$

By this model are obtained the interrelated equations of motion of masses m_1 and m_2 with nonlinear members of inertial and elastic character depending on the expression of directing cosines of Euler angles (in linear or nonlinear forms).

Let's present directing cosines in the form of Euler-Krylov's angles [3] and in their decomposition we shall consider products not higher than second order of infinitesimal (Table 2):

Table 2

	$x_1^{'}$	$y_1^{'}$	$z_1^{'}$
x_1	$1-\psi_{O1}^2/2-\varphi_{O1}^2/$	$-\varphi_{O1}+\psi_{O1}\theta_{O1}$	ψ_{O1}
y_1	φ_{O1}	$1-\theta_{O1}^2/2-\varphi_{O1}^2/$	$-\theta_{O1}$
z_1	$-\psi_{O1}+\varphi_{O1}\theta_{O1}$	$\varphi_{O1}\psi_{O1}+\theta_{O1}$	$1-\psi_{O1}^2/2-\theta_{O1}^2/2$

For definition of dynamic position of the working organ (WO) are used directing cosines, obtained by the expression

$$\left|\lambda_{ij}\right| = \left|\alpha_{ij}\right| \left|\alpha_{ij}^{'}\right|, \qquad (2)$$

where α_{ij} are directing cosines of the angles, caused by initial imperfections $\theta_{oi}, \psi_{oi}, \varphi_{oi}$, and $\alpha_{ij}^{'}$ - by dynamic displacement $\theta_i, \psi_i, \varphi_i$; expansion of the right parts of eq. (2) is carried out by multiplication of lines of the first determinant on columns of the second one:.

$$\lambda_{11} = \alpha_{11}\alpha_{11}^{'} + \alpha_{12}\alpha_{21}^{'} + \alpha_{13}\alpha_{31}^{'},$$
$$\lambda_{12} = \alpha_{11}\alpha_{12}^{'} + \alpha_{12}\alpha_{22}^{'} + \alpha_{13}\alpha_{32}^{'} \quad \text{etc.} \quad (3)$$

In this case position of the coordinate system $O_1^{"}x_1^{"}y_1^{"}z_1^{"}$ relatively to $O_1x_1y_1z_1$ will be defined by table 3 where angles of inclination α and vibration β of the WO are also taken into account (the fragment of the full table is given only on x_1):

Table 3

	$x_1^{''}$	$y_1^{''}$	$z_1^{''}$
x_1	$(1-\psi_1^2/2-\varphi_1^2/2-\varphi_{O1}^2/2-\psi_{O1}^2/2-$ $-\varphi_1\varphi_{O1}-\psi_1\psi_{O1})\cos\alpha+(\varphi_{O1}\theta_{O1}-$ $-\psi_{O1}+\varphi_1\theta_{O1}-\psi_1-\varphi_1\theta_1)\sin\alpha$	$(\psi_1\theta_1-\varphi_1-\varphi_{O1}+\psi_{O1}\theta_{O1}+$ $+\theta_1\psi_{O1})\cos\alpha+(\varphi_1\psi_{O1}+$ $\varphi_{O1}\psi_{O1}+\theta_{O1}-\varphi_1\psi_1+\theta_1)\sin\alpha$	$(\psi_{O1}-\theta_1\varphi_{O1}+\psi_{O1})\cos\alpha+$ $+(1-\psi_1\psi_{O1}-\theta_1\theta_{O1}-\psi_1^2/2-$ $-\theta_1^2/2-\psi_{O1}^2/2-\theta_{O1}^2/2)\sin\alpha$

Rotations of other coordinate systems will be defined in similar way.

In view of speeds (1) and directing cosines according to Table 3 the expressions of kinetic energy of motion of masses ***m_1*** and ***m_2*** will be decomposed on coordinate axes $O_1x_1y_1z_1$.

The basic differences of the considered system from the classical n-mass spatial system are: a) specificity of the mass ***m_2*** (various materials one-sided connected to the mass ***m_1***); b) defined initial location of masses m_1 and m_2 (angles α and β, initial angular deviations $\theta_{oi}, \psi_{oi}, \varphi_{oi}$, eccentricities of exciting power transfers $-e_x, e_y, e_z$), that makes asymmetric the general sequence of construction of the mathematical model; c) particularities of interaction of masses ***m_1*** and ***m_2***, connected with each other by conditional elastic liaison depending on characteristics of the technological load(TL); d) consideration of deviation of elastic liaisons from undeformed state (at great deformations).

Let's present the differential equations of spatial motion of masses ***m_1*** and ***m_2***, obtained with the use of Lagrange method.

Equations for the WO (m_1) (two equation for translational and rotary motions are given):

$$
\begin{aligned}
&(m_1+m_2)\dot{x}_1+m_2[(\ddot{\psi}_1 z_2+2\dot{\psi}_1\dot{z}_2-\ddot{\varphi}_1 y_2-2\dot{\varphi}_1\dot{y}_2-\ddot{y}\varphi_1+\ddot{z}_2\psi_1)\cos\alpha_1+\\
&+(\cos\alpha_1+\psi_{01}\sin\alpha_1)\ddot{x}-(\varphi_{01}\cos\alpha-\theta_{01}\sin\alpha_1)\ddot{y}_2+(\psi_{01}\cos\alpha+\sin\alpha_1)\ddot{z}+(\ddot{\theta}_1 y_2+\\
&+\ddot{x}_2\psi_1-2\dot{\theta}\dot{y}_2-2\dot{x}_2\dot{\psi}_1-\ddot{\psi}x_2+\ddot{y}_2\theta_1)\sin\alpha_1]=Q_{x_1}+Q_{x_1}^{'};
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
&A_{1\theta}\ddot{\theta}_1+A_{2\theta}\ddot{\psi}_1\varphi_1+A_{3\theta}\dot{\varphi}_1\dot{\psi}_1+m_2(\ddot{z}_1 y_2\cos\alpha_1+\ddot{x}_1 y_2-\ddot{y}_1 z_2+y_2\ddot{z}_2-z_2\ddot{y}_2)+\\
&+A_{4\theta}\ddot{\psi}_2\varphi_1+A_{5\theta}\dot{\varphi}_3\dot{\psi}_1+A_{6\theta}\ddot{\varphi}_2\psi_1+A_{7\theta}\dot{\psi}_2\dot{\varphi}_1+A_{8\theta}(\ddot{\theta}_2+\ddot{\psi}_2\varphi_{01}+\ddot{\varphi}_2\psi_2-\\
&-\ddot{\varphi}_2\psi_{01}-\ddot{\varphi}_2\psi_1)=M_{x_1}-M_{x_1}^{'},
\end{aligned}
$$

where coefficients $A_{1\theta}$, $A_{2\theta}$, $A_{3\theta}$....., etc. are functions of the sums of moments of inertia of masses with respect to corresponding axes, as for example

$$A_{1\theta}=J_{x_1}^{O_1}+J_{x_2}^{O_2};\ A_{2\theta}=J_{x_1}-J_{y_1}+J_{z_2}-J_{y_2};$$
$$A_{3\theta}=J_{x_1}-J_{y_1}+J_{z_2}-J_{y_2}+J_{z_2}+J_{z_1}.$$

Q_q and M_q - forces and moments of elastic (potential) character; $Q_q^{'}$ and $M_q^{'}$ - external forces and moments.

The items of the left side of eq. (4) contain inertial forces arose in the result of interaction of masses m_1, m_2. At that, nonlinear members (products) in the first equation have arisen due to the relative movement of the mass m_2 and due to considering of errors of fabrication and assemblage of the real machine ($\theta_{01}, \psi_{01}, \varphi_{01}$, etc). In the case, when m_2 =0 the mentioned items vanish. There are non-linear members in the next equation (rotary motion) from both, the rotary motion of the mass m_2 and the proper movement of the mass m_1, that follows from the general rule of drawing up of mathematical expression of the rigid body movement [4].

Differential equations of motion of TL (m_2), carrying out relative motion in the coordinate system $O_1x_1y_1z_1$ and absolute motion - in *Oxyz* will have the following form (two equations for translational and rotary motions are given):

$$m_2[\ddot{x}_2+(\ddot{x}_1-\ddot{z}_1\varphi_{01}-\ddot{z}_1\psi_1)\cos\alpha_1-(\ddot{x}_1\psi_{01}+\ddot{x}_1\psi_1+\ddot{z}_1)\sin\alpha_1+\ddot{y}_1\varphi_{01}+\ddot{y}_1\varphi_1+\ddot{\psi}_1 z_2- \quad (5)$$

$$-2\dot{\psi}_1\dot{z}_2-\ddot{\varphi}_1 y_2-2\dot{\varphi}_1\dot{y}_2]=Q_{x_2}+Q'_{x_2};$$

$$C_{1\theta}(\ddot{\theta}_2+\ddot{\theta}_1)+C_{2\theta}\ddot{\psi}_2\varphi_{01}+C_{3\theta}\ddot{\psi}_2\varphi_1+C_{4\theta}\dot{\psi}_2\varphi_1-C_{5\theta}\ddot{\varphi}_2\psi_2+C_{6\theta}\dot{\varphi}_2\dot{\psi}_2+C_{7\theta}\ddot{\varphi}_2\psi_{01}+$$

$$+C_{8\theta}\ddot{\varphi}_2\psi_1+C_{9\theta}\dot{\psi}_1\dot{\varphi}_2+C_{10\theta}\ddot{\psi}_1\varphi_1+C_{11\theta}\ddot{\varphi}_1\psi_1-C_{12\theta}\ddot{\psi}_1\varphi_{01}+C_{13\theta}(\ddot{\varphi}_1\psi_1-\ddot{\varphi}_1\psi_{01}-$$

$$-\ddot{\varphi}_1\psi_2)=M_{x_2}+M'_{x_2}.$$

Similarly to the previous equations, coefficients $\boldsymbol{C_{iq}}$ are functions of the sums of the moments of inertia of masses.

Equations (4) and (5) are interconnected by nonlinear items of potential and inertial character, and the form of liaison for both systems of the equations is similar; difference is in the presence of the sum of masses $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ in system (4) where as in (5) only one mass $\boldsymbol{m_2}$ appears. The principle of vibratory motion of one body with respect to another is just in such form of dependence.

It should be noted, that systems (4) and (5) describe a movement of the mass $\boldsymbol{m_2}$ relative to $\boldsymbol{m_1}$ at their constant interconnection; potential forces in the form of Q_q are elastic and damping characteristics of the mass $\boldsymbol{m_2}$ (at the friable material) and depending on its type and location relative to $\boldsymbol{m_1}$, the characteristics will vary. Besides, a dynamical relation between $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ can be uneven and in this case systems of equations (4) and (5) are not true any longer without corresponding corrections [4].

Forces Q'_q are not related to deformation of the elastic system or to inertness of the oscillatory system; these are: external forces, force of gravity and resistant force of external friction type. Consider their decomposition on coordinate axes in view of initial angular deviations of manufacturing and installing of the machine $\theta_{oi},\psi_{oi},\varphi_{oi}$ and corresponding eccentricities of application of the exciting power e_x, e_y, e_z (Fig.1b).

Different approaches are applied to the description of friction forces between WO and TL for materials of loose type [1, 2, 4], the idea of which is that the reaction of TL on WO is proportional to the speed and deformation of TL; at that both, the internal resistance and the resistanse of the environment in which the movement is realized, are taken into account

$$N_q = f(\dot{q}, q).$$

At spatial motion (translational or rotary)of TL, besides normal reactions, as a result, the moments of these forces arise

$$(F_{fr})_q = fN_q; \quad (M_{fr})_q = (F_{fr})_q \cdot r_q. \quad (6)$$

where f is a coefficient of friction of TL on the WO surface (f is usually taken variable in each cycle of the motion, depending on dynamic state of TL – sliding on the surface, stops etc.); $\boldsymbol{r_q}$ – distance from the surface of friction up to the centre of gravity of TL in the direction of coordinates (Fig.2); the components of friction forces can be given in this way

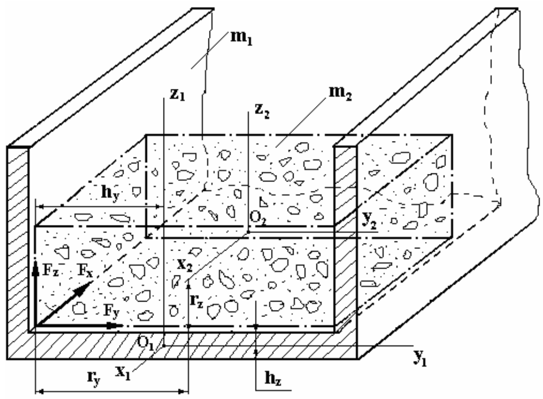

Fig. 2. Model of TL(m2) on WM (m1)

$$F_{x_2} = f_x N_z sign(\dot{x}_2);$$

$$F_{y_2} = f_y N_y sign(\dot{y}_2); \quad (7)$$

$$F_{z_2} = f_z N_z sign(\dot{z}_2),$$

where f_x , f_y , f_z are friction coefficients between TL and WM along directions x, y, z; N_y – normal reaction of the load on the lateral surfaces; N_z – normal reaction of the load on the botton urface; the function *sign* is non-linear and is determined depending on the sign of the velocity V: *sign*=1 at $V < 0$ and *sign* = -1 at $V > 0$.

Ther moments of the friction forces with respect to axes of the system $O_2x_2\, y_2\, z_2$ have the form:

$$(M_{fr})_{x_2} = (F_{z_2} r_y - F_{y_2} r_z) sign(\dot{\theta}_2);$$

$$(M_{fr})_{y_2} = (F_{x_2} r_z) sign(\dot{\psi}_2); \quad (8)$$

$$(M_{fr})_{z_2} = (F_{x_2} r_y) sign(\dot{\varphi}_2);$$

Where r_y, r_z are distances from friction surfaces to axes of the system $O_2x_2\,y_2\,z_2$.

The moments of the friction forces relative to axes of the co-ordinate system $O_1x_1\,y_1\,z_1$ (axes of the WO) have the following form

$$(M_{fr})_{x_1} = (F_{z_2}h_y - F_{y_2}h_z);$$
$$(M_{fr})_{x_1} = F_{z_2}h_z; \quad (M_{fr})_{z_1} = F_{x_2}h_y, \qquad (9)$$

where h_y, h_z are distances from friction surfaces to axes of the system $O_1x_1\,y_1\,z_1$.

In the considered case the WO is restricted from two sides (with planes $O_1x_1\,y_1$ and $O_1\,z_1x_1$) and in the direction O_1x_1 it is oupen; in this case a friction force on the surface $O_1x_1\,y_1$ is absent and consequently items with multipliers in expressions (8) and (9) are also absent.

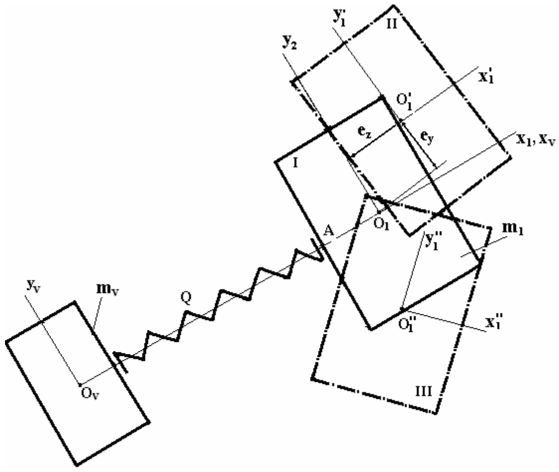

Fig. 3. Location of the WO relative to vibro-exciter (m_v): in ideal position (I); after assemblage (II); after dynamical displacement (III)

The form of the exciting force (Q) depend on the vibro-exciter type. In any case a direction and point of application of the force are known.

The WO is presented in three different positions in the Fig.3: I – initial (ideal) position, when direction of the Q coincides with the axis of the non-deformed spring and passes through the centroid of the WO – O_1; II – corresponds to the real position, i.e. considering deviations determined by tolerances on fabrication and assemblage of the machine and by eccentricities of transfer of the force Q; III – corresponds to the position after dynamical displacement.

Considering the mentioned initial and dynamic deviations, projections of the force Q on the coordinate axes of the system $O_1x_1y_1z_1$, will be:

$$\left.\begin{aligned} Q_{x_1} &= Q[(\psi_{01}+\psi_1)\sin\alpha_1 + \cos\alpha_1; \\ Q_{y_1} &= -Q[(\theta_{01}+\theta_1)\sin\alpha_1 + (\varphi_{01}+\varphi_1)\cos\alpha_1]; \\ Q_{z_1} &= Q[(\psi_{01}+\psi_1)\cos\alpha_1 + \sin\alpha_1]. \end{aligned}\right\}(10)$$

The moments of the force Q are determined according to theory of the vector algebra []. If we determine co-ordinates of the point O_1” through which really passes the vector of the force Q; then the moments of thi vector relative to axes of the system $O_1x_1\,y_1\,z_1$ will have the form:

$$M_{x_1} = e_{y_1}Q_{z_1} - e_{z_1}Q_{y_1}\ ; \quad M_{y_1} = e_{z_1}Q_{x_1} - e_{x_1}Q_{z_1}\ ;$$
$$M_{z_1} = e_{x_1}Q_{y_1} - e_{y_1}Q_{x_1}\,. \qquad (11)$$

The equations (4) and (5) in view of eqs. (6), (10) and (11) enable us to carry out comprehensive investigation of the vibratory technological processes.

Particularly, a stady of influence of spatial vibrations of the machine WO on the technologic process of vibro-transportation is realized by means of resounding this or that spatial vibrations at fixed parameters of the remaining ones.

Results of numerical experiments

In Figures 4, 5, 6 are presented relations of velocities of the friable load displacement to variation of frequency (amplitude) of separate spatial oscillations of the WO of the resonant vibratory machine, obtained by computer modeling. As it is shown, at increase of the amplitude of one or another spatial (non-working) oscillation of the WO above limits of the certain size, the speed of basic (on the longitudinal axis) displacement of the TL essentially varies: at one moment reduces and at another - increases; It is especially noticeable at passing of frequency through resonance (**ω** = 50 Hz) when the phase of the fluctuation changes into opposite one.

It should be noted that in all cases the operating conditions of the machine are the same (ω_x=50 Hz) and additional resonance oscillations were introduced into the process simultaneously.

The given dependences show that in the resonance vibratory machines, accompanying the basic operating conditions, spatial oscillations break regularity of the vibratory motion of the loose material. Thus, at passing through the resonance, speed of the material sharply varies and in some areas of frequency (amplitude) change of speed is commensurable with its calculating magnitude.

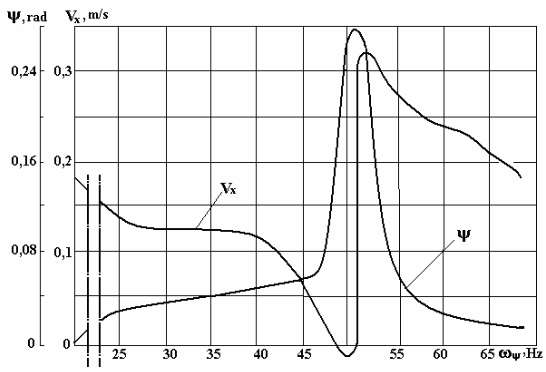

Fig. 4. Dependence of the speed (V_x) of the TL in the longitudinal direction on the resonant rotary oscillations (ψ) of the WO

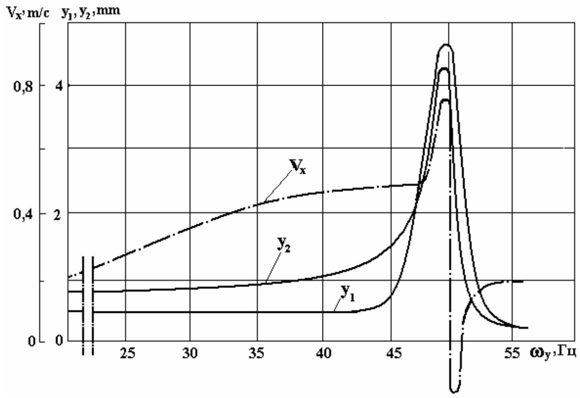

Fig. 5. Dependence of the speed (V_x) of the TL in the longitudinal direction *and trajectories (Y_2)* on the *cross-section resonant oscillations (y_1)* of the WO

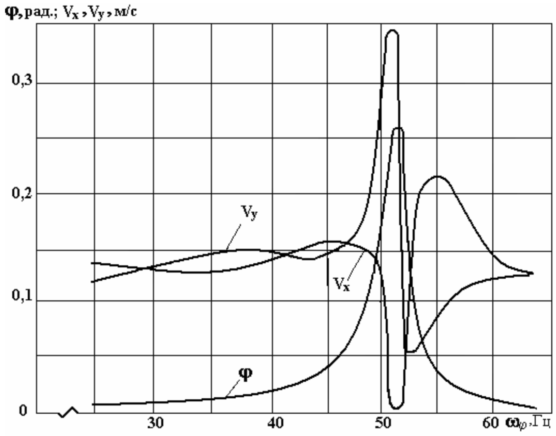

Fig. 6. Dependence of the speed of the TL in the longitudinal (V_x) and cross-section (V_y) directions on the resonant rotary oscillations (φ) of the WO

Can be concluded that with the use of the given mathematical model (in particular, at realization of combined oscillations of WO) it is possible to control the vibratory technological process and achieve its optimization.

Conclusions

1. The reasons and the mechanism of arising of spatial fluctuations in the vibratory machine are considered and analyzed.
2. The generalized pattern of spatial movement of the vibratory machine with a technological load is developed and the appropriating differential equations are deduced.
3. Influence of various spatial resonant fluctuations of the vibromachine working member on behaviour of the friable technological load is investigated.
4. It is established, that the combination of separate partial resonant fluctuations with the working fluctuation causes essential change of speed of the material transportation. Basically, these variations have a negative character, however, in some cases there is an increase of speed of transportation.

At analysis of results of the research it is planned to create a design of the vibratory exciter with two-componental (combined) oscillatory operating conditions.

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