405. Generating of sub harmonic resonant oscillations and problems of their stability

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Abstract. In a defined frequency area of any nonlinear systems two stabile resonance operating conditions are generated with high or low amplitudes of oscillations. Together with that, if the nonlinearity of the exciting force (for instance electromagnetic traction force) is a reason of the system nonlinearity then in a system can be established a parametric type of sub harmonic resonant operating conditions. Sub harmonic oscillations are causing interest because it ensures to carry out many technological operations where low frequency oscillations are needed. Regrettably these types of oscillations up to date are not studied on the sufficient level. In the field of the research of nonlinear resonant operating conditions, the amplitude frequency characteristic (AFC) gives more significant information on stability and reliability of the got amplitudes. A study of the nonlinear systems and their related processes meets much difficulty. Especial follows to note that difficulties existed under mathematical simulation, when the frequency of the external force or technological load happened to change discrete. The carried out researches enables us to obtain the stable operating conditions of non-linear electromagnetic vibrators.

Keywords: sub harmonic resonant oscillations, electromagnetic vibratory machine, electromagnetic force, nonlinear resonant operating conditions, differential equation, mathematic simulation.

Introduction

On the base of broad investigations can be said that for obtaining stable sub harmonic resonant operating conditions (for performing many technologic operations is needed 25 Htz and lower frequency of oscillations) not only multiple frequency relation is needed between system rigidity and exciting force but there must be determined also a relation between component parameters of the exciting force as they are: inductive and actual resistances, voltage and clearance. Here must be underlined that the pointed parameters are in a high correlation between themselves that causes certain difficulties to obtain above mentioned relations between them.

For forming a pulsation exciting force, the electromagnet coil must be supplied by alternating voltage. Formed inductive resistance defiantly limits nonlinear dependence of traction force to value of the clearance. At the period of oscillation increase of the value of clearance is accompanied by decrease of inductive resistance of the coil and hereupon by increase of the running currant in the coil and on the contrary. Thereby at the alteration voltage the dependence of traction force of the electromagnet on the value of clearance is something decreased unlike to electromagnet working under direct current which depends quadratically on a value of the clearance. For obtaining sub harmonic operating conditions it is necessary that any change of existed clearance were caused by sharp change of the electromagnet traction force.

Electromagnetic force. For obtaining mechanical oscillations with frequency of 50 Htz the alteration voltage is rectified semi periodically. Aforesaid is approached by means of the successive cut-in diode in an electric circuit that causes constant forming of the electromagnetic force with a corresponding mechanical asymmetric oscillation.

At the condition of defined relation of the traction force (which is formed in a magnet as a permanent component) to the coin active resistance (that is limiting also electric current at the increase of clearance together with increased dispersion of magnetic flow) is formed additional nonlinearity of electromagnet traction force in respect of value of the clearance. So there is formed possibility for generating sub harmonic oscillation. Here must be emphasized that the more becomes amplitudes of sub harmonic oscillations the more bigger becomes system nonlinearity. That is to say there is established an attractive process. Practically to limit constantly increased oscillation amplitudes (up to blow up of electromagnetic parts) is possible by means of adding nonlinear dissipation forces.

On the base of investigations can be said that generating of sub harmonic oscillations require exact co-ordination of aforementioned parameters. For instance direct increase of active resistance of the coil (by means of increase of number of twisted wire or decrease of its diameter or direct cut-in active resistance from outside of the coil) limits magnet powers, but increase of voltage (at the increased clearance) cause sharply diminution cosφ and vibrator's coefficient of efficiency. From the standpoint of economy two low power vibrators working in the parallel operating conditions are not acceptable. In such case sharply are increased dimensions of the vibrator and expensive copper consumption.

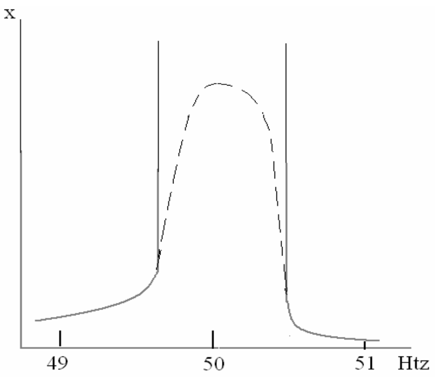


Fig. 1. AEC of the sub harmonic vibration machine. AFC of intermittent line is obtained by means of adding nonlinear dissipation forces

On the base of research can be said that up today condition of approaching electronic technique the generating of sub harmonic operating conditions is effective only for electromagnet vibrators which have 0.5 k wt and more lower power, as to the vibrators of more bigger powers working at low frequencies is reasonable that the electromagnet vibrators were made by trigger controlled or used other type vibrators, for instance more expensive un balanced vibrators

Particularly it should be noted that researches of machines at aforesaid operating conditions by means of mathematic simulation there exists certain difficulties. If the results of experimental and analogy modeling are coincided more or less (anyway) then at the mathematic simulation it is practically imposable to build amplitude and frequency diagrams and research other transitional processes at the condition of variable technologic loading if additionally there are not taken into account border conditions of differential equations.

The work of noted electromagnet vibrators are described by following differential equation

$$\ddot{x} + 2h\,\dot{x} + \omega_0^2 x = a\,\phi^2, \qquad (1)$$

$$\dot{\phi} = b\sin\,\omega t - c(\delta - x)\phi. \qquad (2)$$

herewith

$$\phi = \left|\begin{array}{l} 0 \text{ when } \phi < 0 \\ \phi \text{ when } \phi > 0 \end{array}\right.$$

When mechanical oscillation displacement x is infinitesimal or completely is absent was solved an equation [1,2,3]:

$$\psi = \frac{b\omega}{(c\delta)^2 + \omega^2} e^{-c\delta t} + b\,\frac{c\delta\sin\omega t - \omega\cos\omega t}{(c\delta)^2 + \omega^2}.$$

Analysis of equation (3) shows that the square-law member $a\psi^2$ of equations (1) is conducting not wholly correct role in the process of generating the non-linear resonance and in the similar form is met in the literature. Certainly, when feeding a magnet by only direct currents $\omega=0$ (that the same $b\sin\omega t=u_c$) tractive magnet force $F=a\psi^2$ is changed according to the square-law. However at alternating current ($\omega\neq0$) will not difficult to make sure that because of presence of inductance of spools, variable δ causes inverse proportional changing of the inductive resistance ωL. Due to that tractive force of electromagnet, practically it become independent from δ. For instance, under $\omega=0$ according to the equation (3) $\psi=uo/\delta$, but under $\omega\neq0$ and $\delta=0$ it takes a form:.

$$\psi = \frac{b}{\omega} - \frac{b}{\omega}\cos\omega t. \qquad (3)$$

Equation (4) shows that magnetic flow in a magnet core because of presence in it of inductance of spools is sharply limited. The member b/ω, exists when starting a machine only and is referred to as fading part of equations (3).

Equation (2) is formed for linear magnetization area. Magnetic resistance in a core is ignored [3]. So functional dependence between: ψ, δ and x is linear.

On the base of aforesaid, for determination of influence of air clearance δ on the dynamics of vibration machine, it is reasonable to solve an equation (2) analytically. After starting the machine the afore mentioned transitional process can be considered completed.

$$\psi = -\frac{b}{\omega}\cos\omega t + \frac{bc\delta}{\omega^2}\sin\omega t - \frac{bcA_{(\omega)}\sin\varphi}{\omega^2}\sin\omega t + \ldots \quad (4)$$

Really, with increase of t first member approaches to zero in (3) and consequently, oscillatory processes is supported only by the second member

From the equation (8) also follows that main role in the oscillatory process plays a first member (b/ω)cosωt. For example under $b=0{,}2\div0.3$; $c=300\div500$; $\delta=0.002\div0.004$ and $\omega=314$ corresponding 50 Htz turns out to be 200÷300 times greater in quantity than second or third members. Certain interest causes second or third member's physical nature of which is greatly distinguished from the first member. They practically are additional members of system rigidity with variable factors [3,4]. In the equation (8) the value $A_{(\omega)}\sin\omega t\sin\varphi$ corresponds to a displacement x. It is not difficult to see that equation (1) with (8) is Hillas equation of parameter resonance [4]..

Practically the similar result is obtained by the spreading (factoring) equation (3) in power-mode row of δ:

$$\psi = \frac{b}{\omega} - \frac{bct\delta}{\omega} + \ldots - \frac{b}{\omega}\cos\omega t + \frac{bc\delta}{\omega^2}\sin\omega t + \ldots \quad (5)$$

Thereby, from accomplished analysis of equations (3,8,9), follows highly essential conclusion: when the machine is power supplied by alternating current in spite of the air clearance δ and oscillation amplitude A practically do not render influences upon the value of tractive force and their presence causes parametric resonances.

Raising members of equation (6) or (9) to the second power gives expression for the determination of disturbing force of electromagnetic vibratory machine. From equation (6)

$$\psi^2=\frac{b^2}{\omega^2}-2\frac{b^2}{\omega^2}\cos\omega t-2\frac{b^2c^2(\delta-A_{(\omega)}\sin\varphi)^2t}{\omega^3}\sin\omega t\cos\omega t+\ldots \quad (6)$$
$$\ldots+\frac{b^2}{\omega^2}\cos^2\omega t+2\frac{b^2c(\delta-A_{(\omega)}\sin\varphi)}{\omega^3}\sin\omega t\cos\omega t+\ldots$$

Presence of active resistance r causes a shift on phase φ=arc tg(ωL/r) in circuits of power supply. At semi-periodic rectifying because of reduction of inductive resistance leads to asymmetrical mechanical oscillations.

For the simplification of calculations of the main resonant operating conditions without the essential error it is possible to change a system differential equation by one equation, or to leave in the right part of eq. (1) only main member stimulating main resonance.

$$\ddot{x}+2h\dot{x}+\omega_0^2x=-a\frac{b^2}{\omega^2}\cos^2\omega t, \quad (7)$$

At the research of dynamic processes for aforesaid machines annulment of parameter u causes interruption of ϕ that breaks required condition of needing and sufficiency [4]. Non-execution of the noted case causes inaccuracy and unrealistic results. If there are need to interrupt one of the parameters then there must be taken into account border conditions for the breaking area.

Mathematical simulation. On the fig. 1 are given oscillations of magnet flows ϕ and mechanical system x. The interruption of ϕ is caused by discrete change of argument of acted force fsin(ωt) fig. 1a. The sharp change of ω also causes sharp change of angle of the shift that accordingly gives sharp 2-4 cycling increase or decrease of oscillation amplitudes depending on phase relation of angle of the shift and amplitudes of oscillations (accompanying or opposite).

The correction of angle of the shift of the border condition may be solved very simply. By comparison of conditions $\omega_{vo}t=\varphi_o$ before and after $\omega_{v1}t=\varphi_1$ changing ω can be defined new meaning $t_1$of t ($\omega_{v1}t_1=\varphi_1$ replacing φ_1 on φ_o) so that to carry out condition $t_1=\varphi_o/\omega_{v1}$. That is to say that on a new value of argument ω the angle of the shift φ_o is stayed former. By taking into account the given condition the modeling system is continuing oscillations with a new period smoothly and goes on new operating conditions quietly that is very necessary and decisive for the researches of nonlinear resonant operating conditions for getting real results by mathematic simulation fig. 2b.

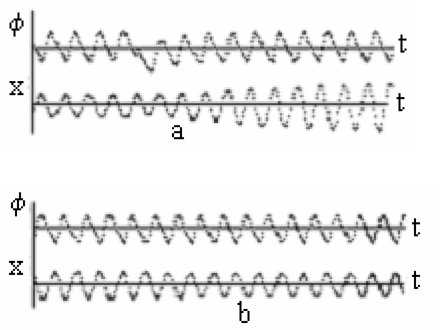
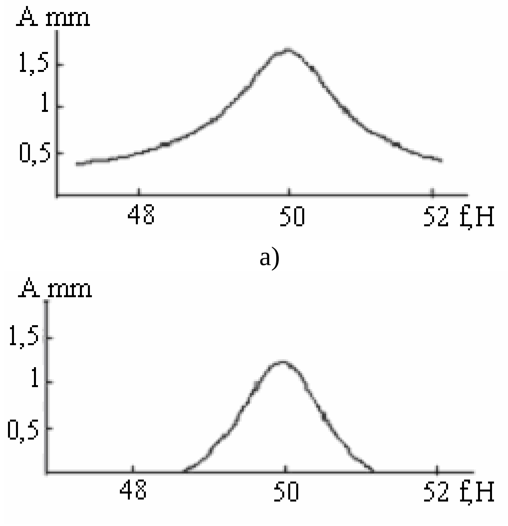


Fig. 2. a) nonstabilar amplitudes by annulment the Voltage; b) stabile amplitudes obtained by means of discrete change of frequency parameter with a shift correction

On the fig. 3b is given amplitude frequency characteristic (AFC) diagram of 50/50 resonance regime at the case of breaking voltage. The stability of process is approached artificially by mean of correction of inductive resistance. Description of the halftime rectified current by means of differential equation 2 (where is formed magnet flow) is not a simple task. As the current is half timely rectified by means of diode (which is successively inserted in electric circuit), so the periodic interrupting of the voltage is inadmissible at the mathematical simulation. Since current and magnet flow are shifted on phase from voltage (90^0 angles) that under annulment of the voltage the current and magnetic flow are not getting a zero meaning in this moment. Thereby at the annulment of the voltage in equation 2 (but at this moment the magnetic flow have not a zero meaning) ϕ begins grows instead of decrease. So, at the moment of voltage annulment the equation 2 is not describing electromagnetic flow.



a)

b)

Fig. 3. a) AFC under cutting magnet flow; b) AFC with a annulled voltage

On the fig. 3a is given amplitude frequency characteristic for the same operating conditions by means of cutting magnet flow. Obtained diagram is very real in contrast with 2b which is received under annulment electric voltage with artificial correction of the inductive resistance.

For solving noted problems it becomes necessary (at the annulment voltage) to change equation 2 by equation describing disappearance of magnetic flow after the cessation of the electric voltage. Certainly, for full-flanged link-up separate areas of equations there must be taking into account the shifts of the phases between electric voltage and magnetic flow before and after the resonant condition of the nonlinear system.

As the direct current and magnet flow are shifted from electric voltage evenly, it becomes possible by certain accuracy that noted problem be solved simpler. Instead of annulment of the electric voltage there is possible to cut (half timely rectified) the magnetic flow taking into account shifts between direct currant and magnetic flow for the linking-up separating areas of equations that is very required for obtaining the real picture at study of nonlinear resonant operating conditions by means of mathematic simulation.

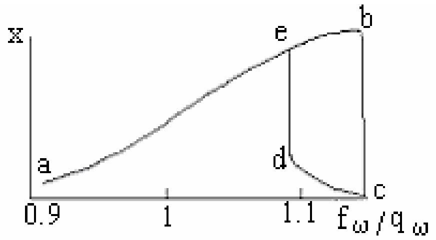
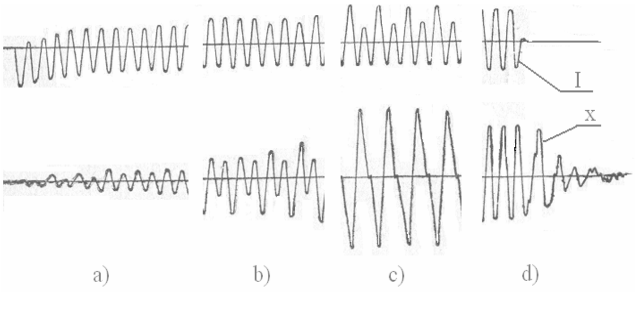


Fig. 4. AEC of the nonlinear vibration machine

On the fig 4 is given AFC which is build by means of aforesaid correction (at discrete changing of the frequency of the exiting forse) with possibility of obtaining eb and cd areas of the curve at the mathematical simulation. Here follows to remind that, arising, generating and stability of the nonlinear amplitudes greatly depends on a springy-friction features [5].



a) b) c) d)

Fig. 5. sub harmonic amplitudes of operating conditions starting a), transition b) stabile and output from pesonance processes

Technological loading (periodically joined and teared away mass m_t) under mathematical simulation was taken into account as follows

$$m = \frac{(m_t + m_1) m_2}{(m_t + m_1) + m_2} \qquad (8)$$

Where m_1 and m_2 are active and reactive mass of the machine accordingly

Conditions of periodic joined and teared away mass m_t from working part of the machine was considered by means of equation (9) together with differential equations (1) and (2).

$$s = s_o + v_o t - g t^2 / 2 \qquad (9)$$

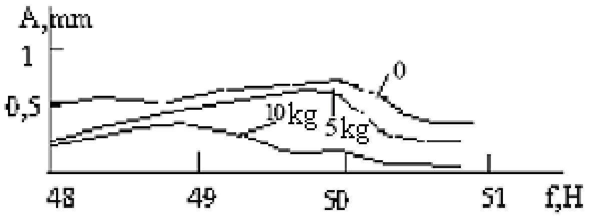


Fig. 6. Influence of technologic load on AFC

On the fig.6 is given AFC obtained by means of mathematical simulation on the two operating condition 30 kg laboratory vibration machine at the different technologic loadings. The technologic loading gives machine nonlinearity. The peak of resonance curve is displaced aside lower frequencies of 1.25 Htz.

Conclusion

By means of mathematical modeling was explored low frequency electric vibratory machines dynamics with provision of technological load. Mistakes and difficulties of mathematical simulation of the dynamic systems with periodic changing parameters are revealed and ways for their overcoming are shown.

Finally it should be noted that in certain operating conditions in electromagnetic, machines, including electric motors and generators presence of air clearance in circuits of the magnetic flow can become a reason of causing both the vibrations and the parametric resonance with increasing amplitudes of oscillations.

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