400. Method of dot mappings in measurement of dynamic parameters of rotary systems

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Abstract. The method of integrated continuous measurement of dynamic parameters of rotary systems is based on natural interrelations between these parameters, which are implied by fundamental mechanics laws, and also on direct measurements of those parameters, which permit such measurement. An integrated continuous measurement of dynamic parameters envisages a direct measurement of distances s_{ij} up to points on a rotor surface also measurement of linear speeds V_{ij} of these points by three sensors D_{ij} (in each supporting plane). The equations system, which allows determining positions and size of radiuses-vectors r_{ij}, and also instant positions of a rotation axis and a symmetry axis of a rotor and instant values of its angular speed, may be made up and solved according to results of such measuring. It is obvious that instant values of parameters, which are determined by direct measurement, by the solution of the equations system and the subsequent calculation (at the solution of an inverse problem), are dot mapping of an instant condition of rotary system. The information, which accrues in computer memory at multiple recurrences of the listed above actions during each rotor revolution, represents the dot mapping of dynamics of rotary system during all work cycle. We can assume that the dot mapping of rotary system dynamics will allow applying simultaneously with identification of dynamic parameters a method of full bifurcation groups for the full analysis of a condition the rotary system, including nonlinear conditions and interim steady states, which these nonlinear conditions generate.

Introduction

Measurement of mechanical parameters of rotors as sequence of the experimental and computing operations which are carried out for the finding purpose of values of physical magnitudes describing a state of a rotating rotor, represents doubtless interest both from the information point of view, and as a basis for construction of control systems, and also for systems of technical diagnostics and reliability prediction.

Existing devices, stands and systems for measurement of mechanical parameters of rotors may be characterized both by variety, and by absence of universality and system-defined approach. Measurement of each parameter is carried out separately from measurement of other parameters; these measurements demand application of highly specialized measuring stands and, accordingly, installation of a rotor on such stand. Such operations exclude an opportunity of continuous measurement of parameters in an operational regime.

However all dynamic parameters of a rotating rotor are determined by dynamics of one and the same physical body moving by fundamental laws of mechanics, and for this reason are interconnected among themselves, forming the single system of parameters. Change of one of parameters entails change of characteristics of movement and dynamics of a rotor, and consequently, change of all other parameters of movement. Consequently, measurement and identification of a complex of dynamic parameters of a rotating rotor demand the systems approach based on mechanics laws [1, 2].

Principle of continuous measurement of dynamic parameters

The natural interrelation of character and parameters of movement of a rotor allows assuming, that systems approach to measurement and identification of parameters of

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movement should be based on the analysis of the differential equations of movement of a rotor [3]:

$$\ddot{\varphi} + 2\mu_\varphi \dot{\varphi} + \Omega^2 \varphi = \frac{M_0}{J} \sin \dot{\varphi} t \qquad (1)$$

$$\ddot{r} + 2\mu_r \dot{r} + \Omega^2 r = \frac{F_0}{m} \sin \dot{\varphi} t \qquad (2)$$

where φ is angular coordinate, $\dot{\varphi} = \omega$ is angular speed, r is radial coordinate, μ_φ and μ_r are damping coefficients of radial and torsional oscillations, $\Omega = \sqrt{cm^{-1}}$ is resonant frequency, c is rigidity of a rotor support, m is a rotor mass, J is the inertia moment of a rotor, t - time, F_0 is the module of driving force, M_0 is the module of the moment of driving force.

It follows from the equations (1) and (2), the system of continuous non-contact measurement and identification of mechanical parameters of a rotating rotor may be based on direct measurement of radial and angular moving with the subsequent calculation of parameters values on the regularities, which are consequence of mechanics laws.

Usually we judge about linear moving and their speed on measurement base of vibratory displacement and vibratory speed, though any of these parameters does not allow defining true position of a rotor in space and change of this position at rotation of a rotor.

True position of an axis of rotation and axis of symmetry of a rotating rotor can be determined at measurement of fluctuations of a rotor [2] as follows.

Three standard sensors D_{ij} (fig. 1) are seated in each of supporting planes I and II (or close to them). Continuous non-contact measurement of a complex of dynamic parameters is based on measurement of distances s_{ij} between these three sensors D_{ij} and three corresponding points A, B, C on a surface of a rotor (or its shaft) at simultaneous measurement of linear speeds V_{ij} of these points (fig. 2) [2].
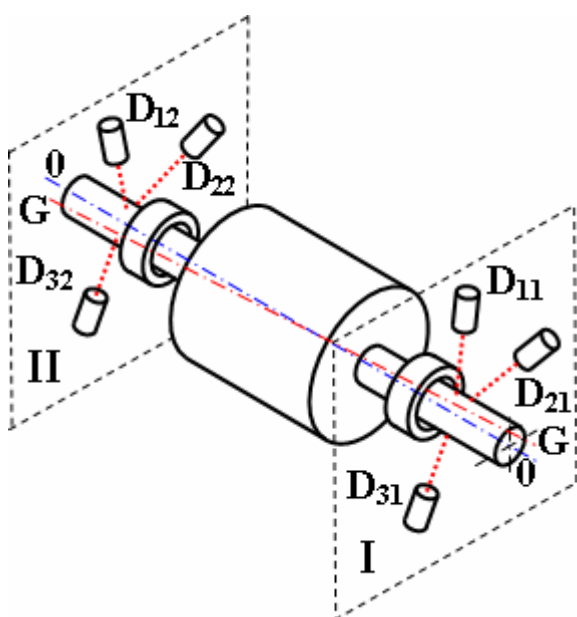

Fig. 1. A disposition of sensors D_{ij}

Measurement of these parameters allows forming and solving the equations system determining radiuses-vectors r_{ij} of points A, B, C and angles χ_{ij} between them, and also to determine instantaneous angular speed of a rotor [1, 2].
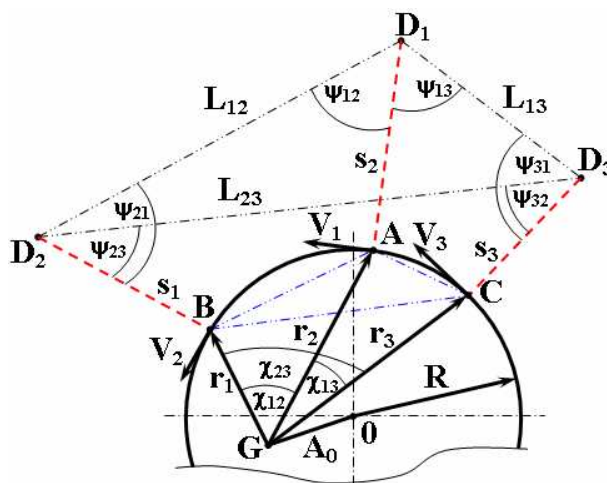

Fig. 2. Definition of radiuses-vectors r_{ij}, positions of a rotation axis and a symmetry axis of a rotor

Identification of points A, B, C on a rotor surface allows identifying unambiguously a circle of corresponding rotor section and, hence, unambiguously to identify the center 0 rotor section. Identification of the sections centers in two support planes (or close to them) allows identifying instantaneous spatial position of a rotor symmetry axis (or its shaft), and also to trace position change of the symmetry axis during a full working cycle of a rotor.

Measurement by sensors D_{ij} of distances s_{ij} up to points A, B, C on a rotor surface (or its shaft) at simultaneous measurement of linear speeds V_{ij} of these points allows forming and solving an equations system (3) – (8), in which radiuses-vectors r_{ij} of points A, B, C and angles χ_{ij} between them are the sought values; and measured magnitudes – linear speeds V_{ij}, distances s_{ij} from sensors D_{ij} up to points A, B and C, distances L_{ij} between sensors D_{ij} and also angles ψ_{ij} between them – are coefficients and constants (at the given moment of time):

$$\cos \chi_{23} = \cos \chi_{12} \cos \chi_{31} - \qquad (3)$$
$$- \sqrt{\left(1 - \cos^2 \chi_{12}\right)\left(1 - \cos^2 \chi_{31}\right)};$$

$$r_{11}^2 \left(1 + \frac{V_{21}^2}{V_{11}^2} - \frac{2V_{21}}{V_{11}} \cos \chi_{12} \right) = \qquad (4)$$
$$= L_{12}^2 + s_{11}^2 + s_{21}^2 + 2s_{11}s_{21}\cos(\psi_{12} + \psi_{21}) -$$
$$- 2L_{12}\left(s_{11}\cos\psi_{12} + s_{21}\cos\psi_{21}\right);$$

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$$r_{11}^2\left(1+\frac{V_{31}^2}{V_{11}^2}-\frac{2V_{31}}{V_{11}}\cos\chi_{31}\right)= \qquad (5)$$

$$=L_{31}^2+s_{11}^2+s_{31}^2+2s_{11}s_{31}\cos(\psi_{13}+\psi_{31})-$$
$$-2L_{31}(s_{11}\cos\psi_{13}+s_{31}\cos\psi_{31})$$

$$r_{11}^2\left\{\frac{V_{21}^2}{V_{11}^2}+\frac{V_{31}^2}{V_{11}^2}-\frac{2V_{21}V_{31}}{V_{11}^2}\left[\cos\chi_{12}\cos\chi_{31}-\right.\right.$$
$$\left.\left.-\sqrt{\left(1-\cos^2\chi_{12}\right)\left(1-\cos^2\chi_{31}\right)}\right]\right\}= \qquad (6)$$
$$=L_{23}^2+s_{21}^2+s_{31}^2+2s_{21}s_{31}\cos(\psi_{23}+\psi_{32})-$$
$$-2L_{23}(s_{21}\cos\psi_{23}+s_{31}\cos\psi_{32});$$

$$r_{21}=r_{11}\frac{V_{21}}{V_{11}}; \qquad (7)$$

$$r_{31}=r_{11}\frac{V_{31}}{V_{11}}. \qquad (8)$$

Values calculation of radiuses-vectors r_{ij} of points A, B, C on a rotor surface and also calculation of angles χ_{ij} between them allows defining the crossing point G of a rotation axis with a plane of the rotor section, determined by points A, B, C. Definition of values r_{ij} and χ_{ij} in two support planes allows identifying instantaneous spatial position of a rotor rotation axis, and also to trace change of its position during a full working cycle of a rotor.

Identification of instantaneous spatial positions of a symmetry axis and rotation axis of a rotor allows defining distance between these axes. This distance define an amplitude of radial oscillations of a rotor (its projections to orthogonal planes of sensors disposition D_{ij} are defined by lengths $A_0=0G$, fig. 2).

It is obvious, the solution of the equations system (3) – (8) and all subsequent actions allow defining instantaneous value of amplitude A_0, or, otherwise, dot value of parameter A_0. Continuous positions tracking of a rotation axis and a symmetry axis, and also a temporal change of these positions, demands multiple (during each rotor revolution) solution of the equations system (3) – (8) and multiple definition of positions of a rotation axis and a symmetry axis of a rotor.

Let's assume, the rotor makes harmonious oscillations, and the radial deviation of a rotor at the point of time t_i is determined by results of the solution of the equations system (3) – (8) and of subsequent positions identification of a symmetry axis and a rotation axis:

$$A_i=A_0\sin\varphi_i, \qquad (9)$$

where φ_i is a turn angle of a rotor at the point of time t_i.

It is obvious, some time interval Δt_i is needed for distances measurement s_{ij} between sensors D_{ij} and points A, B, C on a rotor surface, for the solution of equations system

(3) – (8) by computer complex of a measuring system, also for the subsequent positions identification of a symmetry axis and a rotation axis of a rotor. The rotor will turn upon some angle $\Delta\varphi_i$ during time interval Δt_i, therefore the following measuring of a rotor deviation under oscillations action may be defined by expression:

$$A_{i+1}=A_0\sin(\varphi_i+\Delta\varphi_i) \qquad (10)$$

Performance of condition $\Delta t\rightarrow_i 0$ allows passing on from the equation (10) to the differential equation of a rotor movement

$$m\ddot{r}+b_r\dot{r}+cr=F_0\sin\dot{\varphi}t, \qquad (11)$$

where m is a rotor mass, φ is an angle of a rotor turn at its rotation, F_0 is the module of compelling force, $b_r=2m\mu_r$, μ_r is a damping coefficient of radial oscillations, $c=m\Omega^2$ is rigidity, Ω is frequency of free oscillations.

It is obvious, that the equation (11) is identical to the equation (2).

The solution of the equation (11) allows defining amplitude of the compelled oscillations of a rotor in the form [4]:

$$A=\frac{\varepsilon\omega^2}{\sqrt{\left(\Omega^2-\omega^2\right)^2+4\mu_r^2\omega^2}}, \qquad (12)$$

where ε is mass eccentricity of a rotor and $\omega=\dot{\varphi}$ is angular speed of a rotor.

Method of dot mappings in measurement of dynamic parameters of rotors

The condition $\Delta t_i\rightarrow 0$ is impracticable in real measuring system (in view of the before-mentioned reasons), therefore interpolation of received dot solutions (10) is necessary for transition toward the differential equations of movement and subsequent use of the differential equations of movement for the solution of a inverse problem and continuous definition (identification) of dynamic parameters of a rotating rotor [4 – 7]. Amplitude-frequency characteristic of a
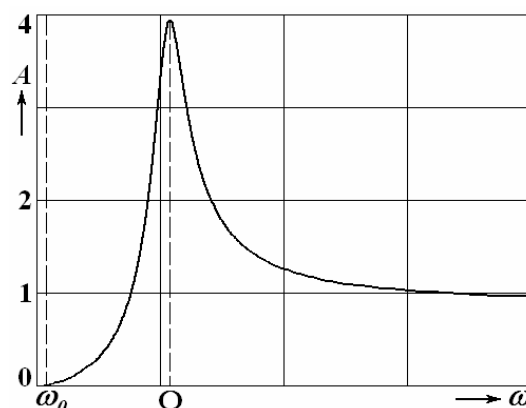

Fig. 3. Amplitude-frequency characteristic of a rotor at movement under the linear law

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rotor (fig. 3) may be constructed also by results of interpolation (for example, by a method of cubic splines).

We must notice in view of above: each subsequent value of amplitude A_{i+1} may be submitted in the form:

$$A_{i+1} = f(A_i).\qquad(13)$$

Expression (13) is the discrete (dotted) mapping of dependence (12).

Let's consider that A is arbitrary value of amplitude of the compelled oscillations described by the equation (13). We shall set small perturbation $\tilde{a}$ so, that $A_i = A + \tilde{a}_i$. Linearization of mapping (13) allows presenting the small perturbation $\tilde{a}_{i+1}$ in the form:

$$\tilde{a}_{i+1} = f'(A)\,\tilde{a}_i.\qquad(14)$$

It is obvious, perturbation $\tilde{a}_{i+1}$ diminishes under condition $|f'(a)| < 1$. Result from this is the system stability in a point A. Perturbation $\tilde{a}_{i+1}$ increases under condition of $|f'(a)| > 1$, and it signifies the system instability in the point A.

The occurrence reason of rotor instability can be, for example, change of contact rigidity at dot or linear contact of working surfaces of support. As against constant value of rigidity $c = m\Omega^2$ in the differential equation of movement (11), the rigidity coefficient which is taking into account mentioned change, can be submitted as:

$$c = c_0 + c_1 r^2,\qquad(15)$$

where c_0 is average value of rigidity and c_1 is the rigidity caused by change of load at radial displacement r.

Equation (11) with the account (15) gets the next form:

$$m\ddot{r} + b_r\dot{r} + (c_0 + c_1 r^2)r = F_1 + F_0\sin\dot{\varphi}t,\qquad(16)$$

where F_1 is average value of compelling force, F_0 – oscillations amplitude concerning average value of force F_1.

It follows from the worded above, the variable rigidity coefficient (15) is the reason of the equation nonlinearity (16) of rotor movement.

The rotor amplitude-frequency characteristic, which is determined at the solution of the equation (16), is submitted on fig. 4 [3].

The analysis of the amplitude-frequency characteristic presented on fig. 4 allows disclosing its some features. At increase of frequency $\omega = \dot{\varphi}$ from some value located on a section $0AB$ of amplitude-frequency characteristic, up to value ω_1, the amplitude suppression and transition to a point C of curve take place. The decrease of frequency ω from some value, which is situated to the right of a point C (fig. 4),

leads to sudden change of amplitude when value $\omega = \omega_2$. It follows from this; section BD of the amplitude-frequency characteristic fits unstable regimes [3].
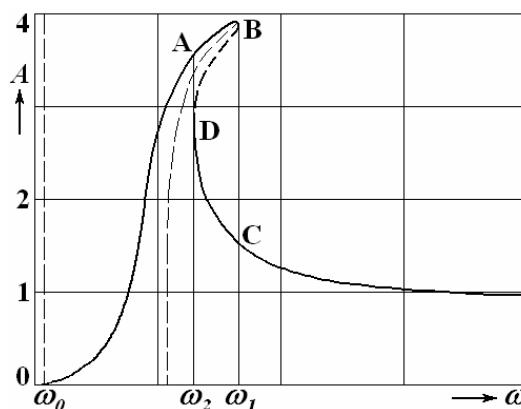

Fig. 4. Amplitude-frequency characteristic of a rotor at inconstant rigidity coefficient

The above problem, which is presented as an example, is one of problems pertaining to nonlinear dynamics and the nonlinear theory of oscillations. Various approximate analytical methods [8] are used for solution of such problems.

However «many important regimes remain unnoticed at the traditional analytical modeling or numerical methods, despite modern opportunities of using of high-speed computers» [9]. It is caused by multimodiness of nonlinear systems and impossibility of system detection of a full set of all nonlinear regimes by traditional methods of nonlinear dynamics.
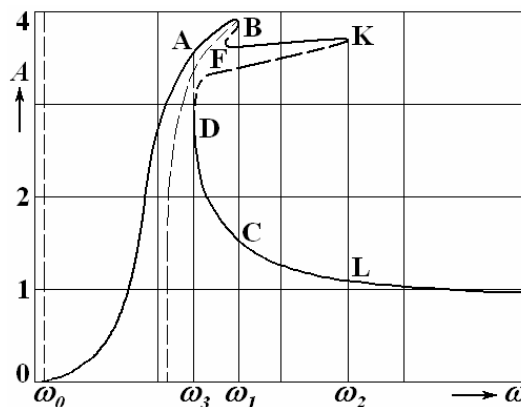

Fig. 5. Bifurcation in an instability zone of nonlinear amplitude-frequency characteristic

The reason of such multimodiness may be conditioned by bifurcation occurrence (fig. 5) in the instability zone [9, 10]. Bifurcation occurrence and caused by it protuberance *FKD* in the zone of instability *BD* leads to occurrence of the dangerous interim stable state mapped by section *FK* of the amplitude-frequency characteristic, and that is characteristic for multimodiness systems . The amplitude of a rotor oscillations is close to resonant value at this case, but, as against the phenomenon of a resonance (see fig. 3 and

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fig. 4), which disappears promptly at increase of angular speed ω such interim stable state remains during a prolonged interval of time under substantial growth of angular speed ω. It is obviously, long-term regime with the anomalously big oscillations amplitude can lead to catastrophic destruction of rotary system. The probability of catastrophic destruction grows inasmuch as the zone of instability *BD* may generate the bifurcations set, and each of these bifurcations is capable to generate the interim stable state similar to section *FK* (fig. 5).

Diagnostics of interim stable states, which are caused by bifurcations and are capable to lead to catastrophic destruction of rotary systems, are complicated by next circumstance: bifurcations occurrence occurs seldom enough at particular combination of constructive and phase parameters (not repeating at each working cycle). It was already mentioned, detection of interim stable states by traditional methods of nonlinear mechanics is impossible [9, 10]. At the same time simple change of a working regime can deduce the rotary system from a dangerous interim stable state and to prevent catastrophe.

The alternative approach to the problem solution is the method of full bifurcation groups [9], which is based on the method of dot mappings [11].

Let's analyze from this point of view a method of integrated continuous measurement of dynamic parameters of rotary systems (fig. 1 and fig. 2).

Direct measurement by three sensors D_{ij} of distances s_{ij} up to points *A*, *B*, *C* on a surface of a rotor (fig. 1 and fig. 2) and measurement of linear speeds V_{ij} of these points lays in a basis of this method. The subsequent solution of the equations system (3) – (8) (or analogous equations system) allows defining instantaneous positions and value of radiuses-vectors r_{ij} and also instantaneous positions of a rotation axis and a symmetry axis of a rotor and instantaneous values of its angular speed. It is obvious, all instantaneous parameters values which are determined both by direct measurement and by solution of the equations system (3) – (8) and by the subsequent calculation (by solution of the inverse problem) is dot (discrete) mapping of an instantaneous state of rotary system. The information accrued in computer memory at multiple recurrence of the listed above actions during each rotor revolution, represents the dot mapping of rotor dynamics during all working cycle. The same conclusion follows from the analysis of the above mentioned equations (9), (10), (13) and (14).

It follows from above-stated, the method of complex continuous measurement of dynamic parameters of rotary systems [1, 2, 4 – 7] allows simultaneously identifying dynamic parameters of rotor and applying a method of full bifurcation groups [9] for the full analysis of the rotary systems, including nonlinear regimes and interim stable states, generated by them.

Conclusion

The method of integrated continuous measurement and identification of dynamic parameters of the rotary systems,

which is based on direct measurement of distances from sensors up to a surface of a rotor (or its shaft) and on the subsequent calculation of all complex of parameters according to their natural interrelation implied from fundamental laws of mechanics, is investigated from the point of view of dot mapping of a rotor state during all work cycle. It is shown; that one-momentary measurement of the mentioned distances and linear speeds, and also the subsequent calculation of a complex of parameters characterize an instant state of a rotor, and their multiple recurrences during each revolution of a rotor defines a rotor monitoring and simultaneously is dot mapping of its state. Dot mappings of a rotor state during all work cycle allow realizing simultaneously continuous identification of a complex of dynamic parameters and the method of full bifurcation groups for the full analysis of a rotor state, including nonlinear regimes and interim steady states generated by them.

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