399. Some nonlinear effects of machine dynamics

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Abstract: An overview of some problems related to the elimination of following negative dynamic effects is presented: the effects arising from the nonlinear geometrical characteristics of mechanisms; the effect arising from the joint action of nonlinear position function and clearances; the effects of excitation vibratory regimes arising from overcoming the threshold level of nonlinear dissipation.

Keywords: mechanism, vibration, dissipation, stability.

1. Introduction

The problem of reducing vibratory activity is one of the most important problems of modern machine dynamics. In this paper we have presented an overview of some problems, among which we should single out the elimination of following negative dynamic effects: the effects arising from the nonlinear geometrical characteristics of mechanisms; the effect arising from the joint action of nonlinear position function and clearances; the effects of excitation vibratory regimes arising from overcoming the threshold level of nonlinear dissipation.

2. Effects arising from nonlinear geometrical characteristics of mechanisms

Preliminary remarks. A distinctive property of machine drives are the motion transformation and programmable displacement of actuators according to nonlinear position function $\Pi(\varphi)$, where φ is the coordinate of an input link [1,2]. The mechanisms realized programmed motion ("cyclic" mechanisms) are playing a double role in the vibratory system on the one hand being the source of vibration excitation, and on the other hand being a critical object to vibration protection.

For example, a typical dynamic model is presented on Fig.1. Similar models were created by combining the block diagrams of vibratory mechanical systems with certain kinematic analogs Π_i that determine the kind of connections between the input and output links . In the diagrams of Fig. 1. J_i, c_i, ψ_i denote moments of inertia, stiffnesses and energy dissipation coefficients of the corresponding links in the kinematic chain.

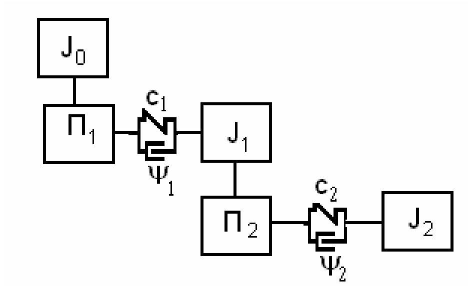

Fig. 1.

Any coordinate in the absolute motion φ_i is a combination of the "large" coordinate $\Pi_i(\varphi_0)$, realizing the motion of the absolutely rigid drive, and the "small" coordinate $\Delta\varphi$, whose ensemble corresponds to number of vibratory system degrees of freedom *H*.

In this case the set of differential equations is nonlinear. However, with representing the position functions as truncated Taylor series the linearization in the vicinity of the current value of φ_0 is carried out. In doing so the position functions retained their nonlinear properties relative to the large coordinate, and only small deformations entered the corresponding expressions in a linear fashion. After the transformation to the quasinormal coordinates the original system can be described by differential equations

$$\ddot{y}_r + 2n_r(t)\dot{y}_r + p_r^2(t)y = W_r(t) \quad (r = \overline{1,H}) \qquad (1)$$

Using the method of conventional oscillator (Vulfson, 1969) the decision y_r has the following structure [2,3]:

$$y_r = \mu_r \sum_{j=0}^{s-1} D_{jr} \exp\left[-\int_{t_j}^{t} n_r dt\right] \sqrt{\frac{\Omega_r(t_j)}{\Omega_r(t)}} \sin\Psi_{jr} + Y_{sr}, \quad (2)$$

$$\left.\begin{aligned} &\mu_r = \left[1 - 2\exp\left(-\vartheta_r \bar{N}_r\right)\cos 2\pi \bar{N}_r + \exp\left(-2\vartheta_r \bar{N}_r\right)\right]^{-0,5};\\ &Y_{sr} = \frac{1}{\sqrt{\Omega_r(\tau)}} \int_{t_{s-1}}^{t} \frac{W_r(u)}{\sqrt{\Omega_r(u)}} \exp\left[-\int_u^t n_r d\xi\right] \sin\left[\int_u^t \Omega_r(\xi) d\xi\right] du;\\ &\Psi_{jr} = \int_{t_j}^{t} \Omega_r(\tau) d\tau, \end{aligned}\right\} \quad (3)$$

where $\Omega_r(t) = \bar{p}_r \exp z_r$ is conventional “natural” frequency; $\bar{p}$ is an optional parameter with the dimension of a frequency; $N_r = \bar{p}_r / \omega$.

According to the method of conventional oscillator the relation between the function z_r and variable frequency $p_r(t)$ will have the form of the following differential equation, responding to the particular "conventional oscillator" with the excitation $2p_r^2(t)$ [2,3]:

$$\ddot{z}_r - 0,5\dot{z}_r^2 + 2\bar{p}_r^2 e^{2z} = 2p_r^2(t). \quad (4)$$

By slow changes of $p_r^2(t)$ the dynamic components in the Eq:(4) is small in relation to the static one. For this case, $\Omega_r \approx p_r$, and the solution (2) corresponds to the *WKBJ* approximation method.

Violating of dynamic stability conditions by slow change of parameters. According to (2) the amplitude of free and accompanying vibrations is changing proportional to the function

$$S_r = p_r(t)^{-0,5} \exp[-\int_0^t n_r(\xi) d\xi]. \quad (5)$$

With changing parameters it may happen that $dS_r / dt > 0$, therefore the customary decrease of amplitudes can be disturbed. In the similar case the amplitude modulation exists, where the zone of decrease alternates of the zone of increase. Therefore, contrary to a parametric resonance we do not experience the unlimited increase of amplitudes. Under some unfavorable conditions the increase of amplitudes may become rather intensive. Using (5) the dynamic stability conditions on any time interval can be written as [2–5]

$$n_r + 0,5\dot{p}_r / p_r > 0. \quad (6)$$

It is possible to show that condition (6) can also be obtained by the direct Lyapunov method and it is, consequently, the sufficient condition for asymptotic stability. Compliance with this condition removes also the instability in the zones of parametric resonances.

A dynamic effect caused by a sudden temporary change in the “natural” frequency of a system (so called "parametric impulse") is considered in [2,6] using the method of conventional oscillator.

The effect arising from the joint action of nonlinear position function and clearances. For cyclic mechanisms the clearance-effect leads to possibility of vital distortion of kinematical characteristics and increase the drives vibroactivity. Two cases are revealed. In the first case the clearance proves as a nonlinear element to which a possibility of generating vibratory impact modes is connected. In the second case reaction to a clearance manifests itself as an impulse in linear systems. This dynamic effect is equivalent to impact arising from disruption of a continuity of the function $d\Pi / d\varphi_0$. Some dynamic criterions that allow forecasting the excitation of vibratory impact regimes are offered [5, 7–9].

In the linkages the clearance effect sometime softens due to the conjugate action between the contracting surfaces of hinge (Fig. 2). On many researches of this problem it is supposed, that the vibration excitation at elimination of breaks of the kinematic contact in clearances does not arise. However at parametrical pulses the arising effect is close to impact. This effect, named *pseudo-impact*, under certain conditions is transformed to the impact with disruption of contact of a kinematic circuit.

The relation between linearized tangential and normal stiffnesses of "links" $A'A'', B'B''$ – c_i^τ, c_i^n can be presented as [9]

$$c_i^\tau / c_i^n = \left|R_i^*\right| / (0,5 c_i^n s_i + \left|R_i^*\right|), \quad (7)$$

where R_i^* (i=1,2) is the reaction in the corresponding hinge calculated at the kinetostatic level; s_i is the value of clearance.

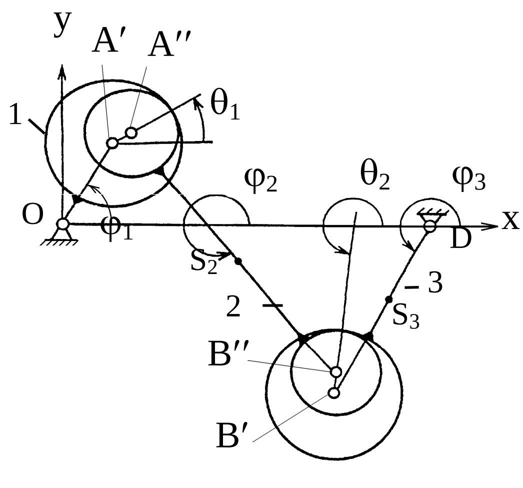

Fig. 2.

The offered model of clearance-joint (Fig. 3) is submitted as a pendulum that oscillates in a rotating power field about the elastic support [9]. The analysis of this model allows to define the conditions of stability on the limited time intervals and critical values of parameters of system at which the excitation close to impact takes place.

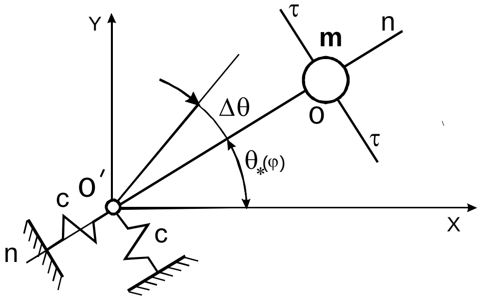

Fig. 3.

In order to obtain comparable results in varying the system parameters the quasielastic coefficients were scaled by the stiffness coefficient $c_0 = J_3 p_0^2 / l_3^2$, where p_0 is the partial frequency realized conditionally for a zero pressure angle and clearance-free elastic joint between coupler and the rocker; J_3 is the reduced moment of inertia of the rocker; $l_3 = BD$. For the closed kinematic chain, an increase of natural frequency parameter $\eta_0 = p_0 / \omega_1$ results in the slow frequency variation being interrupted by intense parametric pulsations. In turn this may result in a considerable growth in the level of dynamic errors and the vibration activity of the mechanism [9]

The dynamic effect under analysis is illustrated on Fig.4 by the plots of functions φ_{2*}'', φ_{3*}'', which are proportional to the ideal angular accelerations of coupler and rocker (curves 1), and φ_2'', φ_3'', calculated taking into account the clearances and elasto-dissipative properties of hinges (curves 2). We see that at a rather high value of $\eta_0 = 50$ (Fig. 4. a) free accompanying vibrations are intensively excited in the zones of parametric pulses and cause a marked grows of the maximum accelerations and vibration activity of mechanism. At $\eta_0 = 10$ (Fig. 4. b) the effect is considerably attenuated.

An analysis shows that the dynamic stability of the system over a finite time interval is an important factor determining the system behavior in the zones of parametric pulses. The corresponding sufficient conditions (6), based on the method of conditional oscillator, lead to the form [2]

$$\vartheta_r > |\vartheta_r^0| = |\ln(\eta_r^+ / \eta_r^-)|, \qquad (8)$$

where $\vartheta_r, |\vartheta_r^0|$ are logarithmic decrement and its critical value for the mode r , ,

$$\eta_r^+ = \eta_r(\varphi_1 + \Delta\varphi_r), \; \eta_r = \eta_r^-(\varphi_r - \Delta\varphi_r),$$

$$\Delta\varphi_r = \pi / \eta_r(\varphi_1).$$

Real damping is determined by the effective value of the logarithmic decrement $\vartheta_r^* = \vartheta_r - |\vartheta_r^0|$. The value $\max \vartheta_r^0$ can be used as an efficient criterion for characterizing the level of vibration. The envelopes shown in Fig.4a (curves 3) correspond to the logarithmic decrement decreasing from ϑ_i =0,2 to 0,06. We see that a decrease in ϑ_i intensifies the dynamic instability and vibration excitation.

For η_0=10 and ϑ_i =0,2 , the value ϑ_i^* remains nearly zero for a rather long time. In this case the compensating of the dissipative factors takes place (Fig.4b).

The transformation the pseudo-impact to impact is evidently visible at comparison of phase trajectories (Fig.5)

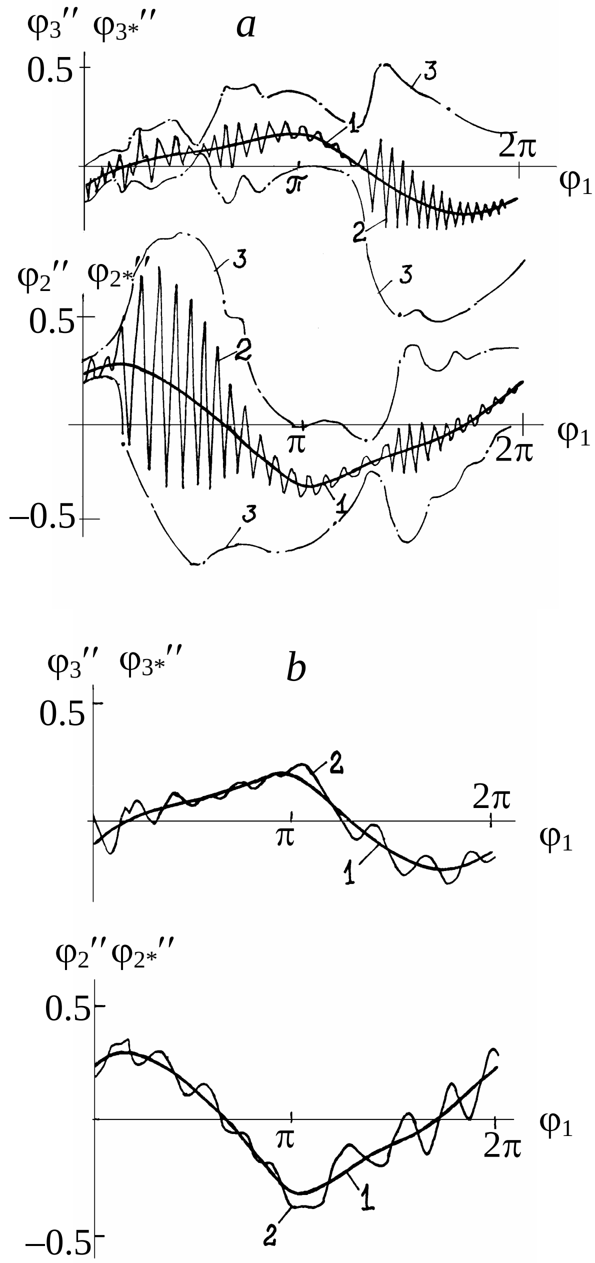

Fig. 4.

The effect of excitation owing to fall out of synchronism in multi-sections drives with lattice structure. The researched effect in drives with regular structure is observed. Similar drives are used in machines for realization of repeating technological and transport operations [2,5,10]. The theory of regular oscillatory systems is based on the analysis of the lattices consisting of masses and springs. For the first time this problem was considered by Born and Karman with reference to the analysis of the heat capacity of crystals. The basic directions of the further development of this theory are reflected in [11].

With reference to machines with cyclic mechanisms the theory of regular oscillatory systems requires additional development. Dynamic models of drives have more complex internal structure of each repeating module formed not only a simple connected chain, but also brunched and ring structured vibratory systems with nonlinearities and nonstationary dynamic connections [2,5,10,12]. Should be noted, that in some cases the conditions of regularity are realized only approximately.

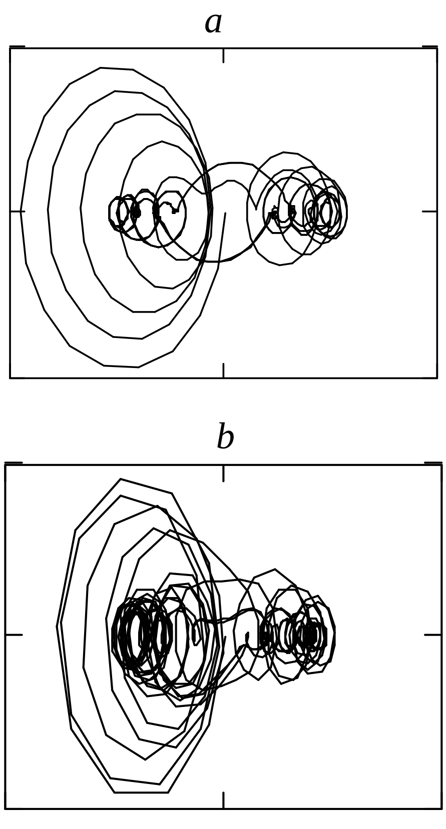

Fig. 5.

Father we consider the dynamic model of a drive (Fig. 6), consisting subsystems of the main shaft ($k = 1$) and an actuator ($k = 2$), which is connected to the main shaft by n cyclic mechanisms. Each of mechanisms is submitted as consecutive connection of the elements, subject to there dissipative, inertial, and kinematic characteristics, and clearances. The following symbols are accepted: $J_{j,k}$ are moments of inertia; $c_{j,k}$, c_j are factors of rigidity; $\psi_{j,k}$, ψ_j are factors of dissipation; $\Pi(\varphi_{j,1})$ is the position function. It is supposed, that dynamic characteristics of the main shaft and the actuator are given as inlet and outlet parts of cyclic mechanisms. Besides angular speed ω on "input" is accepted constant, that corresponds usually as first approximation to real machines at a rational choice of characteristics of the electric drive and transmission mechanisms. The considered oscillatory system has $2n+1$ degrees of freedom. As the generalized coordinates we accept the dynamic mistakes equal to deviations of absolute coordinates in the appropriate sections of inertial elements from coordinates of program motion. Thus, for the main shaft $q_1 = \varphi_{1,1} - \varphi_0$, $q_{2(j-1)} = \varphi_{j,1} - \varphi_0$,

where $\varphi_0 = \omega t$, $j = \overline{1, n+1}$ and $q = \varphi - \Pi(\varphi)$ $(j \geq 2)$ for the actuator. The accepted dynamic model is described by the set of nonlinear differential equations with slowly varying factors [12]. By $\Pi' = r \sin\varphi$ the computer simulation with variation of number of identical mechanisms n and other parameters of system was carried out in work [12].

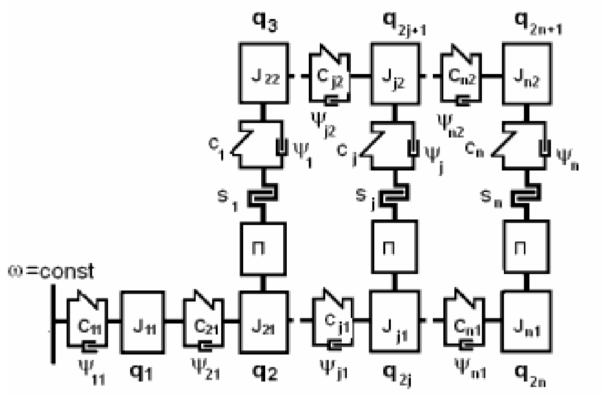

Fig. 6.

Under favorable conditions the queasy-synchronous mode of actuator vibrations is realized (a mode of type 1, Fig.7a). At this form the elastic element c_{j2} is not deformed. For modes of type 1 the "natural" frequency is close to a case with $c_{j1} \to \infty$ (the rigid main shaft) and is described by dependence

$$p_m \approx p\sqrt{1 + 2\varsigma_2[1 - \cos((m-1)\pi/n)]},$$

$(m = \overline{1, n}; \ \varsigma_2 = c_{2j}/c_j)$

The synchronous motion corresponds to the lowest frequency (m=1).

The conditions in which the synchronous form of vibrations of an output link is strongly broken were received [12]. Usually this corresponds to the most deformed site of the main shaft (a mode of type 2, Fig.7b). In this case the plots q_i for first three mechanisms $(i = 3, 5, 7)$ differ a little, but for the coordinate q_9 not damping vibrations and violation of synchronism are observed.

Strong additional excitation of the mechanism n is caused by a specific influence of the subsystem formed by the previous mechanisms, whose energy is partially "pumped over" in the asynchronous form. Thus a large role plays the violating of dynamic stability conditions on finite time intervals (see above). At vibratory impact modes the spectrum of frequency can be change, and the level of vibration increase. For analysis of these phenomena the method harmonious linearization of force was used [12].

One of ways for the constructive decision of the problem for elimination of asynchronous modes with large amplitudes is the transition to branched-ring structured drive. In this case the separate sections with limited number of mechanisms replaced the long actuator.

3. Effects of excitation vibratory regimes caused by overcoming the threshold level of nonlinear dissipation

To the theory of nonlinear dissipation under polyharmonic excitation. The problem of the nonlinear dissipation at polyharmonic oscillations of mechanical systems has not yet been solved completely. The complexity of this problem is caused by the nonlinear character of dissipative forces and also by the fact that usually it is possible to determine only the integral characteristics of dissipation. To solve the problem are used two approaches based on the idea of the separation of motions.

The first approach is based on the ideas of vibrational rheology implying the separation of motions, dividing them into the fast and slow ones. This approach has been developed by I.I. Blekhman (1973 and later) [13]. By this approach it is proposed, that the mathematical description of dissipative force is known.

The second approach bases of motions separate, dividing them into two groups: the resonant motions (and some other oscillations, whose frequencies are close to the natural frequency), and the non-resonant components. This approach for frictional force of sliding by M.Z.Kolovsky (1963) [14], and for position force of resistance by I.I. Vulfson (1968 and later) [15-18] was developed. According to this approach the modes can be classed into "basic" and "additional". Note, that the engineering determination of nonlinear position dissipative forces is based usually on the limited initial information, such as energy damping factor ψ_0 or logarithmic decrement ϑ_0, which for typical objects at monoharmonic vibratory modes are received experimentally. If the energy transfer between the modes, caused by nonlinear dissipative forces, is small, we have

$$\mathbf{R} = -\mathbf{b}\dot{\mathbf{q}}\ ;\ \ \mathbf{b} = (\boldsymbol{\alpha}^{\mathrm{T}})^{-1}\mathbf{b}^*(\boldsymbol{\alpha})^{-1},$$

where $\mathbf{b}^* = \mathrm{diag}[b_1^*,...,b_H^*]$, $b_r^* = \psi_r c_r/(2\pi k_r)$, $\boldsymbol{\alpha}$ is a matrix of the coefficient of mode.

At use of this procedure for polyharmonic regimes can be result in essential mistakes. In works [15–18] the essential decrease of an effective level of dissipation is established with additional movement of system. Physical preconditions for this effect are related with the occurrence of so-called minor hysteresis loops located inside a loop, appropriated to vibration with the basic frequency. The total area of minor hysteresis loops is proportional to the work of forces of the resistance, carried out due to additional motion. Thus, the effective area of a loop of the basic movement decreases (Fig. 8).

In the analytical form the marked effect for position force of resistance results in the following corrective:

$$\psi = \psi_0\Phi(z)\ ;\ \ \vartheta = \vartheta_0\Phi(z)\,. \qquad (9)$$

Here $\Phi(z)$ is the factor of decrease of the equivalent dissipation characteristics, z is the ratio of speed's amplitudes of the basic and additional movement. The general relationship of function $\Phi(z)$ for the widespread forms of loops of a hysteresis can described by the following form:

$$\Phi = \frac{\int_0^{2\pi}\beta(\varphi)\xi(\varphi,z)\cos\varphi\, d\varphi}{\int_0^{2\pi}\beta(\varphi)\cos\varphi\ d\varphi}\ , \qquad (10)$$

where

$$\xi(\varphi,z) = \frac{1}{2\pi}\int_0^{2\pi}\mathrm{sign}\left(z\cos\varphi + \cos\theta\right)d\theta =$$

$$= \begin{cases} \dfrac{2}{\pi}\arcsin\left(z\cos\varphi\right) & \left(\left|\cos\varphi\right| \le z^{-1}\right); \\ \mathrm{sign}\left(\cos\varphi\right) & \left(\left|\cos\varphi\right| > z^{-1}\right). \end{cases}$$

The function $\beta(\varphi)$ depends from the form of hysteresis loop [15]. However, as the analysis shows, the function $\Phi(z)$ depends only slightly on the form of the hysteresis loop. For typical cases the following approximating dependence may be applied as (Fig. 9):

$$\Phi(z) = z(0,4 + 0,5z)/(1 + 0,5z^2). \qquad (11)$$

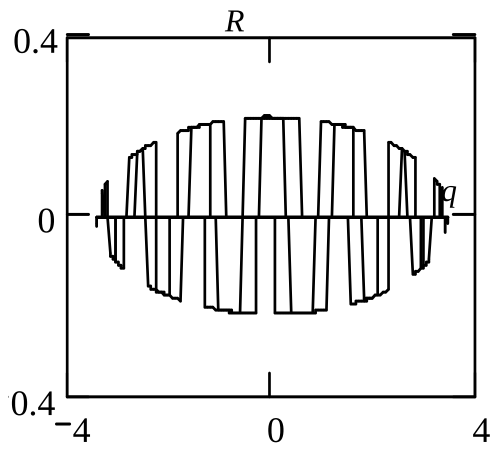

Fig. 8.

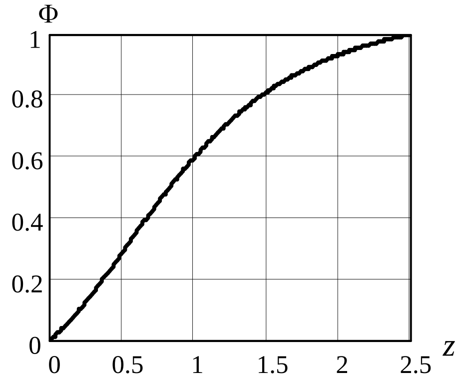

Fig. 9.

Based on the method of linearization of the function of distribution, offered by M.Z. Kolovsky and A.A. Pervozvansky [19], the proposed procedures can be also used in the case of polyharmonic excitation [16]. It can be shown that in this case

$z \approx A_1 k_1 /(\sigma_v \sqrt{2})$.

Here $\sigma_v^2 = \sum_{(N)} A_j^2 \omega_j^2$ is the dispersion of velocities in additional motion ($j \neq 1$) .

The obtained results can be extended on the case of dry friction. Then, the resulting dynamic effect with additional condition ($z < 0,4$) corresponds to the vibrational linearization. The results of the analytical investigation of dissipation are in good agreement with experimental data.

Increase of the resonant amplitude with allowance for additional excitation. The factor of increase of resonant amplitude χ in comparison with results of the analysis without additional excitation is defined as [15,17]

$$\chi = A_1 / A_{10} = \Phi^{-1}(\chi z_0), \quad (12)$$

where $z_0 = v_0 / v_{ad}$; v_0, $A_{10} = \pi A_* / \vartheta_0$ are the velocity and resonant amplitudes without additional excitation; v_{ad} is amplitude of velocity at additional vibration.

The differences between results of computer simulation and analytical investigation are less than 10%.

Specified conditions of dynamic stability at joint action of parametrical and forced excitation. Let us consider the initial differential equation

$$m\ddot{q} + cq = -\mu\left[R(q,\dot{q}) - \varepsilon cq \sin\Omega t\right] + F_1 \sin\omega_1 t , \quad (13)$$

where ε is the depth of parametric pulsation, F_1 are the amplitude of disturbing force, $R(q,\dot{q}) = |R(q)| \operatorname{sign}\dot{q}$ is the position dissipative force, μ is the small parameter.

We have considerate the problem of the action the high-frequency excitation ($\omega_1 \gg k = \sqrt{c/m}$) on the dynamic stability in a zone of a main parametric resonance ($\Omega \approx 2k, F_1 = 0$). Using the method discussed above, the modified differential equation can be represented in the form

$$\ddot{q}_1 + \pi^{-1} k \vartheta_0(A)\Phi(z)\dot{q}_1 + k^2(1 - \varepsilon \sin\Omega t)q_1 = 0, \quad (14)$$

where

ϑ_0 is the logarithmic decrement without additional excitation ($\Phi(z) = 1$); $z = Ak/(A_1\omega_1)$; A, A_1 –are the amplitudes of vibrations with frequencies k and ω_1 .

The condition of dynamic stability we write down in energy form: $\Delta E = \Delta E_+ - \Delta E_- < 0$, where $\Delta E_+, \Delta E_-$ - are accumulated and absorbed energy. Then, the condition of dynamic stability in the zone of main parametric can be presented as [18]

$$\eta_0 > \eta_* = \Phi^{-1}(z), \quad (15)$$

where $\eta_0 = 2\vartheta_0 /(\pi\varepsilon)$ is the coefficient of stability margin.

If $\eta_0 < 1$ the system is unstable independently of presence of additional vibration; if $1 < \eta_0 < \Phi^{-1}$ the amplitude grows up to an output of stability border; at $\eta_0 > \Phi^{-1}$ the solution is always absolute stable.

According to (15) we have three areas in the plane of parameters η_0 and z (Fig.10). In the area 1 the system is always unstable independently of the action of additional vibrations. In the area 2 the system is stable for the case 1 and unstable for the case 2. Thus under the action of additional excitation the area of dynamic instability is increased. In the area 3 the system is always stable. If $\vartheta_0 = \text{const}$, or $\partial\vartheta_0 / \partial A > 0$, in area 2 the amplitude grows up to the border of asymptotic stability (curves 1,2).

If $\partial\vartheta_0 / \partial A < 0$ (for example by action of dry friction) there are two or one cross-points with the border of dynamic stability (curve 3). In this case the lower point corresponds to the stable mode of vibrations. If the lower cross-point is absent, we have $A \to 0$.

Specified conditions of subharmonic resonance excitation. It is established that the additional high frequency or low frequency excitation exerts essential effect on the level of dissipation, determining the conditions of occurrence of subharmonic resonances [20].

As applied to analytical investigation of this problem the efficiency of transformation the initial differential equation to the modified type is established. As this takes place the additional excitation is replaced with energetically equivalent correction of dissipative forces. Fore some characteristics of hysteresis loops the analytical solutions are received. This allows predicting the occurrence of subharmonic resonances.

The initial differential equation is presented as

$$\ddot{q} + |R(q)| \operatorname{sign}\dot{q} + P(q) = w_1 \sin(\Omega_1 t + \varphi) + a\Omega_2^2 \sin\Omega_2 t, \quad (16)$$

where q is the generalized coordinate; $R(q) = -|R(q)| \operatorname{sign}\dot{q}$ is the positional dissipative force; $-P(q)$ is the elastic force; the terms of the right-hand side of Eq.(16) correspond to the disturbing forces with frequencies Ω_1 (basic frequency) and $\Omega_2 \gg \Omega_1$ or $\Omega_2 \ll \Omega_1$ (additional frequency); all forces are related to unit of mass. It is assumed that $P(q) = k_0^2(1+\alpha q^2)q$ and the order of subharmonic resonance is 1/3.

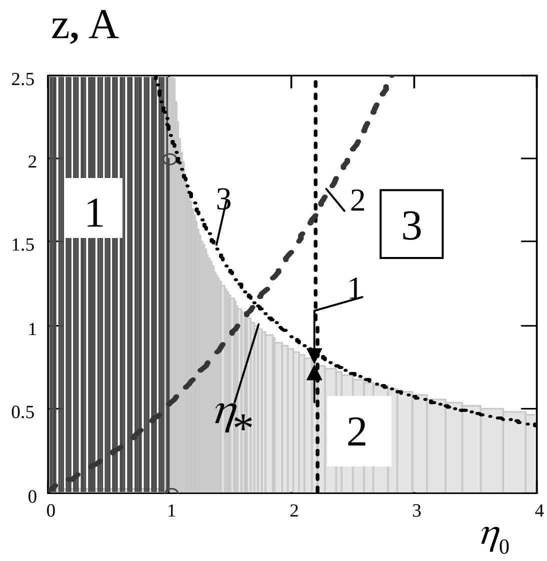

Fig. 10.

The condition of existence of subharmonic resonance is defined as $\Delta E_- < \Delta E_+$, where ΔE_+ is the accumulated energy, ΔE_- is the absorbed energy. The dissipative forces establish some barrier of energy. If to overcome this barrier the occurrence of subharmonic resonance is possible.

In the modified equation the additional excitation with frequency Ω_2 is absent but the correction of dissipative component is done. Comparison of decisions of both equations testifies to efficiency of the offered approach in analytical research at fast and slow additional excitation.

Fore some characteristics of hysteresis loops the analytical solutions are received that allow to predict the occurrence of subharmonic resonances [20].

Nonlinear resonant oscillations of a drive at the amplitude-modulation frequency of high-frequency excitation. For linear dissipation this problem was discussed in [14]. The specific dynamic effect in the nonlinear elastic-dissipative characteristics of a drive and in the harmonic amplitude pulsation of high-frequency disturbing forces is investigated [21]. The conditions corresponding to the excitation of low-frequency resonant oscillations are obtained, taking account of the significant correcting influence of polyharmonic excitation on the effective dissipation parameters. Following the considered approach we transform the initial equation in modified form.

Initial differential equation:

$$q'' + |R(q)| \operatorname{sign} q' + P(q) = w \sin\omega\tau \cos\Omega\tau\,, \quad (\Omega \gg \omega)\,. \tag{16}$$

Modified differential equation:

$$q_1'' + \vartheta(q_1', \tau)\pi^{-1} q_1' + p_1^2(1 - \varepsilon\cos 2\omega\tau) q_1 + \alpha q_1^3 = 0\,, \tag{17}$$

Where

$$\vartheta = \vartheta_0 \sigma(|q'| - |w\Omega^{-1} \sin\omega\tau \cos\Omega\tau|) \approx \vartheta_0 \Phi(z)\,;$$

$$P(q) = p_0^2(1+\alpha q^2)q\,,\ p_1^2 = p_0^2(1+\gamma)\,,$$

$\gamma = 0,75\alpha w/\Omega^2$, σ is the unit function.

On plots of amplitude-frequency characteristic the intervals limited to curves 1-1 $(\delta = 0)$ or curves 2-2 $(\delta = 0,03)$ correspond to conditions of existence of the regime being studied (Fig. 11). The curve 3 corresponds to the case $p_1 = \omega$; the continuous curves – to the stable solutions and dotted curves – to unstable.

For various typical nonlinearities, the results of analysis good agree with the data of numerical experiments. On Fig.12 the comparison of the results received by computer simulating (Eq:14, Fig.12a,b)and with an analytical method (Eq:15, Fig 12c) is shown. At the accepted data the resonant mode at low frequency arises when the factor dissipation does not exceed the value $\vartheta = 0,35$ (Fig.12*a*). Thus, beats of excitation transforms to the biharmonical vibration.

Given the physical character of this effect, it should be noted that without nonlinearities $(\alpha = 0)$ the low-frequency resonance is not present. In this case and by rather large dissipation the beats of excitation lead to the similar response of vibration (Fig.12b). Only in nonlinear vibratory system the amplitude pulsation of disturbing forces leads to eigenfrequency pulsations and to possibility of parametric excitation with frequency ω.

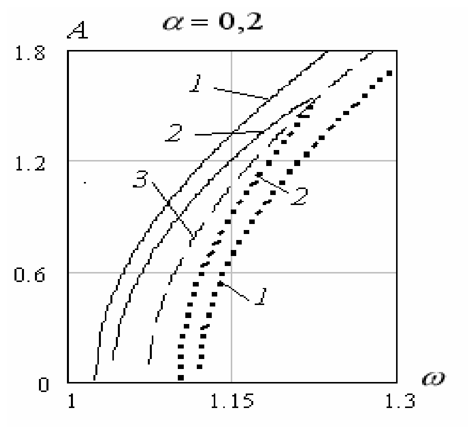

Fig. 11.

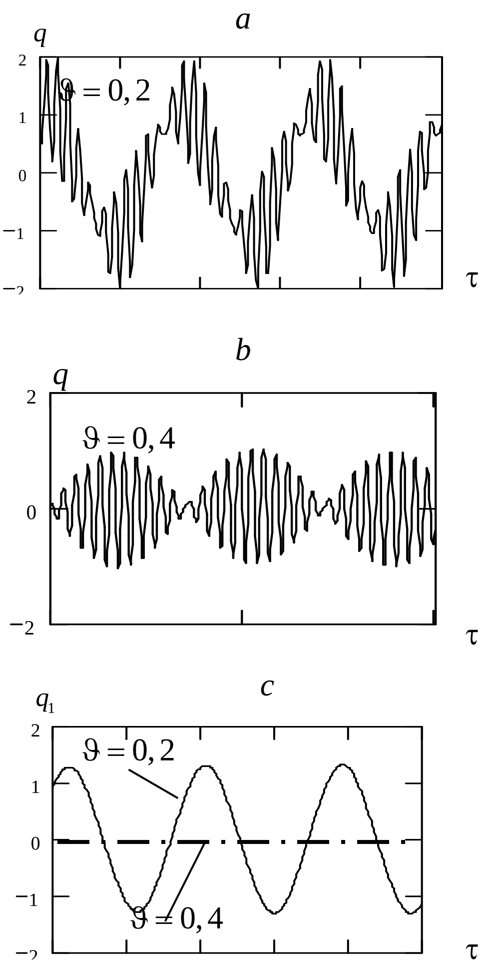

Fig. 12.

Critical condition of frictional self-oscillations excitation. At the account of additional excitation a critical condition of excitation of frictional self-oscillations can be submitted as

$$v_* = \Delta R_0 \sqrt{\Phi(z)[1-\alpha\psi_0\Phi(z)]/(\psi_0 c)}, \qquad (18)$$

where v_r is the maximal critical speed, below which the frictional self-oscillations do not arise; ΔR_0 is the difference in dry friction forces referred at rest and motion; c is the elastic coefficient of a drive; $\alpha = 5/6 - 11/12$.

Formula (18) indicates to opposing tendencies with decrease of function Φ. This is connected with the fact that the effective values of difference of forces of friction ΔR_0 and ψ decrease simultaneously. The first of these factors leads to decrease v_*; the second factor leads to increase. The correcting function $K(\Phi)$ has the form

$$K(\Phi) = \frac{v_*(\Phi)}{v_*(1)} = \sqrt{\frac{\Phi(1-\psi_0\Phi)}{1-\psi_0}}.$$

The similar situation came in cyclic mechanisms with "nonreversible" kinematic pairs, in particular, where the line of reaction does not coincide with the direction of the force of friction [22].

4. Conclusion

The development and perfection of modern machinery raise many complicated technical problems for engineers. Some of them are related to the reducing vibroactivity of machines. This is connected with toward intensification of process and transport operation. In the paper some negative dynamic effects arising due to nonlinear kinematic and dissipative characteristics were analyzed. The ways of their elimination are offered.

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