382. RESEARCH OF DYNAMICS OF ROTARY VIBRATION ACTUATORS BASED ON MAGNETIC COUPLING

K. Ragulskis[1, a], **R. Jonušas**[2, b], **K. Kanapeckas**[3, c], **K.Juzėnas**[4, d]
Kaunas University of Technology,
Kęstučio str. 27, 44025 Kaunas, Lithuania
E-mail: [a]*kazimieras3@hotmail.com*, [b] *remigijus.jonusas@ktu.lt*,
[c] *kestutis.kanapeckas@stud.ktu.lt*, [d] *kazimieras.juzėnas@ktu.lt*

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Abstract. The article analyses a new type of a mechanical rotary oscillator – vibrator, the base of which is made of steady magnets. The scheme of a vibrator's regulated power is submitted to excite the rotary oscillations of a turning frame. Some of the dynamical characteristics of a vibrator have been researched.

Keywords: rotary oscillation, rotational vibrator, steady magnet, dynamical property.

1. Introduction

Since 1978, a few types of vibrators, which operate on the principle of two uninteractive frames with steady magnets of a relative motion, have been patented. Such vibrators can change from a micro up to a macro size and have different power.

The article analyses the vibrators of a similar type (Fig.1) with an easily regulated power, which can be achieved by relative shifting in an axial direction of the rotary frames comprising magnets.

2. The scheme of rotary oscillations vibrator

Owing to these qualities we have chosen this type of an magnetic vibrator. The equipment (Fig.1) comprises external ring 2 immovably fastened on electric motor shaft 1 which provides a rotational motion. From external ring 2 with fastened magnetic poles 5 on it the rotational motion is transmitted to analyzed body - spindle 4 on which internal disk 3 is fastened. Transmission of the torque is secured by the interaction of magnetic forces between magnetic poles 5.

When the electric motor does not work, equilibrium steadies in the magnetic system and magnets do not generate the torque. The electric motor being switched on, its rotor 1 and external ring 2 start rotating. The poles N and S having moved towards each other in the magnetic system, the torque emerges and tends to return the system to the state of equilibrium. In this way, rotational fluctuations arise and they are transmitted to analyzed body 4. Magnets have been taken in order to get the more simple construction compared to the previous one [4].

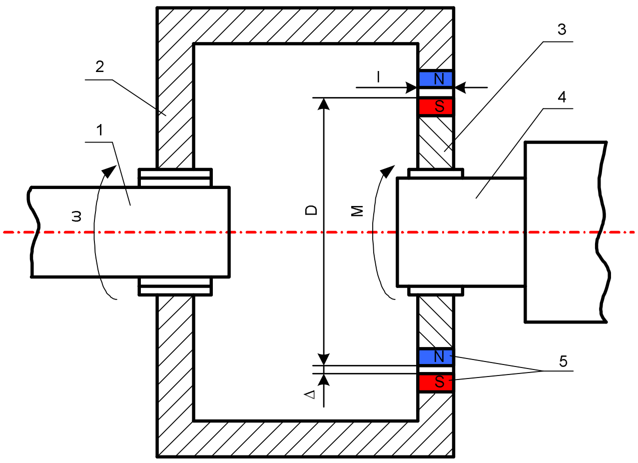

Fig. 1. The scheme of rotary oscillations vibrator: 1-rotor; 2- external ring; 3-internal disc; 4- spindle of the research object; 5-magnets

3. The dynamic model of the system and its investigation

The systems dynamic model (Fig.1), consisting of two (incoming and outgoing) links of rotary movement with steady magnets, can approximately be described by the following differential equations of motion:

$$I_1\ddot{\varphi}_1 + M_{12}\left[n(\varphi_1-\varphi_2)\right] + H_1\dot{\varphi}_1 = M_1\ ,$$
$$I_2\ddot{\varphi}_2 - M_{12}\left[n(\varphi_1-\varphi_2)\right] + H_2\dot{\varphi}_2 = M_2\ , \qquad (1)$$

here I_1, I_2 – moments of inertia of incoming and outgoing links; φ_1, φ_2 – angles of turning links; M_{12} – moment of magnetic forces, proceeding in one magnetic pair; n – the number of pair magnets; H_1, H_2 – coefficients of viscous friction; M_1, M_2 – moments of external forces.

Equations become simpler when the motion of one component is defined, for example:

$$\varphi_2 = \omega t \,, \tag{2}$$

Then if

$$\varphi = \varphi_1 \,,$$
$$\frac{1}{I_1} M_{12}[n(\varphi_1 - \varphi_2)] = -a \cos n(\omega t - \varphi) \,. \tag{3}$$

Differential equation of the motion is going to be:

$$\ddot{\varphi} + h\dot{\varphi} = a \cos n(\omega t - \varphi) + m \,, \tag{4}$$

where $h = \frac{H_1}{I_1}$, $m = \frac{M_1}{I_1}$.

Case 1: conservative system, i.e. when in the equations (1)

$$H_1 = H_2 = M_1 = M_2 = 0 \,.$$

Indicating:

$$\phi = \varphi_1 - \varphi_2 \,,$$

we get:

$$\ddot{\varphi} + \nu M(n\phi) = 0 \,, \tag{5}$$

where $\nu = \frac{I_1 + I_2}{I_1 I_2}$.

The solution of an equation is:

$$\dot{\phi} = \pm\sqrt{C - 2\nu \int M(n\phi) d\phi} \,, \tag{6}$$

where C is a constant of integration.
The equation (6) calculates separatrises.
Then the equation (4) is being analyzed.

Case 2: constant slow motion, when:

$$\varphi = \Omega t + \beta \,, \tag{7}$$

where $\Omega \ll \omega$. (8)

Having estimated (7,8) the equation is changed into:

$$\ddot{\beta} + h\dot{\beta} = a \cos n(\delta t - \beta) + m - h\Omega \,, \tag{9}$$

where $\delta = \omega - \Omega$. (10)

The equation (9) for calculating a slow motion is modified:

$$\ddot{\beta} + h\dot{\beta} = a \cos n\delta t + \varepsilon\{a[\cos n(\delta t - \beta) - \cos n\delta t] + \\ + m - h\Omega\}, \tag{11}$$

where ε- a small parameter in the end of calculations is 1. E shows the analyzed regime, when the motion doesn‘t bend a lot from the motion ε=0

The steady motion regime appears in the form of degree series:

$$\beta = \beta_0 + \varepsilon\beta_1 + \ldots, \tag{12}$$

where $\beta_i (i = 1,2,\ldots)$ are periodical functions.

Considering

$$\beta_0 = -\frac{a}{n(\omega - \Omega)[n^2(\omega - \Omega)^2 + h^2]} \cdot \\ \cdot [n(\omega - \Omega)\cos n(\omega - \Omega)t - h \sin n(\omega - \Omega)t]. \tag{13}$$

From (11, 12) β_1 is calculated in the following way:

$$\ddot{\beta}_1 + h\dot{\beta}_1 = a \cos n[(\omega - \Omega)t - \beta_0] - \\ - a \cos n(\omega - \Omega)t + m - h\Omega. \tag{14}$$

The periodicity condition of β_1 is:

$$\overline{a \cos n[(\omega - \Omega)t - \beta_0] - a \cos n(\omega - \Omega)t +} \\ + m - h\Omega = 0. \tag{15}$$

In the equation (15) we mark only the linear part according to β_0. Having estimated (12), we get:

$$\frac{ha^2}{2(\omega - \Omega)[n^2(\omega - \Omega)^2 + h^2]} + m - h\omega = 0 \,. \tag{16}$$

When the speed (8) of the motion is far smaller than the speed of the excitation wave, we can calculate:

$$\omega - \Omega = \omega - \varepsilon\Omega \,. \tag{17}$$

And to find the linear part of an answer (16) according to ε:

$$\Omega = \Omega_0 + \varepsilon\Omega_1 + \ldots, \tag{18}$$

where $\Omega_i\,(i = 0,1,2...)$ is constant.

From equations (16,17,18) we get:

$$\Omega_0 = \frac{a^2}{2\omega(n^2\omega^2 + h^2)} + \frac{m}{h},$$

$$\Omega_1 = \frac{h}{\omega^2(n^2\omega^2 + h^2)}\Omega_0, \tag{19}$$

so only confining those two approximations:

$$\Omega \approx [\frac{a^2}{2\omega(n^2\omega^2 + h^2)} + \frac{m}{h}][1 + \frac{h}{\omega^2(n^2\omega^2 + h^2)}]. \tag{20}$$

Case 3: steady motion when the system is moving in the speed of an excitation wave:

$$\varphi = \omega t + \overline{\varphi}, \tag{21}$$

where $\overline{\varphi}$ - constant.

From equations (4, 5) we get

$$a\cos n\overline{\varphi} + m - h\omega = 0. \tag{22}$$

The conditions of the existence and stability are the following ones:

$$\left|\frac{m - h\omega}{a}\right| \rangle 1,$$

$$na \sin n\overline{\varphi} \rangle 0. \tag{23}$$

Case 4: steady motion proceeds in an average speed, which is bigger than the speed of the excitation wave.

In this case:

$$\omega - \Omega = \varepsilon(\Omega - \omega)\rangle 0. \tag{24}$$

The zero approximation is:

$$\Omega_0 = \frac{m}{h}, \tag{25}$$

and confining two approximations:

$$\Omega \approx \frac{m}{h} - \frac{a^2}{2(\frac{m}{h} - \omega)[n^2(\omega - (\frac{m}{h})^2) + h^2]}. \tag{26}$$

Eq. (1) describes a well known rotary motion transfer mechanism [2] when $m = 0$. Also, Eq. (1) represents a system describing dynamics of a dendritic neuron firing process [3].

Evaluating $m = \omega = 1$ and change of variables $y = t - \varphi$ yields:

$$\ddot{y} + h\dot{y} + a\sin y = h - m. \tag{27}$$

This is a classical mathematical pendulum with external constant drag. There is no external drag when $m = h$. Equation (2) can be expressed in a form of two first order ordinary differential equations:

$$\begin{cases} \dfrac{dy}{dt} = z, \\ \dfrac{dz}{dt} = h - m - hz - a\sin y. \end{cases} \tag{28}$$

When $m \le h$, equation (3) produces 2 sets of equilibrium points:

$$P_1 = \left(z = 0; y = \arcsin\left(\frac{h-m}{a}\right) + 2\pi n\right);$$

$$P_2 = \left(z = 0; y = \pi - \arcsin\left(\frac{h-m}{a}\right) + 2\pi n\right),\ n \in Z. \tag{29}$$

The necessary (but not sufficient) condition for the existence of an equilibrium point is

$$-a \le h - m \le a. \tag{30}$$

Characteristic linearized equations in the surroundings of the points P_1 and P_2 will be:

$$\begin{vmatrix} -\lambda & 1 \\ -h & -\sqrt{a^2 - (h-m)^2} - \lambda \end{vmatrix} = 0;$$

$$\begin{vmatrix} -\lambda & 1 \\ -h & \sqrt{a^2 - (h-m)^2} - \lambda \end{vmatrix} = 0. \tag{31}$$

Both roots of the characteristic equation for P_1 have negative real parts. P_1 is a stable knot when $a^2 - (h-m)^2 - 4h \ge 0$; it is a stable focus when $a^2 - (h-m)^2 - 4h < 0$.

Characteristic roots for P_2 have always different signs of their real parts; P_2 is a saddle point. Basin boundaries of attractors can be constructed by reverse time integration from the surroundings of the unstable saddle points for different sets of system parameters (Fig. 2). It can be seen that two different stable attractors coexist – a stable equilibrium point and a stable limit cycle. The system's attractors can also be represented in the phase plane $(\dot{x};\ddot{x})$

(Fig. 3). Limit cycles are represented as closed loops in the phase plane $(\dot{x};\ddot{x})$. We skip the transients and plot only the steady state limit cycles at increasing values of the parameter *m*. Limit cycles get closer to the homoclinic orbit as the parameter *m* increases until, finally, the stable limit cycle disappears and only the stable equilibrium point (representing the motion by the velocity of the propagating wave) is left. We plot also a transient orbit for the equilibrium point for better visualization.

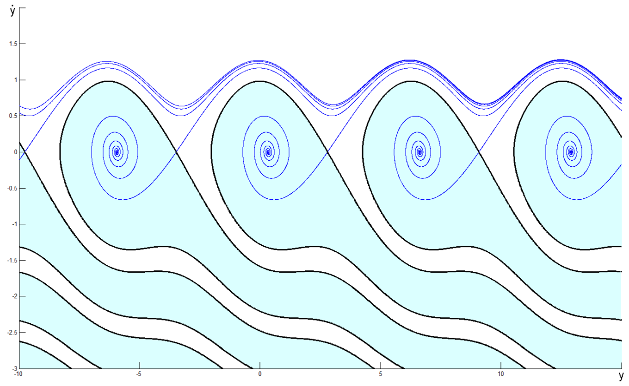

Fig. 2. Basin boundaries at h = 0.1; a = 0.3; m = 0.01. Gray-shaded regions mark the basins of attraction of the stable equilibrium point

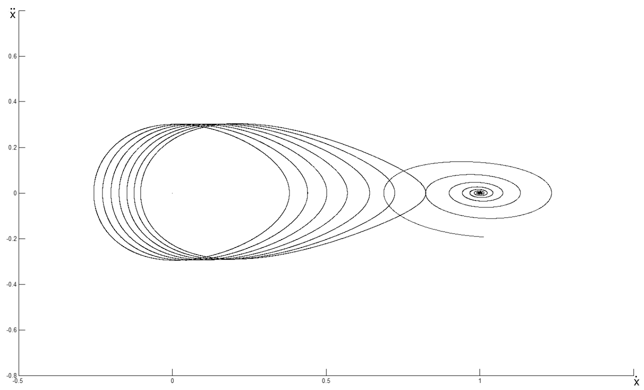

Fig. 3. Limit cycles in the phase plane $(\dot{x};\ddot{x})$ plotted at h = 0.1; a = 0.3; $m = 0.004i$; $i = 1,\ldots,10$

Fourier amplitude spectrums of limit cycles (of $\dot{x}$) are plotted in Fig. 4 at m = 0 and m = 0.03. It can be seen that the amplitude spectrum becomes more complex as the limit cycle approaches the homoclinic trajectory.

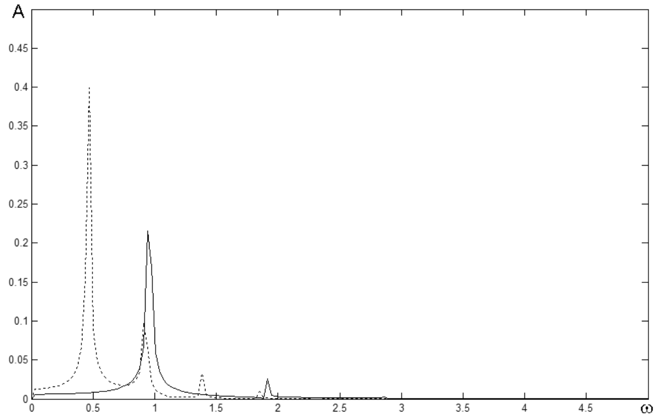

Fig. 4. Fourier amplitude spectrum of limit cycles at h = 0.1; a = 0.3; m = 0 (solid line) and m = 0.03 (dotted line)

4. Conclusions

The article analyses a mechanical rotary vibrator, the base of which is made of steady oscillatory magnets that move unstoppably and generate rotary oscillations. Such vibrators compared to the known ones can change from a micro up to a macro size and may have different power.

The system's dynamic characteristics have been found in the analytical and numerical methods. Specific features have been revealed, when the motion starts from the smaller speed and alters to the bigger speed of the excitation wave.

Harmonic analyze shows, that we can get different types of amplitude-frequency characteristics.

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