375. ANALYSIS OF A MODEL OF THE ADHESIVE LAYER BETWEEN TWO SHEETS OF PAPER

G. Petriaszwili, J. Pyrjew
Warsaw University of Technology,
Department of Production Engineering, Institute of Printing,
Konwiktorska st. 2, 00-217, Warsaw, Poland
E-mail: j.petriaszwili@wip.pw.edu.pl

(Received 2 June 2008, accepted 29 August 2008)

Abstract. There are cases of endurable books production. Paper proposes an improved mathematical model for calculating the stresses in the adhesive spine layer during the opening of the book.

Keywords: adhesive layer, book spine, stresses.

Introduction

Nowadays, the bookbinding with adhesives (*Perfect Binding*) is the widest practiced bookbinding technology in printing industry. Usage of different types of adhesives and paper poses for bookbinders many problems. This is the reason that there are cases of endurable books production, called by the readers, as „disposable". Formulating of a mathematical model and a numerical algorithm of the stresses within the adhesive layer emerging in the spine of books is therefore crucial.

Earlier research enabled development of the mathematical model and the analytical evaluation of the stresses [1], but the complexity of the adhesive layer phenomena requires more in-depth and coherent approach with more determinants taken into account. This paper proposes an improved mathematical model for calculating the stresses in the adhesive layer spine during the opening of the book.

Background

During the use of the book, adhesive layer is strained, which results in the form of internal stresses of the layer. Opening of the book is the main reason for the increase of internal stresses. Excessive stresses can aggravate contact between paper and adhesive layer, which leads to the damage of a book.

The structure of the adhesive binding of paper sheets in the adhesive layer is presented on Figure 1. This paper analyses the phenomena associated with the adhesive layer between two sheets of a book (Figure 2). We will mainly focus on evaluating the stresses occurring during the use of the book, more specifically when the book is opened to the angle $2\theta_0$. The following assumptions were made: paper is a non-flexible material; adhesive is a homogeneous, isotropic, elastic material with the following properties: E – Young's modulus, ν - Poisson ratio. This model is valid only for small deformations.

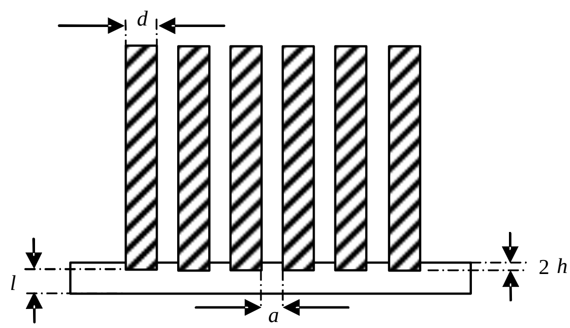

Fig. 1. The structure of the adhesive binding of the paper sheets: *d* – thickness of the paper sheet, *l* – thickness of the adhesive layer in the book spine, *a* – distance between paper sheets, *h* – thickness of the adhesive layer in the binding area of the paper sheet

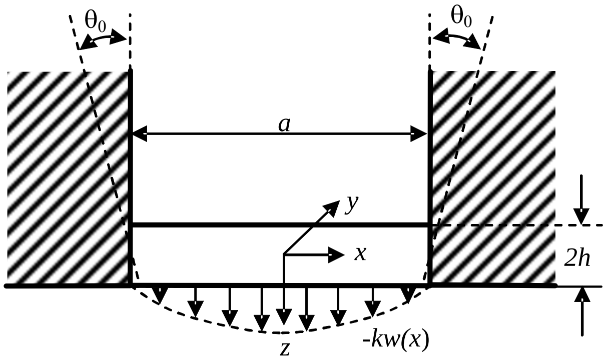

Fig. 2. An adhesive layer model between two sheets of paper

The differential equation of adhesive layer on elastic base as a deflection of elongated, rectangular plate $w(x)$ (i.e. arching) [2], $\frac{d^4w(x)}{dx^4}+4\alpha^4w(x)=0$, and formulas descri-bing the deflection of elastic plate can be written as follows.
Deflection of adhesive layer:

$$w(x)=\frac{\theta_0}{Z(\alpha a)}\left(-W_3(\frac{a}{2})W_1(x)+W_1(\frac{a}{2})W_3(x)\right)\cdot \quad (1)$$

Opening angle θ defined as follow:

$$\theta(x)=-\frac{dw(x)}{dx}=-\frac{\theta_0}{Z(\alpha a)}\left(4\alpha^4W_3(\frac{a}{2})W_4(x)+W_1(\frac{a}{2})W_2(x)\right)\cdot \quad (2)$$

Bending moment:

$$M_x(x)=-D\frac{d^2w(x)}{dx^2}=-D\frac{\theta_0}{Z(\alpha a)}\left(4\alpha^4W_3(\frac{a}{2})W_3(x)+W_1(\frac{a}{2})W_1(x)\right)\cdot \quad (3)$$

Stresses:

$$\sigma_x(x)=\frac{z}{2h^3/3}M_x(x)\cdot$$

The maximum deflection of adhesive layer:

$$w_{\max}=-w(0)=\frac{2\theta_0}{\alpha}\frac{\sin(\frac{\alpha a}{2})\sinh(\frac{\alpha a}{2})}{\sin(\alpha a)+\sinh(\alpha a)}\cdot \quad (4)$$

The maximum bending moment:

$$M_{\max}=-D\frac{d^2w(\frac{a}{2})}{dx^2}=-D2\alpha\theta_0\frac{\cos(\alpha a)+\cosh(\alpha a)}{\sin(\alpha a)+\sinh(\alpha a)}\cdot \quad (5)$$

Bending moment in the middle of the adhesive layer (plate):

$$M(0)=-D\frac{d^2w(0)}{dx^2}=-D4\alpha\theta_0\frac{\cos(\frac{\alpha a}{2})\cosh(\frac{\alpha a}{2})}{\sin(\alpha a)+\sinh(\alpha a)}, \quad (6)$$

where: $D=\frac{2Eh^3}{3(1-\nu^2)}$ - cylindrical stiffness of a plate, α - coefficient, which could be found from equation $4\alpha^4=k/D$, where $k=E/l$.
Specific solutions of this differential equation are given by functions:

$$W_1(x)=\cosh(\alpha x)\cos(\alpha x), \quad (7)$$

$$W_2(x)=\frac{1}{2\alpha}\left(\cosh(\alpha x)\sin(\alpha x)+\sinh(\alpha x)\cos(\alpha x)\right), \quad (8)$$

$$W_3(x)=\frac{1}{2\alpha^2}\sinh(\alpha x)\sin(\alpha x), \quad (9)$$

$$W_4(x)=\frac{1}{4\alpha^3}\left(\cosh(\alpha x)\sin(\alpha x)-\sinh(\alpha x)\cos(\alpha x)\right)\cdot \quad (10)$$

After introducing the dimensionless coordinate: $\xi=\frac{x}{a}$, $\left(-\frac{1}{2}<\xi<\frac{1}{2}\right)$ the dimensionless parameter of adhesive layer Λ can be written as:

$$\Lambda=\frac{\alpha a}{2}=\frac{a}{2}\sqrt[4]{\frac{3(1-\nu^2)}{l(2h)^3}}\cdot \quad (11)$$

Instead of equations (7)-(10) the following equations are obtained:

$$w_1(\xi)=\cosh(2\Lambda\xi)\cos(2\Lambda\xi), \quad (12)$$

$$w_2(\xi)=\cosh(2\Lambda\xi)\sin(2\Lambda\xi)+\sinh(2\Lambda\xi)\cos(2\Lambda\xi), \quad (13)$$

$$w_3(\xi)=\sinh(2\Lambda\xi)\sin(2\Lambda\xi), \quad (14)$$

$$w_4(\xi)=\cosh(2\Lambda\xi)\sin(2\Lambda\xi)-\sinh(2\Lambda\xi)\cos(2\Lambda\xi), \quad (15)$$

Deflection of adhesive layer in this case:

$$w(\xi)=\frac{\theta_0 a}{\Lambda}\frac{1}{z^*}\left(-w_3^*w_1(\xi)+w_1^*w_3(\xi)\right) \quad (16)$$

where:

$$w_1^*=\cosh(\Lambda)\cos(\Lambda), \quad (17)$$

$$w_2^*=\cosh(\Lambda)\sin(\Lambda)+\sinh(\Lambda)\cos(\Lambda), \quad (18)$$

$$w_3^*=\sinh(\Lambda)\sin(\Lambda), \quad (19)$$

$$w_4^*=\cosh(\Lambda)\sin(\Lambda)-\sinh(\Lambda)\cos(\Lambda), \quad (20)$$

$$z^*=2(w_1^*w_2^*+w_3^*w_4^*)=\sin(2\Lambda)+\sinh(2\Lambda)\cdot \quad (21)$$

Figure 3 presents the influence of the dimensionless coordinate $\xi=\frac{x}{a}$ and the dimensionless parameter of adhesive layer Λ on the dimensionless deflection of adhesive layer $w(\xi)/(\theta_0 a)$ [3].
Opening angle θ of the paper page is defined as follow:

$$\theta(\xi)=-\frac{dw(\xi)}{ad\xi}=-2\theta_0\frac{1}{z^*}\left(w_3^*w_4(\xi)+w_1^*w_2(\xi)\right)\cdot \quad (22)$$

The formula for bending moment takes the form:

$$M_x(\xi) = -D\frac{d^2 w(\xi)}{a^2 d\xi^2} = -D\frac{8\theta_0\Lambda}{a}\frac{1}{z^*}\left(w_3^* w_3(\xi) + w_1^* w_1(\xi)\right). \quad (23)$$

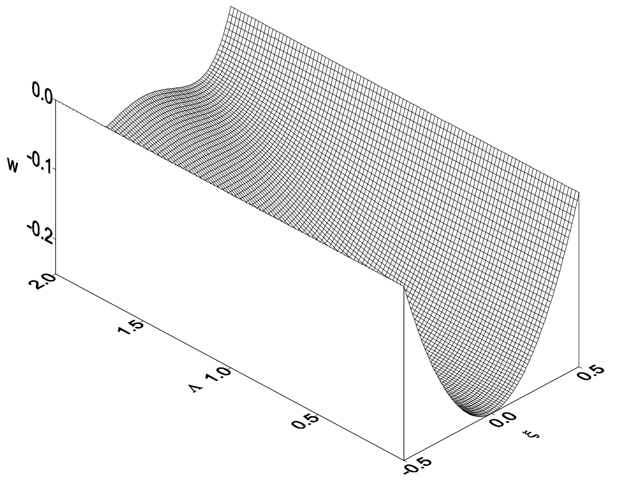

Fig. 3. The influence of the dimensionless coordinate ξ and parameter Λ on the dimensionless deflection of adhesive layer

The stresses in adhesive layer is calculated according to:

$$\sigma_x(\xi) = \frac{z}{2h^3/3}M_x(\xi). \quad (24)$$

Figure 4 presents the influence of the dimensionless coordinate ξ and dimensionless parameter Λ on the dimensionless opening angle $\theta(\xi)/\theta_0$.

The maximum deflection of adhesive plate:

$$w_{\max} = -w(0) = \frac{a\theta_0}{\Lambda}\frac{\sin(\Lambda)\sinh(\Lambda)}{\sin(2\Lambda)+\sinh(2\Lambda)}. \quad (25)$$

The maximum bending moment is:

$$M_{\max} = -D\frac{d^2 w(\frac{a}{2})}{a^2 d\xi^2} = -D\frac{4\Lambda\theta_0}{a}\frac{\cos(2\Lambda)+\cosh(2\Lambda)}{\sin(2\Lambda)+\sinh(2\Lambda)}. \quad (26)$$

Moment in the middle of the adhesive plate is:

$$M(0) = -D\frac{d^2 w(0)}{a^2 d\xi^2} = -D\frac{8\Lambda\theta_0}{a}\frac{\cos(\Lambda)\cosh(\Lambda)}{\sin(2\Lambda)+\sinh(2\Lambda)}. \quad (27)$$

It was demonstrated that $M_{max} > M(0)$ for all values of the parameter Λ.

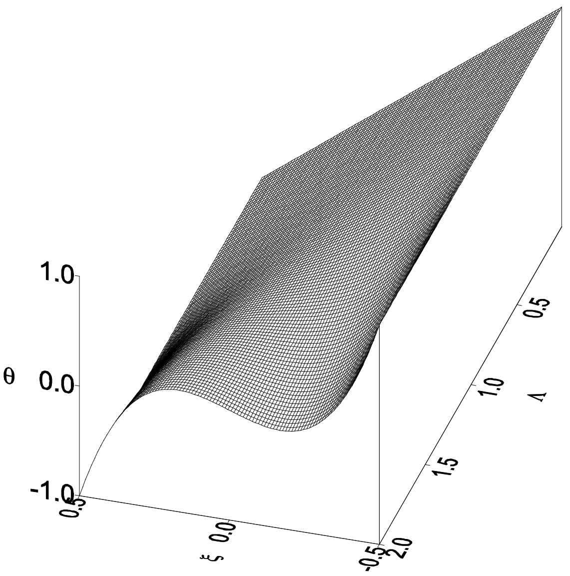

Fig. 4. The influence of the dimensionless coordinate ξ and parameter Λ on the dimensionless opening angle

This paper proposes an improved mathematical model and suggests a solution to the system of equations. Consequently, it provides the means to evaluate the strength of spine adhesive layer in the stage of book structure development.

References

[1] **Kornilov I. K.** Books Construction Design –Moscow: MSUEE, 2001, -212 p. (in Russian)

[2] **Pisarenko G. S., Agajev V. A., Kvitka A. L. et al.** Strength of Materials – Kiev: Vischa Skola, 1986, - 775 p. (in Russian).

[3] **Petriaszwili G., Pyrjew J.** An analysis of the stresses in adhesive layer during books operational use. VIII Seminar in Graphic Arts, Conference Proceeding, University Pardubice, - 2007. p. 193-197.