# 372. CONSTANT MAGNET MOTORS 

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#### Abstract

Motors or actuators with autonomic source of energy, such as for example with constant magnets have scientific and practical value. The transformation of energy of magnets into mechanical motion is performed by using various mechanisms. In the analysed case the wave profile with the elastic member is used. Such systems can be noted by their small dimensions. The expression of the driving force and some characteristics are presented.


Keywords: actuator, magnets, transformation of energy, mechanical motion.

## 1. Introduction

Earlier it was shown that constant magnets are suitable for the creation of generators of mechanical vibrations of various types. Those magnets are used for the transformation of continuous motion of the input member into vibrations of the output member. On this topic several patents were obtained, for example [1] and others. In this paper the actuators based on the energy of constant magnets themselves are analysed. The latter energy is transformed into mechanical motion by mechanisms of various types. Here the mechanism of wave type with elastic member is analysed.

## 2. Model

The analysed actuator consists from the input member 1 and the output member 2 (Fig. 1).


Two mutually parallel rows of constant magnets (or other sources of energy) 3 and 4 and also the wave (curvilinear) profile 5 are attached to the input member 1. The magnets in the rows 3 and 4 are located by opposite poles one against another with constant step. The wave profile is a periodic function of displacement parallel to the lines of location of the magnets 3 and 4 . The output member 2 is kinematically connected to the wave profile 5 , which may move with respect to the input member parallel and perpendicular to the directions of location of magnets 3 and 4. Constant magnets are attached to the output member 2 parallel to the rows of magnets 3 and 4 . In all the parallel rows 3,4 and 2 magnets with equal distances between poles are attached. All magnets are of the same power. The position of equilibrium of the output member $2 \mathrm{y}=0, \dot{\mathrm{x}}=0$ is ensured by the elastic dissipative member 6 .

## 3. Analysis

In Fig. 2 and Fig. 3 the forces acting to the output member 2 are shown.


Fig. 2. The force by which the row 3 of magnets of the input member 1 acts to the output member 2 in the direction of the axis Ox when the members 1 and 2 do not move with respect to one another. $\mathrm{F}_{24 \mathrm{x}}$ is the same when there is respectively the same distance, but with opposite sign

Fig. 1. The scheme of the actuator


Fig. 3. The force by which the row of magnets 3 of the input member 1 acts to the output member 2 in the direction of the axis Oy when the members do not move with respect to one another. $\mathrm{F}_{24 \mathrm{y}}$ is the same when the respective distance is the same, but with opposite sign

The force of magnets of the input member 1 acting to the output member 2 in the direction of the axis $O x$
$\mathrm{F}_{\mathrm{mx}}=\mathrm{nF}_{23 \mathrm{x}} \cdot \mathrm{z}(\mathrm{y})+\mathrm{nF}_{24 \mathrm{x}} \cdot \mathrm{z}(-\mathrm{y})=\mathrm{nF}_{23 \mathrm{x}}[\mathrm{z}(\mathrm{y})-\mathrm{z}(-\mathrm{y})]$, (1)
in the direction of the axis Oy
$\mathrm{F}_{\mathrm{my}}=-\mathrm{nF}_{23 \mathrm{y}} \cdot \mathrm{z}(\mathrm{y})-\mathrm{nF}_{24 \mathrm{y}} \cdot \mathrm{z}(-\mathrm{y})=-\mathrm{nF}_{23 \mathrm{y}}[\mathrm{z}(\mathrm{y})+\mathrm{z}(-\mathrm{y})]$, (2)
where n is the number of pairs of the poles of magnets of the output member 2,
$\mathrm{F}_{24 \mathrm{x}}=-\mathrm{F}_{23 \mathrm{x}}$,
$\mathrm{F}_{24 \mathrm{y}}=\mathrm{F}_{23 \mathrm{y}}$,
$z(y)$ and $z(-y)$ take the strength of interaction into account depending on distance of the poles of magnets.

The wave (curvilinear) profile of the input member is a periodic function as well as the forces of magnets from $x$, that is
$y=y(x)$.
The differential equation of motion of the output member is obtained by taking into account the dry and the viscous friction acting to the output member 2 at the contact point of the input member 1 profile 5:
$\mathrm{F}_{\mathrm{x}}+\mathrm{y}_{\mathrm{x}}^{\prime} \mathrm{F}_{\mathrm{y}}+\left(-\mathrm{y}_{\mathrm{x}}^{\prime} \mathrm{F}_{\mathrm{x}}+\mathrm{F}_{\mathrm{y}}\right) \mathrm{f}_{0} \operatorname{sign} \dot{\mathrm{x}}+\mathrm{f}_{1}\left(1+\mathrm{y}_{\mathrm{x}}^{\prime 2}\right) \dot{\mathrm{x}}=0$,
where
$F_{x}=m \ddot{x}+H_{x} \dot{\mathrm{x}}+\mathrm{Q}_{\mathrm{x}}-\mathrm{F}_{\mathrm{mx}}$,
$F_{y}=m \ddot{y}+H_{y} \dot{y}+C_{y}-F_{m y}$,
$f_{0}$ and $f_{1}-$ coefficients of dry and viscous friction, $\dot{\mathrm{s}}_{21}=\left(1+\mathrm{y}_{\mathrm{x}}^{\prime 2}\right) \dot{\mathrm{x}}-$ velocity of slippage of the member 2 with respect to the profile $5, \mathrm{~m}$ - mass of the output member 2,
$H_{x}, H_{y}$ - coefficients of viscous friction according to the axes Ox and Oy respectively,
$\mathrm{Q}_{\mathrm{x}}$ - the external force of resistance according to the axis Ox,

C - the coefficient of stiffness of the spring, $\cdot=\mathrm{d} / \mathrm{dt}$, ${ }^{\prime}=\mathrm{d} / \mathrm{dx}$.

From the equation (5) by taking into account (6) and after dividing by m it is obtained

$$
\begin{aligned}
& \left(1+y_{x}^{\prime 2}\right) \ddot{\mathrm{x}}+\mathrm{y}_{\mathrm{x}}^{\prime} \mathrm{y}_{\mathrm{xx}}^{\prime /} \dot{\mathrm{x}}^{2}+\left(\mathrm{h}_{\mathrm{x}}+\mathrm{y}_{\mathrm{x}}^{\prime 2} \mathrm{~h}_{\mathrm{y}}\right) \dot{\mathrm{x}}+\mathrm{q}_{\mathrm{x}}^{*}+\mathrm{p}^{2} \mathrm{yy}_{\mathrm{x}}^{\prime}- \\
& -\frac{1}{\mathrm{~m}} \mathrm{~F}_{\mathrm{mx}}-\frac{1}{\mathrm{~m}} \mathrm{y}_{\mathrm{x}}^{\prime} \mathrm{F}_{\mathrm{my}}+\left[\mathrm{y}_{\mathrm{xx}}^{\prime /} \dot{\mathrm{x}}^{2}+\mathrm{y}_{\mathrm{x}}^{\prime}\left(-\mathrm{h}_{\mathrm{x}}+\mathrm{h}_{\mathrm{y}}\right) \dot{\mathrm{x}}+\mathrm{p}^{2} \mathrm{y}-\right. \\
& \left.-\mathrm{y}_{\mathrm{x}}^{\prime} \mathrm{q}_{\mathrm{x}}^{*}+\frac{1}{\mathrm{~m}} \mathrm{y}_{\mathrm{x}}^{\prime} \mathrm{F}_{\mathrm{mx}}-\frac{1}{\mathrm{~m}} \mathrm{~F}_{\mathrm{my}}\right] \mathrm{f}_{0} \operatorname{sign} \dot{\mathrm{x}}+\mathrm{f}_{1}^{*}\left(1+\mathrm{y}_{\mathrm{x}}^{\prime 2}\right) \dot{\mathrm{x}}=0
\end{aligned}
$$

(7)
where
$\mathrm{q}_{\mathrm{x}}^{*}=\frac{\mathrm{Q}_{\mathrm{x}}}{\mathrm{m}}, \mathrm{p}^{2}=\frac{\mathrm{C}}{\mathrm{m}}, \mathrm{h}_{\mathrm{x}}=\frac{\mathrm{H}_{\mathrm{x}}}{\mathrm{m}}, \mathrm{h}_{\mathrm{y}}=\frac{\mathrm{H}_{\mathrm{y}}}{\mathrm{m}}, \mathrm{f}_{1}^{*}=\frac{\mathrm{f}_{1}}{\mathrm{~m}}$.
Further the case is analysed when the wave (curvilinear) profile and the magnetic forces acting to the output member are harmonic functions of the argument $x$, and $z(y)$ is varying according to the line. On this basis it is assumed

$$
\begin{align*}
& \mathrm{y}=\mathrm{A} \sin \frac{\pi \mathrm{x}}{l}, \\
& \mathrm{~F}_{\mathrm{mx}}=\mathrm{F}_{\mathrm{x}_{\mathrm{o}}} \sin \frac{\pi \mathrm{x}}{l}[\mathrm{z}(\mathrm{y})-\mathrm{z}(-\mathrm{y})],  \tag{8}\\
& \mathrm{F}_{\mathrm{my}}=-\mathrm{F}_{\mathrm{y}_{\mathrm{o}}} \cos \frac{\pi \mathrm{x}}{l}[\mathrm{z}(\mathrm{y})+\mathrm{z}(-\mathrm{y})] .
\end{align*}
$$

In this case it is assumed
$\mathrm{z}(\mathrm{y})=0.5\left(\frac{\mathrm{y}}{\mathrm{L}}+1\right)$,
$z(y)-z(-y)=\frac{y}{L}$,
$\mathrm{z}(\mathrm{y})+\mathrm{z}(-\mathrm{y})=1$.
The equation (7) by taking into account $(8,9)$ and $\mathrm{f}_{0}=\mathrm{f}_{1}^{*}=0$ takes the following form
$\left[1+\left(\frac{\pi \mathrm{A}}{l}\right)^{2} \cos ^{2} \frac{\pi \mathrm{x}}{l}\right] \ddot{\mathrm{x}}+\left[\mathrm{h}_{\mathrm{x}}+\mathrm{h}_{\mathrm{y}}\left(\frac{\pi \mathrm{A}}{l}\right)^{2} \cos ^{2} \frac{\pi \mathrm{x}}{l}\right] \dot{\mathrm{x}}+$
$+\left(\frac{\pi \mathrm{A}}{l}\right)^{2} \frac{1}{\pi}\left[\mathrm{p}^{2}-\left(\frac{\pi}{l}\right)^{2} \dot{\mathrm{x}}^{2}\right] \cos \frac{\pi \mathrm{x}}{l} \sin \frac{\pi \mathrm{x}}{l}-\mathrm{q}_{\mathrm{x}}^{*}+$
$+\frac{\mathrm{A}}{\mathrm{m}}\left(-\frac{\mathrm{F}_{\mathrm{x}_{\mathrm{o}}}}{\mathrm{L}} \sin ^{2} \frac{\pi \mathrm{x}}{l}+\frac{\pi \mathrm{F}_{\mathrm{y}_{\mathrm{o}}}}{l} \cos ^{2} \frac{\pi \mathrm{x}}{l}\right)=0$.
10)

For the case of rotors
$l=\frac{\pi \mathrm{D}}{\mathrm{n}}$,
where D is the diameter of rotors in which the centers of poles of the magnets are located, n is the number of pairs of poles magnets. The following notations are introduced
$\mathrm{n} \varphi=\frac{\pi \mathrm{x}}{\mathrm{D}}, \lambda=\frac{\pi \mathrm{A}}{l}=\frac{\mathrm{nA}}{\mathrm{D}}, \mathrm{q}_{\mathrm{x}}=\frac{\mathrm{Q}_{\mathrm{x}}}{\mathrm{mD}}, \mathrm{f}_{\mathrm{x}_{0}}=\frac{\mathrm{A}}{\mathrm{mLD}} \mathrm{F}_{\mathrm{x}_{0}}$,
$\mathrm{f}_{\mathrm{y}_{0}}=\frac{\pi \mathrm{A}}{\mathrm{m} l \mathrm{D}} \mathrm{F}_{\mathrm{y}_{0}}$.
The equation (10) by taking into account $(11,12)$ takes the following form
$\left(1+\lambda^{2} \cos ^{2} \mathrm{n} \varphi\right) \ddot{\varphi}+\left(\mathrm{h}_{\mathrm{x}}+\mathrm{h}_{\mathrm{y}} \lambda^{2} \cos ^{2} \mathrm{n} \varphi\right) \dot{\varphi}+$
$+\lambda^{2} \frac{1}{\mathrm{n}}\left(\mathrm{p}^{2}-\mathrm{n}^{2} \dot{\varphi}^{2}\right) \cos \mathrm{n} \varphi \sin \mathrm{n} \varphi-\mathrm{q}_{\mathrm{x}}-\mathrm{f}_{\mathrm{x}_{0}} \sin ^{2} \mathrm{n} \varphi+$
$+\mathrm{f}_{\mathrm{y}_{0}} \cos ^{2} \mathrm{n} \varphi=0$.
From the equations $(10,13)$ it follows that when
$-\mathrm{f}_{\mathrm{x}_{0}}+\mathrm{f}_{\mathrm{y}_{0}} \neq 0$, or $-\frac{\mathrm{F}_{\mathrm{x}_{0}}}{\mathrm{~L}}+\frac{\mathrm{n}}{\mathrm{D}} \mathrm{F}_{\mathrm{y}_{0}} \neq 0$,
the magnets create the attractive force.
For the investigation of equation (13) in the steady state regime it is assumed
$\varphi=\omega t+\beta$
and the equation (13) takes the following form
$\alpha_{1} \ddot{\beta}+\alpha_{2} \dot{\beta}+\alpha_{3} \sin 2 \omega \mathrm{t}+\alpha_{4} \cos 2 \omega \mathrm{t}+\varepsilon\left[\alpha_{2} \omega+\alpha_{5}+\right.$
$\left.+\alpha_{6}(\omega t+\beta, \omega t)\right]=0$,
where $\varepsilon$ is a small parameter at the end of calculations assumed equal to one,
$\alpha_{1}=1+0.5 \lambda^{2}, \alpha_{2}=h_{\mathrm{x}}+0.5 \lambda^{2} \mathrm{~h}_{\mathrm{y}}$,
$\alpha_{3}=0.5 \lambda^{2} \frac{1}{\mathrm{n}}\left(\mathrm{p}^{2}-\mathrm{n}^{2} \omega^{2}\right), \alpha_{4}=0.5\left(\mathrm{f}_{\mathrm{x}_{0}}+\mathrm{f}_{\mathrm{y}_{0}}\right)$,
$\alpha_{5}=0.5\left(-\mathrm{f}_{\mathrm{x}_{0}}+\mathrm{f}_{\mathrm{y}_{0}}\right)+\mathrm{q}_{\mathrm{x}}$,
$\alpha_{6}(\omega \mathrm{t}+\beta, \omega \mathrm{t})=\alpha_{6}=\alpha_{3}[\sin 2(\omega \mathrm{t}+\beta)-\sin 2 \omega \mathrm{t}]+$
$+\alpha_{4}[\cos 2(\omega \mathrm{t}+\beta)-\cos 2 \omega \mathrm{t}]+$
$+0.5 \lambda^{2}\left(\mathrm{p}^{2}-\mathrm{n}^{2} \omega^{2}\right) \cos 2(\omega \mathrm{t}+\beta)-$
$-0.5 \lambda^{2} n\left(2 \omega \dot{\beta}+\dot{\beta}^{2}\right) \sin 2(\omega t+\beta)$.
The motion is sough with the help of the power series with respect to $\varepsilon$

$$
\begin{equation*}
\beta=\beta+\varepsilon \beta_{1}+\ldots . \tag{18}
\end{equation*}
$$

From the equations $(16,18)$ it is obtained
$\beta_{0}=\frac{1}{\Delta}\left[\left(2 \alpha_{1} \alpha_{3} \omega-\alpha_{2} \alpha_{4}\right) \sin 2 \omega \mathrm{t}+\right.$
$+\left(\alpha_{2} \alpha_{3}+2 \alpha_{1} \alpha_{4} \omega\right) \cos 2 \omega t$,
where $\Delta=\left(4 \omega^{2} \alpha_{1}^{2}+\alpha_{2}^{2}\right) 2 \omega$.
The condition of periodicity of $\beta_{1}$ is
$\bar{\alpha}_{6}=\overline{\alpha_{6}\left(\omega t+\beta_{0}, \omega t\right)}=0$,
where the upper dash means averaging with respect to time.

From the equations $(16-20)$ it is obtained
$\bar{\alpha}_{6}=\alpha_{2} \omega+\alpha_{5}+\varepsilon \alpha_{7}=0$,
where
$\alpha_{7}=\frac{\alpha_{4}}{\left(4 \omega^{2} \alpha_{1}^{2}+\alpha_{2}^{2}\right) 2 \omega}\left\{\alpha_{2} \alpha_{4}+\left[2(1+n) \alpha_{1} \omega^{2}+\right.\right.$
$\left.\left.+0.5 \mathrm{~h}_{\mathrm{y}} \alpha_{2}\right] \lambda^{2} \omega\right\}$.
Thus from the equation (21) $\omega$ is found with the help of the power series with respect to $\varepsilon$

$$
\begin{equation*}
\omega=\omega_{0}+\varepsilon \omega_{1}+\ldots \tag{22}
\end{equation*}
$$

On the basis of (21-22) it is obtained

$$
\begin{align*}
& \omega_{0}=-\frac{\alpha_{5}}{\alpha_{2}}=\frac{\mathrm{f}_{\mathrm{x}_{0}}-\mathrm{f}_{\mathrm{y}_{0}}-\mathrm{q}_{\mathrm{x}}}{2\left(\mathrm{~h}_{\mathrm{x}}+0.5 \lambda^{2} \mathrm{~h}_{\mathrm{y}}\right)}, \\
& \omega_{1}=-\left.\frac{1}{\alpha_{2}} \alpha_{7}\right|_{\omega=\omega_{0}} . \tag{23}
\end{align*}
$$

## 4. Conclusions

It is determined that constant magnets may ensure steady state motion of the output member. There may be a great number of actuators of this type. Here the structure is analysed which consists from three rows of magnets and a wave mechanism with an elastic member. The regime of resonance makes the implementation of the steady state motion easier.

## References

1. K. Ragulskis, R. Ruzgus. Patent SU 639707 (1978), Patent SU 706241 (1979) and others.
