354. Finite element method: some modelling issues

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Abstract. This paper presents a meaningful discretization scheme for asymmetric object, such as rail section for central loading. Finite element (FE) plane stress analysis of the problem is carried out. The results from FE analysis are post processed to simulate the photoelastic experiment and check the discretization scheme. Issues related to experimental limitation of desired loading and their specification as boundary condition in finite element modeling are discussed.

Keywords: Finite element modelling; mesh discretization; photo-elasticity; fringe contours; isochromatics, central loading.

Introduction

In recent years with advancement in technologies, numbers of numerical and experimental techniques are available for the investigator and researchers for the stress analysis such as microcomputers and image processing. Photoelasticity is an experimental method which yields the principle stress difference and its orientation in the form of fringes known as isochromatics and isoclinics respectively [1]. The finite element method (FEM) is a numerical method for solving a given boundary value problem. The technique is used to solve complex problems. Although the FEM is advantageous for a detailed parametric study of a problem [2], the adequacy of the finite element (FE) approximation needs to be verified at least for one configuration. Since photoelasticity and FEM are whole field techniques, the real potential of the approach is realized in complex problem situation. In such cases identification of stress concentration zones, essential boundary conditions (EBC) and natural boundary conditions (NBC) itself may not be directly evident. The results from a photoelasticity experiment can help to identify stress concentration zones. Further the plotting of fringe contours from FE results has made the comparison very simple and straight forward [3]. To obtain accurate results of stress concentration analysis, the results of a photoelastic experiment can be used as basis for verifying the FE modeling of a given problem [4]. FE modeling is said to be meaningful if post processing of FE results accurately mimics the photoelastic experiments. For accurate numerical analysis the boundary conditions (BC) must be specified based on actual loading condition. Some times although we want that the loading should be applied centrally but due to experimental limitations this may not be possible. Now the BC in FE modeling should be the actual condition existing in experiment, i.e., the BC's specified in FE modeling should take care of experimental limitations in terms of EBC and NBC.

The I section is most widely used cross section from engineering application point of view. It is frequently subjected to central and inclined loadings. As we know when a beam is subjected to bending, the stress is proportional to the distance from neutral axis. Thus for purpose of economy and weight reduction the material must be concentrated as much as possible at the greatest distance from neutral axis, thus the immediate choice turns towards I section.

Discretization of the domain is the first step in FEM in which the given domain is discretized into a collection of pre selected finite elements. Depending on nature of the problem and BC discretization is done. The FE mesh generation has been developing over several decades now. A variety of real-life engineering problems imposes additional requirements on existing mesh generation technologies. For example, local mesh refinement zones around common stress concentrators (mechanical constraints and point loads) must be readily generated as described by Tsvelikh and Axenenko [5]. Lee and Hobbs [6] described the basic principles for the generation of adaptive FE meshes over arbitrary two-dimensional domains using the advancing front technique. In the front technique, nodes and elements are generated one by one to fill up the problem domain. Zienkiewicz and Zhu [7] described automatic mesh generation scheme with aspects such as (i) economical and efficient error estimating process; (ii) close prediction of the refinement necessary for a specific accuracy to be achieved; (iii) implementation of the predicted refinement.

Jung and White [8] presented the results of FE analysis studies of four curved steel I-girders. The models were constructed using a four-node quadrilateral displacement-based shell element with reduced integration. Salem *et al.* [9] determined the ultimate axial capacity of different columns made of slender I-sections using a non-linear FE model.

The generation of digital images mimicking the effect of photoelasticity naturally incorporates into the hybrid iterative procedure enabling effective interpretation of experimental results and provides insight into the physical processes taking place in the analyzed objects. The generation of digital photoelastic images is not a straightforward procedure. It involves such steps as the construction of a numerical model of the analyzed object, FE calculations based on the loading scheme and BC, determination of the nodal values of stress components and their smoothing, and generation of appropriate digital images. Ramesh and Pathak [10] showed the use of photoelasticity to check improper boundary conditions and also to come up with guidelines for discretizing the domain using quadrilateral elements for a class of stress concentration problems. Peindl *et al.* [11] carried out photoelastic stress freezing analysis of total shoulder replacement system. The approach also provides valuable and intuitive visual data conveying stress distribution information to medical and other non-engineering professionals. Ragulskis and Ragulskis [12] described displacement-based FE formulations coupled with stress-based photoelasticity analysis. As the stress field was discontinuous at the inter element boundaries, they introduced smoothing procedure that enabled the generation of high-quality digital images acceptable for hybrid experimental-numerical techniques. Seguchi *et al.* [13] developed a real-time "Computer-Aided FRinge-pattern AN-alyzer" (CAFRAN) system for the analysis of photoelastic fringes.

Discretization scheme for symmetric object has been proposed and validated for class of problems such as circular disk under diametral compression [4,12], rectangular plate with a hole [4]. However studies related to asymmetric bodies are few [8,9]. This paper presents a meaningful discretization scheme for asymmetric object, such as rail section for central loading. FE plane stress analysis of the problem is carried out. The results from FE analysis are post processed to simulate the photoelastic experiment for the same case and check the discretization scheme. Issues related to experimental limitation of desired loading and their specification as BC in FE modeling are discussed. The technique can be used for analysis of dynamics and vibrations of elastic structures.

Finite element modelling and photoelasticity - numerical and experimental tools

In the FE analysis, choice of the element type, number of elements and the density of elements depends on the geometry of domain, problem to be analyzed and degree of accuracy desired. No specific formula exits to get this information. Analysts are mostly guided by their technical background, insight to the physics of the problem and their experience in finite element modeling. There are three sources of error in the FE solution (a) those due to the approximation of the domain; (b) those due to the approximation of the solution; (c) those due to numerical computation (e.g., numerical integration and round - off errors in a computer). The estimation of these errors, in general, is not a simple matter. However, under certain conditions, they can be estimated for a given element and problem [2]. The accuracy and convergence of the FE solution depends on the differential equation, its integral form, and the number of elements used. After imposing the boundary condition the global matrix is solved to get the value of primary variable at different nodes. The solution of FE gives the nodal values of the primary unknown (e.g. displacement, velocity). Post processing of the results gives interpretation of the results to check whether the solution makes sense (an understanding of the physical process and experience are the guides when other solutions are not available for comparison).

Photoelasticity is the optical experimental technique available to study the stresses interior to model. It is one of the oldest methods for experimental stress analysis, but has been overshadowed by the FEM for engineering application over the past three decades. Photoelasticity is an optical technique which provides whole field information in principal stresses, namely contours of ($\sigma_1 - \sigma_2$) in the form of fringes. These fringes are known as isochromatics [1]. The stress analysis in photoelastic technique is usually done by using information from isochromatics and isoclinic patterns. The former gives the information of differences of the principal stresses and the latter the information of their directions. The analyzing procedure, however, includes the determinations of fringe order and its position in the two-dimensional domain under consideration and the fringe patterns are obtained. In photoelasticity a special type of materials which have double refractive properties are used for preparation of models of specimen in which stresses are to be determined. Photoelastic bench is experimental equipment in which the photoelastic models are loaded and stress distribution is obtained. The stress optic law relates the stress information to optical measurement as

$$\sigma_1 - \sigma_2 = \frac{N F_\sigma}{t}, \tag{1}$$

where F_σ is the material stress fringe value with the units N/mm/fringe, N is isochromatic fringe order, t is material thickness.

Photoelastic model loading

Success in photoelastic model preparation depends to a large extent on the selection of material, casting techniques used and precaution taken during fabrication of photoelastic model. The operating procedure followed while conducting the photoelastic experimental work are preparation of photoelastic model, appropriate load application and observed photoelastic fringe pattern. Brief examination of photoelastic literature suggests that most polymeric materials exhibit double refraction. In this work photoelastic stress analysis of I section made of Epoxy resin CY 230 with Hardener HY 951 has been used.

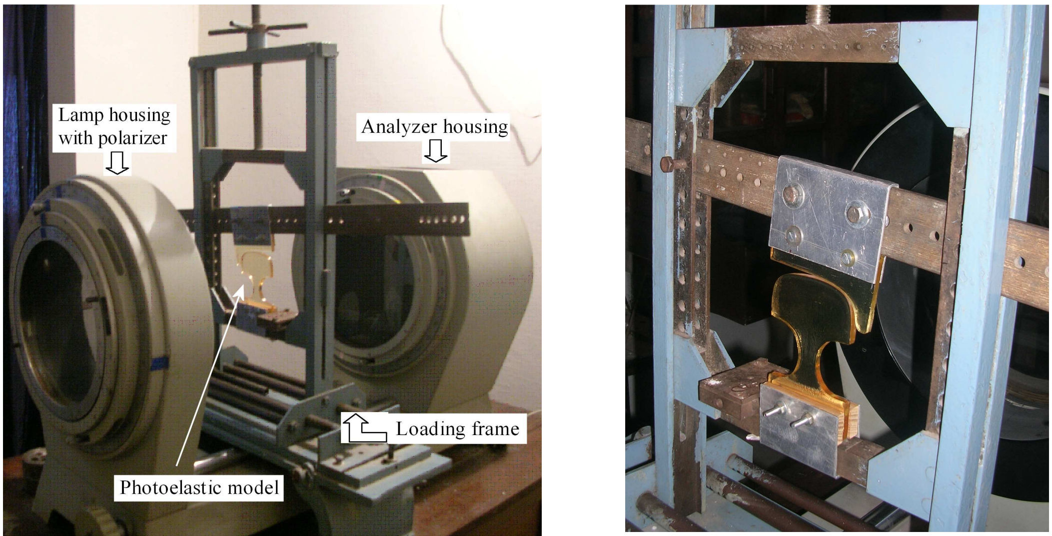
For the appropriate load applications, decided for the photoelastic analysis problem, the compression loading frame has been used as shown in Fig. 1(a). To hold the prepared model properly, clamps and bracket are constructed (Fig. 1(b)). The modified arrangement is used for holding the models. The loads applied are gradually from 0 kg to maximum of 25 kg with increment of 1 kg at the each time. For each load application photoelastic fringe pattern has been observed. The observed photoelastic fringes for different load applications have been photographed.

Meaningful discretization in fe modelling

An important part of FE modeling is the discretization (mesh generation). The discretization of any domain into FEs is important as the value of the variables to be found is very much dependent on it. The manual discretization of any domain is very tedious and time consuming. Hence the development of automatic mesh generation scheme is very important in FE analysis of a problem. Further, a meaningful discretization is very important in evaluating the intended parameters accurately. The concept of meaningful discretization is proposed by Pathak and Ramesh [4]. The basic guideline proposed is that the discretization is meaningful if the fringe pattern observed in a photoelastic experiment is simulated faithfully by FE modeling.

The representation of a given domain by a collection of FE requires engineering judgment on the part of the FE practitioner. The number, type (e.g., linear or quadratic), Shape (e.g., triangular or quadrilateral) and density (i.e., mesh refinement) of elements used in a given problem depends on a number of considerations. The first consideration is to discretize the domain as closely as possible with elements that are admissible. In discretizing a domain, consideration must be given to an accurate representation of the domain, point sources, distributed sources with discontinuities (i.e., sudden change in the intensity of the source), and material and geometric discontinuities, including a reentrant corner. The discretization should include nodes at point sources (so that the point source is accurately lumped at the node), reentrant corners, and element interfaces where abrupt changes in geometry and material properties occur. The second consideration, which requires some engineering judgment, is to discretize the body or portion of the body into sufficiently small elements so that steep gradients of the solution are accurately calculated [1]. Within the above guidelines, the mesh used can be coarse (i.e., have few elements) or refined (i.e., have many elements), and may consist of one or more orders and types of elements. Judicious choice of element order and type could save computational time while giving accurate results.

Generation of meshes of a single element type is easy because elements of same degree are compatible with each other. While refining a mesh, following factors need to be looked into. (i) All previous meshes should be contained in the refined mesh. (ii) Every point in the body can be included with in an arbitrary small element at any stage of the mesh refinement. (iii) The same order of approximation for the solution may be retained. (iv) Large aspect ratio to be avoided. Mesh refinements involve several options. Mesh refinement can be classified as (i) h version: in this the mesh is refined by subdividing existing elements into two or more elements of the same type. (ii) p version: the existing elements can be replaced by elements of higher order. (iii) h, p version: here at some places, h version refinement is done and at other places p version refinement is done [2].



(a) (b)

Fig. 1. (a) Loading of specimen in photoelastic work bench (b) Close up view of specimen loading

Discretization of 'i' section of a rail

In the present work rail section has been discretized using eight noded isoparametric quadrilateral elements and mesh refinement had been done at the most stress concentration area i.e., zones where steep gradient exists (around the point source). 'I' section of rail which has been used for analysis is as shown in Fig. 1(b).

Rail section of width 78.5 mm, central length of 101 mm and thickness of 6 mm is considered. The material fringe value is 11.2 N/mm, E = 3300 MPa, G = 1204 MPa and v = 0.37 under a distributed load of 925.88 N is considered. The problem is analyzed using the software FEM2DM.FOR. In this software the problem is considered as the plane elasticity problem. The input file for this contains the information of mesh along with the specification of essential and natural boundary conditions.

Computer program is flexible as the number of nodes and elements can be changed on the need of accuracy. Computer program generates the mesh using straight line generation logic, requires the user to sketch a desired mesh with certain regularity of node and element numbering. It exploits the regularity to generate the mesh. Node numbering should be regular along the lines and rows being read. Generation of the nodal point coordinates for the specified type meshes is done by using Subroutine msh2dgl. Here whole region has been discretized by same type of element with 'h' version of mesh refinement. Rail section is discretized by drawing straight lines from left to right and then divided into definite ratio of the first element length to last element length. When this type of meshing is used for the case of I section subjected to central loading, in that case one needs to analyse only half of the domain due to symmetry of geometry and loading about vertical axis. However if the model is subjected to inclined loading then the entire domain needs to be discretized.

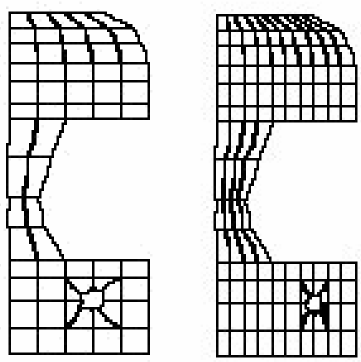
To analyze the central loading case, section is meaningfully discretized using the subroutine msh2dgl. msh2dgl generates the rectangular meshes for rectangular domain. During discretization, section is assumed to be symmetric about Y-axis, so only half of the section is discretized. Section is discretized with coarse and fine mesh. Corresponding meshes are shown in Fig. 2(a) and 2(b). Section has been modeled into 62 elements (coarse mesh) and 120 elements (fine mesh).

Verification of meaningful discretization

For verification of FE discretization contours of isochromatic fringe order are obtained by post processing FEM results. Then FEM results are compared with experimental results obtained for that particular case under the same loading and boundary conditions. This makes comparison between the FEM results and experimental results.

Verification of Discretization for Coarse Mesh Under Central Loading

First of all the coarse mesh discretization is considered by taking 62 elements and 238 nodes as shown in Fig. 2(a).



(a) (b)

Fig. 2. (a) Coarse mesh (No. of elements 62, No. of nodes = 238) (b) Fine mesh (No. of elements 120, No. of nodes = 428)

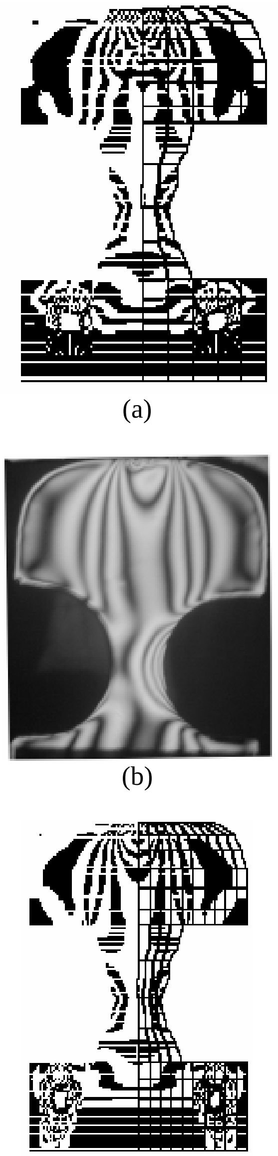
Modeling of the coarse mesh for central loading case has been discussed in previous section. By applying EBC and NBC, analysis for coarse mesh has been done. The EBC v = 0, u =0 is specified at nodes located at the base of the section. Due to vertical axis of symmetry, u = 0 is specified at nodes along vertical axis of symmetry. The NBC is specified at nodes at the point of load application and at the nodes at the reaction point by considering the loading to be uniformly distributed. The reaction is distributed over the upper semicircle of the hole, having four elements. To specify the NBC near the point of load application half of the load i.e. 462.94 N is distributed at the nodes (not shown in Fig. 2(a)) of magnitude -77.15, -308.6 and -77.15 N respectively. At the point of support, a load of 462.94 N appears as the reaction of the applied force. This force is distributed on nodes (not shown in Fig 2(a)) of magnitudes 19.25, 76.98, 38.5, 76.98, 38.5, 76.98, 38.5, 76.98, 19.25 N respectively. By applying this boundary condition FEM analysis has been done and fringe contours are obtained, which are compared with the experimental fringe contours for the same loading. The isochromatics obtained from post processing of FE results for whole section and the experimental results are as shown in Fig. 3 (a) and 3 (b) respectively.

Verification of Discretization for Fine Mesh Under Central Loading

Modeling of the fine mesh for central loading is carried out using 120 elements and 428 nodes as shown in Fig. 2(b). By applying EBC and NBC, analysis for fine mesh has been done. The EBC v = 0 and u = 0 is specified at nodes located at the base of the section. Due to vertical axis of symmetry, $u = 0$ is specified at nodes, along vertical axis of symmetry. To specify the NBC near the point of load application half of the load i.e. 462.94 N is distributed over the span of 5.2 mm of length. The values of the nodal forces at these nodes are -38.57, 154.3, -77.14, -154.3, and -38.57 N respectively. At the point of support, a load of 462.94 N appears as the reaction of the applied force. This force is distributed on nodes along the groove of magnitudes 19.25, 76.98, 38.5, 76.98, 38.5, 76.98, 38.5, 76.98, and 19.25 N respectively.

The isochromatics obtained from post processing of FE results for whole section are as shown in Fig. 3 (c). In the first type of mesh (coarse mesh), fringe pattern is not very well reproduced at the load application, but this is perfectly reproduced with fine mesh. Thus the discretization concept and the loading are properly modeled.

From Fig. 3(a) and 3(c) it is seen that although the discretization of web and top curved portion do not represent the true geometry (due to limitation of discretization by line of curved geometry) the photoelastic countours in experimental results are reproduced well by FE post processed results. Moreover as seen from Fig. 3(b) even a slight variation in the point of application of the applied load in actual case (due to contact between the I section and the object transmitting load) causes the fringes to be unsymmetric as seen from photoelastic fringes in the web. The uncertainty in exact point of application of the load is due to nature of the surfaces in contact with rail section. So while actually loading model in photoelasic work bench every effort must be made to create true experimental conditions (although difficult in this case) and these conditions must be simulated in FE modeling to make the modeling accurate.



(a) (b) (c)

Fig. 3. Over-all fringe contours of $(\sigma_1 - \sigma_2)$ under central loading (a) FEM results for coarse mesh (b) Experimental results (c) FEM results for fine mesh

Conclusions

In this paper, rail section under central loading is analysed. Photoelasticity is used as a tool to discretize the domain and verify the FE modeling. This approach gives the valid and reliable whole field stress analysis. The discretization of domain is guided by some idea of stress distribution inside the domain. Although the FEM is advantageous for detailed parametric study of the problem, adequacy of FE approximation needs to be verified at least for one configuration. The role of experiments to verify the FE modeling is gaining ground. It is to be noted that the FE modeling is validated only in the zones where fringes are seen. It is recommended that every effort must be made to make visible the fringes in all the zones of interest, since comparisons between the experimental results and FE results is good. Thus the discretization concept and the loading for central loading case are properly modeled.

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