# 344. Evaluation of durability of cracked construction elements and estimation of structural strength under vibrations

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**Abstract:** The manuscript deals with the research of relevant problems of cracked elements under cyclic loading. In contrast to the majority of published research work that is dedicated to the case of symmetrical cyclic loading, this paper proposes a methodology for evaluation of influence of asymmetric cyclic loading. First part of the manuscript is concerned with straight crack growth dependence on loading parameters using probabilistic analysis. Speed of crack growth and stress intensity factor values are obtained from experimental studies. Assessment of these parameters can be made using method of Monte Carlo however it is not common and the current research is carried out by applying a more simple averaging method. For elements under asymmetrical loading a general-purpose equation is proposed. Symmetrical cycle and pulsing cycle parameters are used. As a result of these two parameters a general equation is obtained for computational number of loading cycles. This technique enables estimation of durability of elements under various types of loadings with the defined size of the crack.

**Keywords:** strength, durability, vibrations, failure, dynamical loads, stress intensity factor, allowable number of cycles.

## Introduction

Construction elements after failure can be affected by dynamical loads. Dynamical spread of crack challenges solution of these problems:

- To determine stress intensity factor affecting cracks by balanced loads;
- To determine static cracks stress intensity factor, which depends on time and speed of crack extension.

Vibrations of the construction elements, induced by variable load, are inducing high-cycle metal fatigue. Two stages of metal fatigue are distinguished: crack formation and crack extension till critical dimensions. Prediction of construction behavior requires estimation of the loads that induce crack formation as well as speed of crack extension since it can lead to abrupt failures.

Speed of crack expansion was discussed by many authors /1-5/. But the equation of Paris-Erdogan /1/ is widely used.

In this research work we present a probability approach employing equation of Paris-Erdogan for the evaluation of durability. Due to the applicability of Paris-Erdogan equation only for cycles of symmetrical loads, a more general equation was proposed, which encompasses cycles of asymmetrical loads.

## Evaluation of durability subject to conditions of symmetrical loads

Seeking to increase accuracy of crack expansion definition during durability calculations, probabilistic characteristics have to be used.

It is assumed that crack expansion is described by the Paris-Erdogan equation:

$$\frac{dl}{dN} = C(\Delta K)^m, \qquad (1)$$

where constants C and m are random quantities.

Equation (1) is transformed to

$$\frac{dl}{dN} = C_0\left(\frac{\Delta K}{K_0}\right)^m \qquad (2)$$

where $K_0$ is constant magnitude with $\Delta K$ dimension.

Assume that m and $\lg C_0$ generally are independent random quantities described by normal distribution law.

Constant magnitude $K_0$ is a deterministic value. By equations (1) and (2):

$$C = C_0 K_0^m \tag{3}$$

or

$$\lg C = \lg C_0 - m \lg K_0 \quad (\lg C < 0).$$

Here a constant m and lgC are normally distributed and do correlate in interdependent manner.

During experimental tests a plate made of alloy D16T was used (yield strength $\sigma_Y=310$ *MPa*, tensile strength $\sigma_U=450$ *MPa*), with thickness 1 mm, width 70 mm. The plate was subjected to cyclic load $\Delta\sigma= 60$ *MPa*, asymmetry factor $R=0$, *cycle frequency 20 Hz*, temperature of the sample $T=293$ *K*. In total 30 samples were used. Primary central crack with $2l=18{,}0$ *mm*, resulting critical crack with $2l_c=46{,}0$ *mm*. The crack width was measured by applying method of the electric potentials.

Then range of the stress intensity factor is calculated by means of the following equation:

$$\Delta K = \Delta\sigma \sqrt{\pi l \sec\left(\frac{\pi l}{b}\right)}; \tag{4}$$

where b is width of the sample (plate) and $\Delta\sigma$ is the equivalent stress range.

After the equation (4) is inserted into (2), we obtain an equation for the durability calculation, when crack width is from $l = l_1$ to $l = l_c$:

$$N = \frac{1}{C_0}\left(\frac{K}{\Delta\sigma\sqrt{\pi}}\right)^m \int_{l_1}^{l_c} \left(l \sec\frac{\pi l}{b}\right)^{-m/2} dl \tag{5}$$

After η is inserted instead of l/b in equation (5):

$$N = \frac{b}{C_0}\left(\frac{K_0}{\Delta\sigma\sqrt{\pi b}}\right)^m \int_{\eta_1}^{\eta_c} [\eta \sec(\pi\eta)]^{-m/2} d\eta \tag{6}$$

where $\eta_1=l_1/b$
$\eta_C=l_C/b$

If the following integral is expressed by mean magnitude, when $\eta_m(\eta_1< \eta_m < \eta_C)$, then:

$$\int_{\eta_1}^{\eta_c} [\eta \sec(\pi\eta)]^{-m/2} d\eta = (\eta_c - \eta_1)(\eta_m \sec(\pi\eta_m))^{-m/2} \tag{7}$$

Accordingly, equation (5) can be written as follows:

$$N = \frac{l_c - l_1}{C_0}\left[\frac{K_0}{\Delta\sigma\sqrt{\pi b}} \frac{1}{\sqrt{\eta_m \sec(\pi\eta_m)}}\right]^m \tag{8}$$

After the logarithm is taken of the equation (8), then:

$$\lg N = \lg(l_c - l_1) + m \lg\left[\frac{K_0}{\Delta\sigma\sqrt{\pi b}} \frac{1}{\sqrt{\eta_m \sec(\pi\eta_m)}}\right] - \lg C_0 \tag{9}$$

When $l_1=9$ *mm*; $l_c=23$ *mm*, value $\eta_m$ is calculated by using equation (7) and represented in diagram (Fig.1) as a function of $G(m)=\sqrt{\eta_m \sec\pi\eta_m}$ and *m*.

Assume, that parameter $\eta_m$ does represent mean magnitude of *m* (from $l_1=9mm$ to $l_c=23$ *mm*). When $\eta_m=0{,}23$, $G(m)=0{,}515G(m)$ does coincide with $G(\mu_m)$, when $m=2{,}939$.

Quadratic deviation $\sigma^2_m=0{,}247$. Magnitude $lgC_0$ is distributed by normal distribution, too, $\mu_{lgCo}=3{,}49$ and $\sigma^2_{lgCo}=0{,}0424$

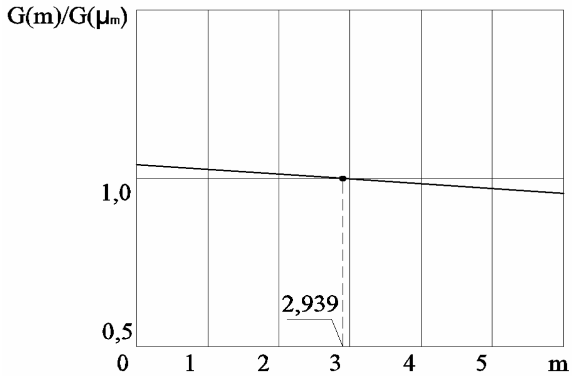


**Fig. 1** $G(m)=\sqrt{\eta_m \sec\pi\eta_m}$ variation with respect to ratio m

When *m* and $lgC_0$ are normally distributed, the first member in the equation (9) will be determinate while second and third members will be distributed by normal law. Then *lgN* will be normally distributed too. Result of the (9) equation is $\mu$ *lg N* - mean magnitude, and variance $\sigma^2_{lgN}$ is calculated by using the following equation:

$$\mu_{\lg N} = \lg(l_c - l_1) + \mu_m \lg\left[\frac{K_0}{\Delta\sigma\sqrt{\pi b}} \frac{1}{\eta_m \sec(\pi\eta_m)}\right] - \mu \lg C_0 \tag{10}$$

$$\sigma^2_{\lg N} = \sigma^2_m \left[\lg\left\{\frac{K_0}{\Delta\sigma\sqrt{\pi b}} \frac{1}{\sqrt{\eta_m \sec(\pi\eta_m)}}\right\}\right]^2 + \sigma^2_{\lg C_0} \tag{11}$$

where ($\mu_m$, $\sigma^2_m$), ($\mu_{lgCo}$, $\sigma^2_{lgN}$) - accordingly mean magnitudes, variances *m* and logarithms taken of constant magnitudes $C_0$.

After these results were inserted into the equations (10) and (11), $\mu_{lgN}$ and $\sigma_{lgN}$ are calculated as a function of *l*.

Obtained results are presented in Figures 2 and 3, which indicate that calculated and experimental results have a good match. The proposed method of approximation is much simpler with respect to the method of Monte Carlo simulations.

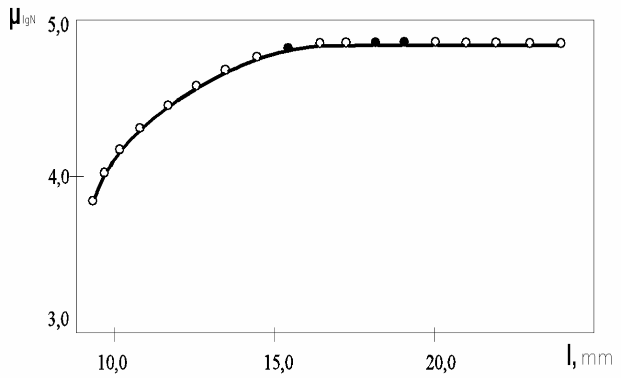


**Fig. 2** Dependence of the mean magnitude *lgN* vs. crack width *l* (alloy D16T)

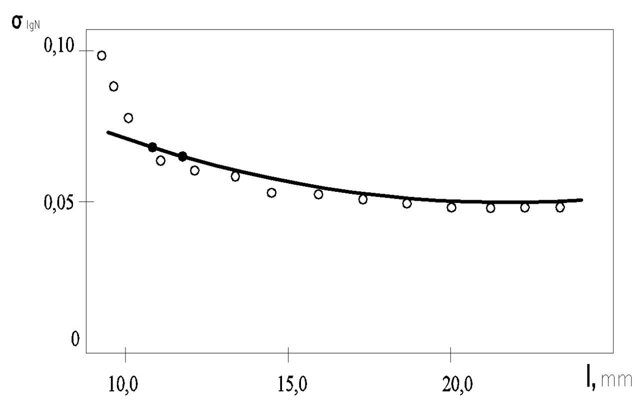


**Fig. 3** Dependence of the variance *lgN* vs. crack width *l* (alloy D16T)

## Evaluation of durability subject to conditions of asymmetrical loads cycle

Previously used method with Paris equation for the evaluation of the speed of crack extension is unsuitable for asymmetrical load cycles. Therefore, in reference [6] an equation with three parameters was proposed:

$$\frac{dl}{dN} = C_{-1} e^{\lambda(R+1)} \Delta K^{m_{-1}} \tag{12}$$

where $C_{-1}$, $m_{-1}$ – constant magnitudes of material subjected to symmetrical cycle (the stress ratio $R=-1$), $\lambda$ - constant magnitude of material. The findings in [6] indicate that $\lambda$ is not dependent on the cycle asymmetry.

If equation (12) is adapted to the pulsing cycle (R = 0), then the equation is transformed as follows:

$$\frac{dl}{dN} = v_0 = C_{-1} e^{\lambda} \Delta K^{m_{-1}} \tag{13}$$

and

$$\lambda = \ln \frac{v_0}{C_{-1} \Delta K^{m_{-1}}} \tag{14}$$

then

$$\frac{dl}{dN} = C_{-1} e^{\ln \frac{v_0}{C_{-1}\Delta K^{m_{-1}}}} (R+1) \Delta K^{m_{-1}} . \tag{15}$$

During tests of samples made of alloy D16T it was determined that $v_0=2{,}34\times10^{-6}$ *m/cycle* constant magnitudes $C_{-1}=4{,}2\times10^{-10}$ *m/cycle*
$m_{-1}=2{,}86$.

When equation (15) is integrated and $N$ is expressed as follows:

$$N_{all} = \int_{l_0}^{l_{all}} \frac{dl}{C_{-1} e^{\ln \frac{v_0}{C_{-1}\left(\Delta\sigma\sqrt{Ml}\right)^{m_{-1}}} (R+1)\left(\Delta\sigma\sqrt{Ml}\right)^{m_{-1}}}}; \tag{16}$$

where
$N_{all}$ – allowable number of cycles
$M$ – constant magnitude of material
$l_{all}$ – allowable crack length

Speed of the crack expansion during pulsing cycle can be expressed as follows:

$$v_0 = C_0 \left(\Delta\sigma\sqrt{Ml}\right)^{m_0} \tag{17}$$

After (17) was inserted into the equation (16):

$$N_{all} = \int_{l_0}^{l_{all}} \frac{dl}{C_{-1} e^{\ln \frac{c_0\left(\Delta\sigma\sqrt{Ml}\right)^{m_0}}{c_{-1}\left(\Delta\sigma\sqrt{Ml}\right)^{m_{-1}}} (R+1)\left(\Delta\sigma\sqrt{Ml}\right)^{m_{-1}}}} . \tag{18}$$

When equation (18) is integrated:

$$N_{all} = \frac{l_{all}^{1-m_0/2} - l_0^{1-m_0/2}}{C_0\left(\Delta\sigma\sqrt{M}\right)^{m_0/2}(R+1)(1-m_0/2)} . \tag{19}$$

Therefore, equation (19) enables the calculation of structural strength when primary and allowable crack lengths are known values.

As it is known, primary crack length can be calculated by using equation of graduated stress intensity factor:

$$K_{th} = \sigma\sqrt{\pi l M} \tag{20}$$

Samples for the fatigue limit tests were made of alloy EI617. The following results were obtained: $m_0=2{,}71$; $c_0=6{,}7\times10^{-12}$ *m/cycle*
Results of the structural strength calculations are presented in Table 1.

**Table 1.** Results of the structural strength calculations

| Equivalent stress ranges $\Delta\sigma$, *MPa* | Primary crack length $l_0$, *mm* | Allowable crack length $l_{all}$, *mm* | Stress intensity range $\Delta K$, $MPa \times m^{1/2}$ | Crack extension speed $v_0$, *m/cycle* | Stress ratio *R* | Allowable number of cycles $N_{all}$ |
|---|---|---|---|---|---|---|
| 0 | 0,8 | 10 | 31,75 | $7{,}8\times10^{-8}$ | 0,1 | $1{,}07\times10^{5}$ |
| 140 | 0,9 | 12 | 30,4 | $6{,}99\times10^{-8}$ | 0,2 | $1{,}3\times10^{5}$ |
| 120 | 1,0 | 14 | 28,2 | $5{,}7\times10^{-8}$ | 0,3 | $1{,}75\times10^{5}$ |
| 100 | 1,1 | 16 | 2,51 | $4{,}16\times10^{-8}$ | 0,4 | $2{,}6\times10^{5}$ |

Tests with samples made of alloy VT8 were also performed. The following results were obtained:

$m_0=3{,}54$ ; $C_0=2{,}1\times10^{-12}$ *m/cycle*

Parameters, calculated when $l_0=0{,}8mm$, are shown in Fig. 4

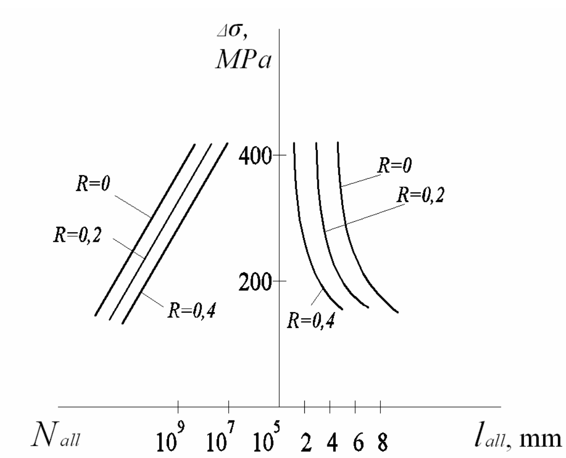


**Fig. 4** Dependencies in between of durability, crack length and loads

## Conclusion

In the case of symmetrical load cycles, when the equation of Paris-Erdogan is used for the determination of constant magnitudes of crack extension speed, normal distribution is preferred.

When integral durability of cracked samples is substituted by mean durability, the obtained dependence of lgN (N – durability) as a function of crack length l has a good match with the testing results. This conclusion is confirmed by lgN deviation variance.

An equation with three parameters was proposed that enables calculation of fatigue limit in the case of various cyclic loads.

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