289. Vibrations of a double wall plane adaptive to electrorheological materials

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Abstract. An analytical study of vibrations response and control capabilities of an Electrorheological (ER) material-filled double wall plate to random excitation pressure is presented. The governing equations of motion are developed by incorporating the ER material constitutive relations to thin face plate equations. A three-parameter viscoelastic model is used to characterize ER material behaviour in pre-yield regime. The effect of electric field strength on the response of the double wall sandwich plate with ER material is investigated. Numerical results include response spectral densities and modal frequencies for several ER material core thicknesses and different electric field levels.

Keywords: electrorheological materials, sandwich plate, random response.

1. Introduction

Electrorheological effect is considered to be a part of electroviscous effect and is observed in certain colloidal suspensions now generally referred to as ER fluids. Upon the application of an electric field, ER fluids experience reversible changes in rheological properties such as viscosity, elasticity and plasticity. The rheological properties of ER fluids can be increased by several orders of magnitude with electric field strength increments of the order of 1 kV/mm. The ER fluids have capacity to change from a fluid type to that of solid-like gel in milliseconds, which makes them very attractive for precise control of stress transfer in a sandwich configuration [1, 2].

The geometry of the prototype of the adaptive structure considered in this study is shown in Fig. 1. The ER material is sandwiched between two parallel aluminium plates. The face plates are rectangular, simply supported on all four edges, sealed along boundaries to contain the ER material. The top plate is exposed to uniformly distributed random pressure that could arise from various sources such as jet noise, machinery noise or convective flow of a turbulent boundary layer. The electric field is assumed to be uniform and applied across the two face plates. The main objective of this study is to examine how the vibration response and natural frequencies of the sandwich structure change with an increasing electric field. Response of ER material is assumed to be in pre-yield region for all levels of random pressure input considered in this study [3]. Numerical results include displacement response spectral densities and modal frequencies for several ER material core thicknesses and different electric field levels.

2. Equations of Motion and Solution Procedure

The behavior of ER material in pre-yield regime is characterized by linear viscoelastic theory where the shear stress τ is related to shear strain γ by complex shear modulus G^* [4]:

$$\tau = G^* \gamma . \tag{1}$$

The complex shear modulus G^* is written in the form:

$$G^* = G^{'} + iG^{''},\ i = \sqrt{-1}, \tag{2}$$

where $G^{'}$ is storage modulus, representing the stiffness of ER material, and $G^{''}$ is loss modulus, a measure of dissipative properties of the material. In the present study, we model the ER material behaviour in pre-yield regime by a three-parameter solid model, referred to as the Zener model which is made up of a spring in series with a Kelvin element as shown in Fig. 2 [5]. The stress-strain relation for a three parameter solid in time domain is given by a differential equation [5]:

$$\dot{\tau} + p_1 \tau = q_1 \gamma + q_2 \dot{\gamma} , \tag{3}$$

where:

$$p_1 = \frac{K_1 + K_2}{C_1}, \quad (4)$$

$$q_1 = \frac{K_1 K_2}{C_1}, \quad (5)$$

$$q_2 = K_2, \quad (6)$$

in which K_1, K_2, and C_1 are functions of electric field strength V in kV/mm.

Using the differential equation for the three-parameter solid given by equation (3) and the complex representation of stress-strain relationship in equation (1), the components of the complex shear modulus can be obtain in frequency domain as:

$$G' = \frac{K_1 K_2 (K_1 + K_2) + K_2 C_1^2 \Omega^2}{(K_1 + K_2)^2 + C_1^2 \Omega^2}, \quad (7)$$

$$G'' = \frac{K_2^2 C_1 \Omega}{(K_1 + K_2)^2 + C_1^2 \Omega^2}, \quad (8)$$

in which Ω is the excitation frequency. Currently experimental data for the values of parameters K_1, K_2 and C_1 as functions of electric field strength V in the three-parameter solid model for ER fluid in pre-yield regime are not available. In the present study, these parameters are calculated using Eqs. 7, 8 and approximate experimental-empirical relations of storage and loss modulus of ER material [3, 5, 6, 7]:

$$G' \approx c_1 V^2, \quad (9)$$

$$G'' \approx c_2 V + c_3, \quad (10)$$

in which c_1, c_2, c_3 are material constants.

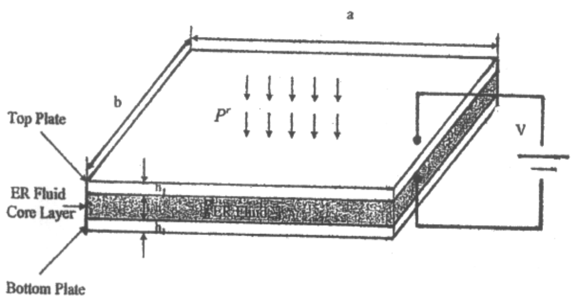

Fig. 1. Configuration of ER material sandwich plate

The equations of motion of sandwich plate vibrations shown in Fig. 1 are developed using a thin plate theory [8]. The two plates are made from isotropic elastic material with thickness of h_1 each. The core layer is the ER material of thickness h_2. An electric field used to tune the rheological properties of ER material is applied across the two face plates. It is assumed that the elastic and shear moduli of the face plates are much larger than the storage modulus of the ER core material. In addition, it is assumed that there is no slipping between face plates and the core, and that continuity of transverse displacement across the thickness of the sandwich plate is guaranteed. The effects of ER material action are incorporated into equations of motion through equilibrium equations and stress-displacement relations [8]. The vibrations of two face plates are coupled through the interaction with the constitutive relations of the ER core material. The four coupled partial differential equations are developed in terms of four geometric variables of the sandwich plate:

$$w = \frac{w^t + w^b}{2}, \quad (11)$$

as effective transverse displacement of the middle plane,

$$e = \frac{w^t - w^b}{2}, \quad (12)$$

as effective transverse deformation of the ER material core layer,

$$\alpha = \frac{u^t - u^b}{h}, \ \beta = \frac{v^t - v^b}{h}, \quad (13)$$

as effective rotations of the normal to the undeformed middle plane. In these equations, the superscripts t and b indicate top and bottom plates. The variables w, u, and v denote transverse, longitudinal and lateral deformations, respectively. The final form of the equations of motion in terms of the four described variables are [8]:

$$-2D_f \Delta^4 w - 2C_f \dot{w} + \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + P^r = = (2\rho_f h_1 + \rho_c h_2) \frac{\partial^2 w}{\partial t^2}, \quad (14)$$

$$-2D_f \Delta^4 e - 2C_f \dot{e} - \frac{4}{h_2}(E_c e + C_c \dot{e}) + P^r = 2\rho_f h_1 \frac{\partial^2 e}{\partial t^2}, \quad (15)$$

$$\dot{V}_x + p_1 V_x = h\left(q_1 + q_2 \frac{\partial}{\partial t}\right) \times \times \left[\alpha + \frac{\partial w}{\partial x} + \frac{h_2^3}{12 h^2 E_c} \frac{\partial}{\partial x}\left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y}\right)\right], \quad (16)$$

$$\dot{V}_y + p_1 V_y = h\left(q_1 + q_2 \frac{\partial}{\partial t}\right) \times \times \left[\beta + \frac{\partial w}{\partial y} + \frac{h_2^3}{12 h^2 E_c} \frac{\partial}{\partial y}\left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y}\right)\right], \quad (17)$$

where:

$$V_x = D\left[\left(\frac{\partial^2}{\partial x^2} + \frac{1-\nu_f}{2_c}\frac{\partial^2}{\partial y^2}\right)\alpha + \frac{1+\nu_f}{2}\frac{\partial^2 \beta}{\partial x \partial y}\right], \quad (18)$$

$$V_y = D\left[\left(\frac{\partial^2}{\partial y^2} + \frac{1-\nu_f}{2_c}\frac{\partial^2}{\partial x^2}\right)\beta + \frac{1+\nu_f}{2}\frac{\partial^2 \alpha}{\partial x \partial y}\right], \quad (19)$$

the biharmonic operator Δ^4:

$$\Delta^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}. \quad (20)$$

The other parameters appearing in these equations are defined as follows:

$$D = \frac{\frac{1}{2}h_1 h^2 E_f}{\left(1-\nu_f^2\right)}, \quad (21)$$

$$D_f = \frac{E_f h_1^3}{12\left(1-\nu_f^2\right)}, \quad (22)$$

where D is effective stiffness of sandwich plate, D_f is stiffness of face plates. C_f, C_c, E_f, E_c, ν_f, ρ_f, ρ_c are damping coefficient of the face plate, damping coefficient of the core layer, modulus of elasticity of face plates, modulus of elasticity of core layer, Poisson's ratio of face plate material, density of the face plates, density of the core material, respectively, and $h=h_1+h_2$.

The details of the derivation of the equations of motion for linear and nonlinear cases can be found in [8].

When subjected to an electric field, the rheological properties of the ER material in the core layer of the sandwich structure will change so that stiffness, damping and natural frequencies will also change accordingly. To examine these changes, the linear response of the sandwich plate can be analyzed in frequency domain. First, the system of governing partial differential equations (14-17) is solved by Galerkin method, i. e. for the simply supported sandwich plate, solutions of effective transverse displacement and rotations are expressed as:

$$w(x,y,t) = \sum_m \sum_n A_{mn}(t) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}, \quad (23)$$

$$e(x,y,t) = \sum_m \sum_n Z_{mn}(t) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}, \quad (24)$$

$$\alpha(x,y,t) = \sum_m \sum_n B_{mn}(t) \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b}, \quad (25)$$

$$\beta(x,y,t) = \sum_m \sum_n C_{mn}(t) \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b}, \quad (26)$$

where A_{mn}, Z_{mn}, B_{mn}, C_{mn} are modal amplitudes.

Substituting the assumed modal solutions into equations (14-17) and making use of orthogonality of modal functions, a system of coupled differential equations for A_{mn}, Z_{mn}, B_{mn}, C_{mn} is obtained. Taking a Fourier transformation of these equations results in a system of algebraic equations that can be solved for the transformed modal amplitude. Then, using these results and Fourier transforms of equations (23-26), solutions for vibration response in frequency domain are determined. Since the input pressure acting on the top plate is random, the response spectral density of the effective transverse displacement w of the middle plate is determined [8, 9]:

$$S_\omega(x,y,\omega) = \sum_m \sum_n \sum_r \sum_s H_{mn} H_{rs}^* X_{mn} X_{rs} S_{mnrs}(\omega), \quad (27)$$

where H_{mn} are the frequency response functions, S_{mnrs} are the cross-spectral densities of the generalized random pressure and X_{mn} are the vibration modes of simply supported plate as specified in equation (23). Similar solutions can be developed for effective displacement e and rotations α, β. The expression for the frequency response function H_{mn} is rather lengthy and it is not included in this paper. However, the details of frequency response function and of natural frequencies of the sandwich construction can be found in [8].

For random pressure uniformly distributed over the top plate surface, the cross-spectral densities of the generalized random forces are:

$$S_{mnrs}(\omega) = \begin{cases} \dfrac{256 S_p(\omega)}{\pi^4 mnrs} & m,n,r,s \quad all\ odd, \\ 0 & otherwise, \end{cases} \quad (28)$$

where $S_p(\omega)$ is the power spectral density of random sound pressure acting on the top plate. In the present study, the random pressure is assumed to be band limited Gaussian white noise with spectral amplitude computed from:

$$S_p(\omega) = S_0 = \frac{p_0^2}{\Delta\omega} 10^{SPL/10}, \quad (29)$$

where p_0=2x10^{-5} N/m^2 is the reference pressure, SPL is the sound pressure level expressed in decibels and $\Delta\omega$ is the selected frequency bandwidth. The frequency range of the selected band limited Gaussian white noise pressure is 0-1000 Hz.

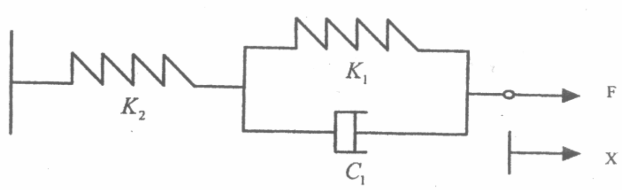

Fig. 2. Three parameter Solid Model

3. Numerical Results

Numerical results are presented for a sandwich plate construction shown in Fig. 1 with the following dimensions: a=500 mm, b=250mm, h_1=0,4 mm, h_2=2,0 mm. The density of ER material is 1060 kg/m^3.

The face plates are made of aluminium with the following material properties: material density=2768 kg/m^3, elastic modulus=6,898x10^{10} N/m^2, Poisson's ratio=0,3. The values of the ER core material parameters K_1, K_2, C_1, E_c, C_c are given in Table 1 for different electric field strengths V.

Table 1. Values of K_1, K_2, C_1, E_c, C_c

Parameter	Electric Field Strength (kV/mm)			
	V=0	V=1	V=2	V=3
K_1 (N/m^2)	282,0	68814,0	486090,0	1445240,0
K_2 (N/m^2)	587,0	181001,0	338828,0	652529,0
C_1 (Ns/m^2)	10,0	50,0	250,0	600,0
E_c (N/m^2)	500,0	44265,9	106965,0	169788,0
C_c (Ns/m^2)	8,0	6,0	4,0	2,0

Modal damping coefficients of the face plates are determined from:

$$\xi_{kl} = \xi_{11}\left(\frac{\omega_{11}}{\omega_{kl}}\right), \qquad (30)$$

where ω_{kl} are the modal frequencies and ξ_{11}=0,01. Displacement response spectral densities corresponding to SPL=110 dB are presented in Fig. 3 for different electric field strengths. These results indicate that the dominant resonant frequencies increase with increasing electric field and consequently the displacement spectral density levels decrease. The sandwich plate becomes more stiff and the dissipation energy of the ER core increases with increasing electric strength. Fig. 4 shows more clearly the trend of change of natural modal frequencies with variation of an applied electric field for the three selected dominant modes of the sandwich structure. The results given in Fig. 5 illustrate the change of first modal frequency at electric field levels of 0.0, 1.0, 2.0, 3.0 kV/mm as a function of ratio of ER material core thickness to the thickness of face plates. As can be observed from these results, the first modal frequency decreased with increasing ratio of h_2/h_1 for cases of no voltage and for low voltage levels. This decrease is mostly due to the added mass effect of the core material. For voltage levels of larger than 1,35 kV/mm, the first modal frequency shows tendency to increase with increasing ratio h_2/h_1. This phenomenon can be explained as follows: the ER material core contributes both mass inertia and stiffness to the sandwich structure upon application of the electric field. The contribution of mass inertia is fixed while the contribution of stiffness increases with increasing electric field. At a certain electric field level, the stiffening effects of the core become more pronounced than the added mass and natural frequency increases with increasing thickness ratio. The results presented in Fig. 5 could be useful in choosing the optimal core thickness for active vibration control with ER materials. The results presented in Fig. 6 illustrate the change in the first modal frequency with increasing electric field strength for several values of core to face plate thickness ratio h_2/h_1. These results also illustrate that for the chosen geometry and material properties of the sandwich construction, the first modal frequency increases more rapidly with increasing ratio h_2/h_1 after electric field strength applied to ER material exceeds about 1,35 kV/mm.

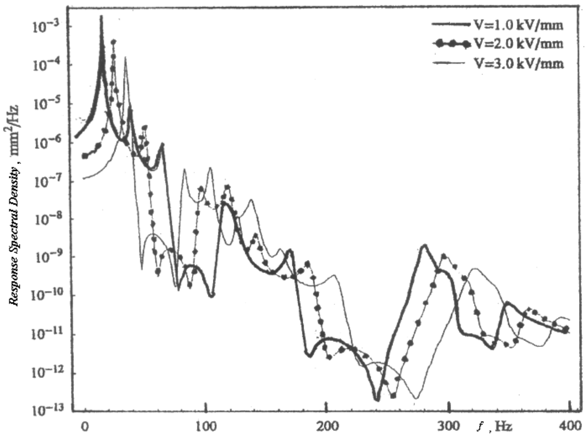

Fig. 3. Displacement spectral density for SPL=110 dB

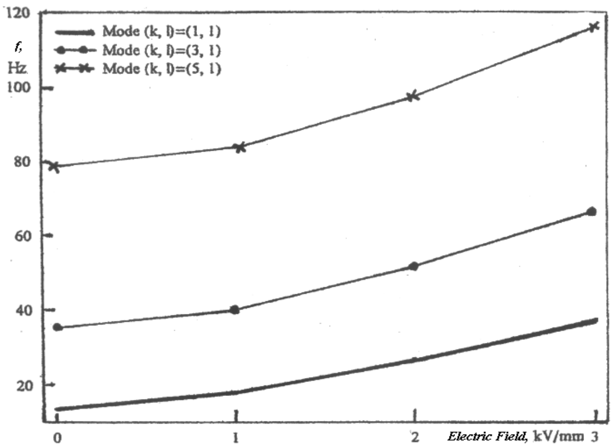

Fig. 4. Modal natural frequencies vs. electric field levels

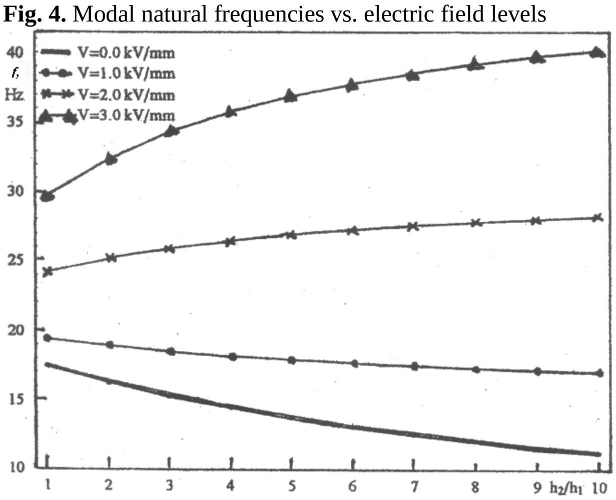

Fig. 5. First modal frequency vs. ER core thickness ratio for different electric field levels

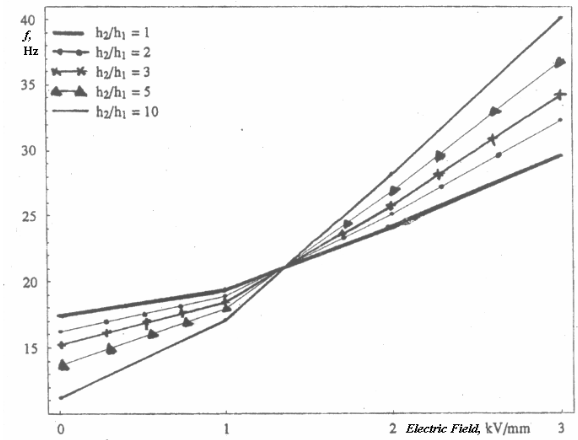

Fig. 6. First modal frequency vs. electric field for different core thickness ratios

4. Conclusions

Displacement vibration amplitudes were significantly reduced as applied electric field to the ER core increased. Modal frequencies shifted to higher values with an increased electric field, which indicated that the sandwich structure becomes more stiff upon application of the electric field. The effect of different ER core layer thickness on structural response could be useful in choosing optimal core thickness for vibration control. Depending on geometric and material properties of the sandwich plate construction, a thicker ER core might not be beneficial for vibration response control at low levels of electric field strength.

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