271. CFAR Vibration Signal Change Test and its Applications to Real-Time Recognition of Developing Cracks in Jet Engine Rotors

N. A. Nechval[1,a], K. N. Nechval[2,b], G. Berzins[1,a] and M. Purgailis[1,a]
[1]University of Latvia, Raina Blvd 19, LV-1050 Riga, Latvia
[2]Transport and Telecommunication Institute, Lomonosov Street 1, LV-1019 Riga, Latvia
E-mail: [a]*nechval@junik.lv*, [b]*konstan@tsi.lv*

(Received 20 April 2007; accepted 15 June 2007)

Abstract. This paper introduces a new technique for early identification of fatigue cracks, namely the constant false alarm rate (CFAR) test. This test works on the null hypotheses that a target vibration signal is statistically similar to a reference vibration signal. In effect, this is a time-domain signal processing technique that compares two signals, and returns the likelihood whether the two signals are similar or not. The system monitors the vibration signal of the rotor as it cycles, and compares that vibration signal with, say, the original vibration signal. The difference vector reflects the change in vibration over time. As a crack develops, the vector changes in a characteristic way. Thus, it is possible, during CFAR test, to determine whether the two signals are similar or not. Therefore, by comparing a given vibration signal to a number of reference vibration signals (for several crack scenarios) it is possible to state which is the most likely condition of the rotor under analysis. The CFAR test not only successfully identifies the presence of the fatigue cracks but also gives an indication related to the advancement of the crack. This test, despite its simplicity, is an extremely powerful method that effectively classifies different vibration signals, allowing for its safe use as another condition monitoring technique.

Keywords: Vibration signal, change, CFAR test, rotor, crack, recognition.

Introduction

The machines and structural components require continuous monitoring for the detection of cracks and crack growth for ensuring an uninterrupted service. Non-destructive testing methods like ultrasonic testing, X-ray, etc., are generally useful for the purpose. These methods are costly and time consuming for long components, e.g., railway tracks, long pipelines, etc. Vibration-based methods can offer advantages in such cases [1]. This is because measurement of vibration parameters like natural frequencies is easy. Further, this type of data can be easily collected from a single point of the component. This factor lends some advantages for components, which are not fully accessible. This also helps to do away with the collection of experimental data from a number of data points on a component, which is involved in a prediction based on, for example, mode shapes.

Nondestructive evaluation (NDE) of structures using vibration for early detection of cracks has gained popularity over the years and, in the last decade in particular, substantial progress has been made in that direction. Almost all crack diagnosis algorithms based on dynamic behaviour call for a reference signature. The latter is measured on an identical uncracked structure or on the same structure at an earlier stage.

Dynamics of cracked rotors has been a subject of great interest for the last three decades and detection and monitoring have gained increasing importance, recently. Failures of any high speed rotating components (jet engine rotors, centrifuges, high speed fans, etc.) can be very dangerous to surrounding equipment and personnel (see Fig. 1), and must always be avoided. Jet engine disks operate under high centrifugal and thermal stresses. These stresses cause microscopic damage as a result of each flight cycle as the engine starts from the cold state, accelerates to maximum speed for take-off, remains at speed for cruise, then spools down after landing and taxi. The cumulative effect of this damage over time creates a crack at a location where high stress and a minor defect combine to create a failure initiation point. As each flight operation occurs, the crack is enlarged by an incremental distance. If allowed to continue to a critical dimension, the crack would eventually cause the burst of the disk and lead to catastrophic failure (burst) of the engine. Engine burst in flight is rarely survivable.

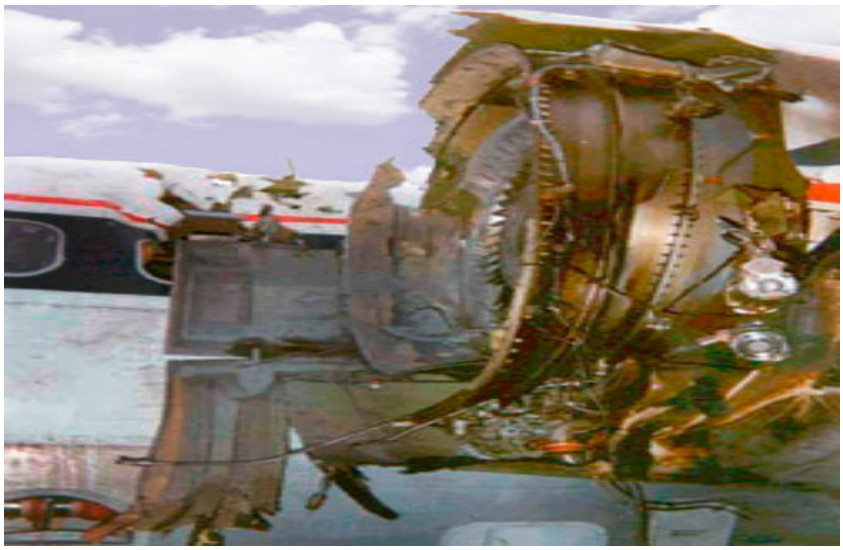

Fig. 1. Jet engine fan section failure

Problem Statement

Suppose that we desire to compare a target vibration signal and a kth reference vibration signal, which have p response variables. Let $x_{ij}(k)$ and y_{ij} be the ith observation of the jth response variable of the kth reference signal and the target signal, respectively. It is assumed that all observation vectors, $\mathbf{x}_i(k)=(x_{i1}(k), ..., x_{ip}(k))'$, $\mathbf{y}_i=(y_{i1}, ..., y_{ip})'$, $i=1(1)n$, are independent of each other, where n is a number of paired observations. Let $\mathbf{z}_i(k) = \mathbf{x}_i(k)-\mathbf{y}_i$, $i=1(1)n$, be paired comparisons leading to a series of vector differences. Thus, in order to compare the above signals, and return the likelihood whether the two signals are similar or not, it can be obtained and used a sample of n independent observation vectors $\mathbf{Z}(k)=(\mathbf{z}_1(k), ..., \mathbf{z}_n(k))$. Each sample $\mathbf{Z}(k)$, $k\in\{1, \ldots, m\}$, is declared to be realization of a specific stochastic process with unknown parameters. It is assumed here that $\mathbf{z}_i(k)$, $i=1(1)n$, are independent p-multivariate normal random variables ($n\geq p+2$) with common mean $\mathbf{a}(k)$ and covariance matrix (positive definite) $\mathbf{Q}(k)$. A goodness-of-fit testing for the multivariate normality is based on the following theorem.

Theorem 1 (*Characterization of the multivariate normality*)**.** Let $\mathbf{z}_i(k)$, $i=1(1)n$, be n independent p-multivariate random variables ($n\geq p+2$) with common mean $\mathbf{a}(k)$ and covariance matrix (positive definite) $\mathbf{Q}(k)$. Let $w_r(k)$, $r = p+2, \ldots, n$, be defined by

$$w_r(k)=\frac{r-(p+1)}{p}\frac{r-1}{r}\left(\mathbf{z}_r(k)-\bar{\mathbf{z}}_{r-1}(k)\right)'\mathbf{S}_{r-1}^{-1}(k)\times$$

$$\times\left(\mathbf{z}_r(k)-\bar{\mathbf{z}}_{r-1}(k)\right)=\frac{r-(p+1)}{p}\left(\frac{|\mathbf{S}_r(k)|}{|\mathbf{S}_{r-1}(k)|}-1\right),$$

$$r=p+2, \ldots, n, \qquad (1)$$

where

$$\bar{\mathbf{z}}_{r-1}(k)=\sum_{i=1}^{r-1}\mathbf{z}_i(k)/(r-1), \qquad (2)$$

$$\mathbf{S}_{r-1}(k)=\sum_{i=1}^{r-1}(\mathbf{z}_i(k)-\bar{\mathbf{z}}_{r-1}(k))(\mathbf{z}_i(k)-\bar{\mathbf{z}}_{r-1}(k))', \qquad (3)$$

then the $\mathbf{z}_i(k)$ ($i=1, \ldots, n$) are $N_p(\mathbf{a}(k),\mathbf{Q}(k))$ if and only if $w_{p+2}(k), \ldots, w_n(k)$ are independently distributed according to the central F-distribution with p and $1, 2, \ldots, n-(p+1)$ degrees of freedom, respectively.

Proof. The proof is similar to that of [2] and so it is omitted here.

Goodness-of-fit Testing for the Multivariate Normality. The results of Theorem 1 can be used to obtain test for the hypothesis of the form H_0: $\mathbf{z}_i(k)$ follows $N_p(\mathbf{a}(k),\mathbf{Q}(k))$ versus H_a: $\mathbf{z}_i(k)$ does not follow $N_p(\mathbf{a}(k),\mathbf{Q}(k))$, $\forall i = 1(1)n$. The general strategy is to apply the probability integral transforms of w_k, $\forall k = p+2(1)n$, to obtain a set of i.i.d. U(0,1) random variables under H_0 [2]. Under H_a this set of random variables will, in general, not be i.i.d. U(0,1). Any statistic, which measures a distance from uniformity in the transformed sample (say, a Kolmogorov-Smirnov statistic), can be used as a test statistic.

Testing for Similarity of the Two Signals. In this paper, for testing that the two signals (target signal and reference signal) are similar, we propose a statistical approach that is based on the generalized maximum likelihood ratio. We have the following hypotheses:

$H_0(k)$: *Similarity is valid for the acceptable range of accuracy under a given experimental frame*;

$H_1(k)$: *Similarity is invalid for the acceptable range of accuracy under a given experimental frame.*

Thus, for fixed n, the problem is to construct a test, which consists of testing the null hypothesis

$$H_0(k)\text{: } \mathbf{z}_i(k) \sim \mathrm{N}_p(\mathbf{0},\mathbf{Q}(k)), \quad \forall i = 1(1)n, \tag{4}$$

where $\mathbf{Q}(k)$ is a positive definite covariance matrix, versus the alternative

$$H_1(k)\text{: } \mathbf{z}_i(k) \sim \mathrm{N}_p(\mathbf{a}(k),\mathbf{Q}(k)), \quad \forall i = 1(1)n, \tag{5}$$

where $\mathbf{a}(k)=(a_1(k), ..., a_p(k))' \neq (0, ..., 0)'$ is a mean vector. The parameters $\mathbf{Q}(k)$ and $\mathbf{a}(k)$ are unknown.

GMLR Statistic

In order to distinguish the two hypotheses ($H_0(k)$ and $H_1(k)$), a generalized maximum likelihood ratio (GMLR) statistic is used. The GMLR principle is best described by a likelihood ratio defined on a sample space Z with a parameter set Θ, where the probability density function of the sample data is maximized over all unknown parameters, separately for each of the two hypotheses. The maximizing parameter values are, by definition, the maximum likelihood estimators of these parameters; hence the maximized probability functions are obtained by replacing the unknown parameters by their maximum likelihood estimators. Under $H_0(k)$, the ratio of these maxima is a $\mathbf{Q}(k)$-free statistic. This is shown in the following. Let the complete parameter space for $\boldsymbol{\theta}(k)=(\mathbf{a}(k),\mathbf{Q}(k))$ be $\boldsymbol{\Theta}=\{(\mathbf{a}(k),\mathbf{Q}(k))\text{: } \mathbf{a}(k)\in\mathbf{R}^p,\ \mathbf{Q}(k)\in \mathrm{Q}_p\ \}$, where Q_p is a set of positive definite covariance matrices, and let the restricted parameter space for $\boldsymbol{\theta}$, specified by the $H_0(k)$ hypothesis, be $\boldsymbol{\Theta}_0=\{(\mathbf{a}(k),\mathbf{Q}(k))\text{: } \mathbf{a}(k)=\mathbf{0},\ \mathbf{Q}(k)\in \mathrm{Q}_p\}$. Then one possible statistic for testing $H_0(k)$: $\boldsymbol{\theta}(k)\in\boldsymbol{\Theta}_0$ versus $H_1(k)$: $\boldsymbol{\theta}(k)\in\boldsymbol{\Theta}_1$, where $\boldsymbol{\Theta}_1=\boldsymbol{\Theta}-\boldsymbol{\Theta}_0$, is given by the generalized maximum likelihood ratio

$$\mathrm{LR} = \max_{\boldsymbol{\theta}(k)\in\boldsymbol{\Theta}_1} L_{H_1(k)}(\mathbf{Z}(k);\ \boldsymbol{\theta}(k)) \Big/ \max_{\boldsymbol{\theta}(k)\in\boldsymbol{\Theta}_0} L_{H_0(k)}(\mathbf{Z}(k);\ \boldsymbol{\theta}(k)). \tag{6}$$

Under $H_0(k)$, the joint likelihood for $\mathbf{Z}(k)$ is given by

$$L_{H_0(k)}(\mathbf{Z}(k);\boldsymbol{\theta}(k)) = (2\pi)^{-np/2}\left|\mathbf{Q}(k)\right|^{-n/2}\cdot \cdot\exp\left(-\sum_{i=1}^{n}\mathbf{z}_i'(k)[\mathbf{Q}(k)]^{-1}\mathbf{z}_i(k)/2\right). \tag{7}$$

Under $H_1(k)$, the joint likelihood for $\mathbf{Z}(k)$ is given by

$$L_{H_1(k)}(\mathbf{Z}(k);\boldsymbol{\theta}(k)) = (2\pi)^{-np/2}\left|\mathbf{Q}(k)\right|^{-n/2}\cdot \cdot\exp\left(-\sum_{i=1}^{n}(\mathbf{z}_i(k)-\mathbf{a}(k))'[\mathbf{Q}(k)]^{-1}(\mathbf{z}_i(k)-\mathbf{a}(k))/2\right). \tag{8}$$

It can be shown that

$$\max_{\boldsymbol{\theta}(k)\in\boldsymbol{\Theta}_j} L_{H_j(k)}(\mathbf{Z}(k);\boldsymbol{\theta}(k)) = = (2\pi)^{-np/2}\left|\hat{\mathbf{Q}}_j(k)\right|^{-n/2}\exp(-np/2), \quad j=0,\ 1, \tag{9}$$

where $\hat{\mathbf{Q}}_0(k)=\mathbf{Z}(k)\mathbf{Z}'(k)/n$,

$$\hat{\mathbf{Q}}_1(k)=(\mathbf{Z}(k)-\hat{\mathbf{a}}(k)\mathbf{u}')(\mathbf{Z}(k)-\hat{\mathbf{a}}(k)\mathbf{u}')'/n,$$

and $\hat{\mathbf{a}}(k)=\mathbf{Z}(k)\mathbf{u}/\mathbf{u}'\mathbf{u}$

are the well-known maximum likelihood estimators of the unknown parameters $\mathbf{Q}(k)$ and $\mathbf{a}(k)$ under the hypotheses $H_0(k)$ and $H_1(k)$, respectively, $\mathbf{u}=(1, ..., 1)'$ is the n-dimensional column vector of units. A substitution of (9) into (6) yields

$$\mathrm{LR} = \left|\hat{\mathbf{Q}}_0(k)\right|^{n/2}\left|\hat{\mathbf{Q}}_1(k)\right|^{-n/2}. \tag{10}$$

Taking the (n/2)th root, this likelihood ratio is evidently equivalent to

$$\mathrm{LR}_\bullet = \left|\hat{\mathbf{Q}}_0(k)\right|\left|\hat{\mathbf{Q}}_1(k)\right|^{-1} = = \left|\mathbf{Z}(k)\mathbf{Z}'(k)\right| / \left|\mathbf{Z}(k)\mathbf{Z}'(k)-(\mathbf{Z}(k)\mathbf{u})(\mathbf{Z}(k)\mathbf{u})'/\mathbf{u}'\mathbf{u}\right|. \tag{11}$$

Now the likelihood ratio in (11) can be considerably simplified by factoring out the determinant of the $p \times p$ matrix $\mathbf{Z}(k)\mathbf{Z}'(k)$ in the denominator to obtain this ratio in the form

$$\mathrm{LR}_\bullet = 1\Big/\left(1-(\mathbf{Z}(k)\mathbf{u})'[\mathbf{Z}(k)\mathbf{Z}'(k)]^{-1}(\mathbf{Z}(k)\mathbf{u})/n\right). \tag{12}$$

This equation follows from a well-known determinant identity. Clearly (12) is equivalent finally to the statistic

$$V_n(k) = \left(\frac{n-p}{p}\right)(\mathrm{LR}_\bullet - 1) = = \left(\frac{n-p}{p}\right)n\hat{\mathbf{a}}'(k)[\mathbf{T}(k)]^{-1}\hat{\mathbf{a}}(k), \tag{13}$$

where $\mathbf{T}(k)=n\hat{\mathbf{Q}}_1(k)$. It is known that $(\hat{\mathbf{a}}(k),\mathbf{T}(k))$ is a complete sufficient statistic for the parameter $\boldsymbol{\theta}(k)=(\mathbf{a}(k),\mathbf{Q}(k))$. Thus, the problem has been reduced to consideration of the sufficient statistic $(\hat{\mathbf{a}}(k),\mathbf{T}(k))$. It can be shown that under H_0, V_n is a $\mathbf{Q}(k)$-free statistic which has the property that its distribution does not depend

on the actual covariance matrix $\mathbf{Q}(k)$. This is given by the following theorem.

Theorem 2 (*PDF of the statistic $V_n(k)$*). Under $H_1(k)$, the statistic $V_n(k)$ is subject to a noncentral F-distribution with p and $n-p$ degrees of freedom, the probability density function of which is

$$f_{H_1(k)}(v_n(k);n,q)=$$
$$=\left[\mathrm{B}\left(\frac{p}{2},\frac{n-p}{2}\right)\right]^{-1}\left[\frac{p}{n-p}\right]^{\frac{p}{2}} v_n(k)^{\frac{p}{2}-1}\times$$
$$\times\left[1+\frac{p}{n-p}v_n(k)\right]^{-\frac{n}{2}}\mathrm{e}^{-nq/2}\,{}_1\times$$
$$\times F_1\left(\frac{n}{2};\frac{p}{2};\frac{nq(k)}{2}\left[1+\frac{n-p}{pv_n(k)}\right]^{-1}\right),\qquad 0<v_n(k)<\infty. \quad (14)$$

where ${}_1F_1(b;c;x)$ is the confluent hypergeometric function, $q(k)=\mathbf{a}'(k)[\mathbf{Q}(k)]^{-1}\mathbf{a}(k)$ is a noncentrality parameter. Under $H_0(k)$, when $q(k)=0$, (14) reduces to a standard F-distribution with p and $n-p$ degrees of freedom,

$$f_{H_0(k)}(v_n(k);n)=\left[\mathrm{B}\left(\frac{p}{2},\frac{n-p}{2}\right)\right]^{-1}\left[\frac{p}{n-p}\right]^{\frac{p}{2}}\times$$
$$\times v_n(k)^{\frac{p}{2}-1}\left[1+\frac{p}{n-p}v_n(k)\right]^{-\frac{n}{2}},$$
$$0<v_n(k)<\infty. \quad (15)$$

Proof. The proof follows by applying Theorem 1 [3] and being straightforward is omitted here.

CFAR Test

The CFAR test of $H_0(k)$ versus $H_1(k)$, based on $V_n(k)$, is given by

$$V_n(k)\begin{cases}\geq h(k), & \text{then } H_1(k)\\ < h(k), & \text{then } H_0(k),\end{cases} \quad (16)$$

where $h(k)>0$ is a threshold of the test which is uniquely determined for a prescribed level of significance $\alpha(k)$. It follows from (15) that this test achieves a fixed probability of a false alarm.

If $V_n(k)>h(k)$ then the kth reference vibration signal is eliminated from further consideration.

If $(m-1)$ reference vibration signals are so eliminated, then the remaining reference vibration signal (say, kth) is the one with which the target vibration signal may be identified.

If all reference vibration signals are eliminated from further consideration, we decide that the target vibration signal cannot be identified with one of the m specified reference vibration signals.

If the set of reference vibration signals not yet eliminated has more than one element, then we declare that the target vibration signal may be identified with the k^*th reference vibration signal, where

$$k^*=\arg\max_{k\in D}(h(k)-V_n(k)), \quad (17)$$

where D is the set of simulation models not yet eliminated by the above test.

Conclusion

The main idea of this paper is to find a test statistic whose distribution, under the null hypothesis, does not depend on unknown (nuisance) parameters. This allows one to eliminate the unknown parameters from the problem.

References

[1] Dimarogonas A. D. Vibration of Cracked Structures: a State of the Art Review, *Engineering and Fracture Mechanics* Vol. 55 (1996), pp. 831-857.

[2] **Nechval N. A. and Nechval K. N.** Characterization Theorems for Selecting the Type of Underlying Distribution, in: *Abstracts of Communications of the 7th Vilnius Conference on Probability Theory and Mathematical Statistics & the 22nd European Meeting of Statisticians, Vilnius*, Lithuania: TEV (1998), pp. 352-353.

[3] Nechval N. A. Radar CFAR Thresholding in Clutter Under Detection of Airborne Birds, in: *Proceedings of the 21st Meeting of Bird Strike Committee Europe*, Jerusalem, Israel: BSCE (1992), pp. 127-140.