270. Vibration control of cantilever beam

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Abstract. The paper describes two approaches to problem of active damping of vibrations of cantilever beam. First one uses standard LTI (linear time invariant) mathematical model of the system and state feedback with the state observer designed by pole placement method. The incomplete pole assignment method is used instead of the standard full assignment. The second one is based on experimental identification of the first mode shape and design dynamic compensator. Experimental results of both methods are compared. The problem of robustness of the compensator by frequency domain method based on the unstructured uncertainty of the model is also addressed.

Keywords: cantilever beam, incomplete pole assignment, robustness, self tuning controller, vibration control.

Introduction

Vibration control of flexible structures is an important issue in many engineering applications. Balancing the stringent performance objectives of modern structures such as superior strength and minimal weight introduces a dynamic component that needs to be considered. Depending on the applications, low structural damping can lead to problems such as measurement inaccuracy of attached equipment, transmission of acoustic noise or structural failure. Various methods to suppress vibrations have been developed and these commonly include active, passive, semi-active and hybrid vibration control systems.

This paper addresses the vibration control of cantilever beam by the methods of linear feedback control. It is concerned with state feedback designed by pole placement method (in our case modification of this method – *incomplete pole assignment*) and by *self tuning controller* [1], [2]. An optimal position is chosen for sensor and actuator by the method shown in [3] which is based on mode shapes – amplitudes and nodal points.

The nominal mathematical model is used for design of the controller based on incomplete pole assignment. Due to this, the problem of robustness needs to be solved to avoid the instability of the closed loop because of the design inaccuracies and truncation errors.

Mathematical model

Mathematical model results from the equation of motion

$$\mathcal{M}\,\ddot{\mathbf{q}}(x,t)+\mathcal{K}\,\mathbf{q}(x,t)=\mathbf{F}(x)\,\gamma(t), \tag{1}$$

where $\mathcal{M}$ is the inertial forces operator, $\mathcal{K}$ stiffness operator, $\mathbf{F}(x)$ vector of forces space distribution, $\gamma(t)$ time function of excitation and $\mathbf{q}(x,t)$ is the vector of displacement.

Solution in frequency domain with respecting orthonormalized eigenfunctions by M-norm

$$\begin{aligned}\left\langle \mathbf{v}_i(x),\mathcal{M}\,\mathbf{v}_j(x)\right\rangle &= \boldsymbol{\delta}_{ij},\\ \left\langle \mathbf{v}_i(x),\mathcal{K}\,\mathbf{v}_j(x)\right\rangle &= \boldsymbol{\delta}_{ij}\ \Omega_i^2,\end{aligned} \tag{2}$$

where $\mathbf{v}_i(x)$ is eigenfunction for i subscript, leads to the displacement vector expressed in frequency domain

$$\mathbf{Q}\,(x,\omega)=\sum_j \frac{\left\langle \mathbf{v}_j(x),\mathbf{F}(x)\right\rangle}{\Omega_j^2-\omega^2}\,\boldsymbol{\Gamma}(\omega)\,\mathbf{v}_j(x). \tag{3}$$

For single force Eq. 3 takes the form

$$\mathbf{Q}\left(x,\omega\right)=\sum_j \frac{\mathbf{v}_j\left(x_0\right)\mathbf{v}_j\left(x\right)}{\Omega_j^2-\omega^2}\mathbf{\Gamma}(\omega), \qquad (4)$$

where x_0 is position of sensor and x position of actuator.

Adding proportional damping

$$\mathcal{M}\,\ddot{\mathbf{q}}(x,t)+\mathcal{B}\,\dot{\mathbf{q}}(x,t)+\mathcal{K}\,\mathbf{q}(x,t)=\boldsymbol{\delta}(x-x_0)\,\gamma(t), \qquad (5)$$

where $\mathcal{B}=\varepsilon_1\mathcal{M}+\varepsilon_2\mathcal{K}$ is assumed, the transfer function from place x_0 to x multiplied by the excitation yields the formula

$$\mathbf{Q}\left(x,\omega\right)=\sum_j \frac{\mathbf{v}_j\left(x_0\right)\mathbf{v}_j\left(x\right)}{\Omega_j^2+2\,D_j\,\Omega_j\,i\omega-\omega^2}\,\mathbf{\Gamma}(\omega), \qquad (6)$$

and can be rewritten into transfer function in Laplace form

$$\mathbf{Q}\left(x,s\right)=\overbrace{\sum_j \frac{\mathbf{v}_j\left(x_0\right)\mathbf{v}_j\left(x\right)}{\Omega_j^2+2\,D_j\,\Omega_j\,s+s^2}}^{\text{Transfer function}}\,\mathbf{\Gamma}(\omega). \qquad (7)$$

Vibration control

In this section, the truncated transfer function described by Eq. 7 is substituted by the equivalent state space model

$$\dot{\mathbf{x}}=\mathbf{Ax}+\mathbf{Bu},$$
$$\mathbf{y}=\mathbf{Cx}, \qquad (8)$$

where $\mathbf{x}\in\mathbf{R}^{2n}$ is the state of the system, *n* is the number of mode shapes considering in truncated model, **u** is the input, $\mathbf{y}$ is the output, and $\mathbf{A}\in\mathbf{R}^{2n\times 2n}$, $\mathbf{B}\in\mathbf{R}^{2n\times 1}$, $\mathbf{C}\in\mathbf{R}^{1\times 2n}$ are system matrices given by

$$\mathbf{A}=\begin{bmatrix}0 & 1 & & & \\ -\beta_1 & -\alpha_1 & & & \\ & & \ddots & & \\ & & & 0 & 1\\ & & & -\beta_n & -\alpha_n\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}0\\ K_1\\ \vdots\\ 0\\ K_n\end{bmatrix},$$
$$\mathbf{C}=\begin{bmatrix}1 & 0 & \cdots & 1 & 0\end{bmatrix}, \qquad (9)$$

$$\left.\begin{aligned}\alpha_i&=2D_i\Omega_i,\\ \beta_i&=\Omega_i^2,\\ K_i&=v_i(x_0)v_i(x)\end{aligned}\right\} i=\left(1,\ldots,n\right) \qquad (10)$$

Further, it is considered that the inputs for disturbances are the same as for those the control actions, $n=1$ is for controller design presuming and for control robustness $n=5$ is assumed.

Incomplete pole assignment. It is well known that the positions of the poles of the system Eq. 8 determine the damping of system responses. For this purpose the slightly damped eigenvalues of the matrix **A** should be properly changed. Especially, the eigenvalues with the minimal absolute values should be located to the suitable positions. Since the complete pole assignment by the state feedback is unrealistic, because of ill conditioning of the corresponding problem, we focus on the incomplete assignment. Thus, only the *m* closed loop poles are required to be assign to the properly chosen locations in the complex plane. For this purpose the standard control configuration with the state observer is used.

Fig. 1. Cantilever beam with sensor and controller

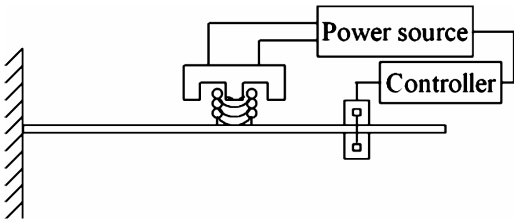

Fig. 2. Schema of cantilever beam with sensor and

The state feedback

$$\mathbf{u}=\mathbf{Fx}, \qquad (11)$$

where $\mathbf{F}\in\mathbf{R}^{1\times 2n}$, gives the closed loop system

$$\dot{\mathbf{x}}=\mathbf{Ax}+\mathbf{BFx}=\left(\mathbf{A}+\mathbf{BF}\right)\mathbf{x}. \qquad (12)$$

The incomplete assignment problem considered above yields the requirement

$$\mathbf{A}+\mathbf{BF}\approx\begin{bmatrix}\mathbf{L} & *\\ \mathbf{0} & *\end{bmatrix} \qquad (13)$$

where the symbol $\approx$ denotes the relation of matrix similarity and $\mathbf{L}\in\mathbf{R}^{m\times m}$ is given matrix with required eigenvalues of the closed loop. In [4], it was shown that any matrix **F** satisfying Eq. 13 can be expressed in the form

$$\mathbf{F}\left(\mathbf{H},\hat{\mathbf{F}}\right)=\mathbf{H}\left[\mathbf{X}^{\mathbf{T}}(\mathbf{H})\mathbf{X}(\mathbf{H})\right]^{-1}\mathbf{X}^{\mathbf{T}}(\mathbf{H})+\hat{\mathbf{F}} \tag{14}$$

where $\mathbf{X}(\mathbf{H})$ is the solution of the matrix equation

$$\mathbf{AX}-\mathbf{XL}+\mathbf{BH}=0 \tag{15}$$

where $\mathbf{H}\in\mathbf{R}^{1\times m}$ and $\hat{\mathbf{F}}\in\mathbf{R}^{1\times 2n}$ is an arbitrary matrix satisfying the condition

$$\hat{\mathbf{F}}\mathbf{X}(\mathbf{H})=0 \tag{16}$$

Moreover in [4] it is proved that $\mathbf{X}(\mathbf{H})$ has fuel rank for almost any $\mathbf{H}$ and $\mathbf{F}\left(\mathbf{H},\hat{\mathbf{F}}\right)$ given by Eq. 14 satisfies Eq. 13. Thus Eq. 14 can be used for computing of the state feedback assigning the eigenvalues of $\mathbf{L}$ to the matrix $\mathbf{A}+\mathbf{BF}$. The freedom in this procedure caused by the free choosing of the matrix $\mathbf{H}$ can be used to obtain the most robust solution by Monte Carlo method.

The observer is described by

$$\begin{aligned}\dot{\hat{\mathbf{x}}}&=\mathbf{A}\hat{\mathbf{x}}+\mathbf{Bu}+\mathbf{K}(\hat{\mathbf{y}}-\mathbf{y})\\ \hat{\mathbf{y}}&=\mathbf{C}\hat{\mathbf{x}}\end{aligned} \tag{17}$$

where $\hat{\mathbf{x}}$ is the estimation of the state $\mathbf{x}$, $\hat{\mathbf{y}}$ is the estimation of the output $\mathbf{y}$ and $\mathbf{K}\in\mathbf{R}^{2n\times 1}$ is the appropriate gain matrix obtained from the eigenvalues assignment problem for the observer matrix $\mathbf{A}+\mathbf{KC}$ (for details see [4]).

Robustness. There is difference between nominal system – mathematical model and real system. It is useful to find the family of real systems, for which the nominal controller can be used.

The transfer function of the controller has following form

$$C(s)=-\mathbf{F}\left[s\mathbf{I}-\left(\mathbf{A}+\mathbf{BF}+\mathbf{KC}\right)\right]^{-1}\mathbf{K} \tag{18}$$

where $s=i\,\omega$ and system, observer and controller matrices are defined above.

The transfer function of the system takes a form

$$P(s)=\mathbf{C}\left(s\mathbf{I}-\mathbf{A}\right)^{-1}\mathbf{B} \tag{19}$$

and the transfer function of open loop system is described by the equation

$$\begin{aligned}L(s)&=C(s)P(s)=\\ &=-\mathbf{F}\left[s\mathbf{I}-\left(\mathbf{A}+\mathbf{BF}+\mathbf{KC}\right)\right]^{-1}\mathbf{KC}\left(s\mathbf{I}-\mathbf{A}\right)^{-1}\mathbf{B}\end{aligned} \tag{20}$$

As was shown in [6], the family of systems, which fulfill the condition of robust performance with nominal controller, is specified in the frequency domain by the system of circles. The radius of the circle of uncertainty is for a given frequency ω defined by

$$\Delta=\frac{1}{C(i\,\omega)}\left(\left|1+L_n(i\,\omega)\right|-r\right) \tag{21}$$

where $L_n(i\omega)$ is the transfer function of nominal open loop system and r is the radius of the circle corresponding admissible upper limit of the sensitivity function. More in [6]. The open loop of nominal system with nominal controller distance with point [-1,0] is $r=0{,}74413$. The bounds for $r=0{,}6$ are depicted in the Fig. 4.

Self tuning controller. The self tuning controller SC2FA is a special controller for vibration damping from the control system REX [1], [2]. This function block provides all necessary steps to design the control law for suppressing the first mode shape. Particularly, it provides the automatic identification of the first mode dynamics, design of the state feedback and state observer according to the user design specification. Also, the corresponding control law is implemented within this block.

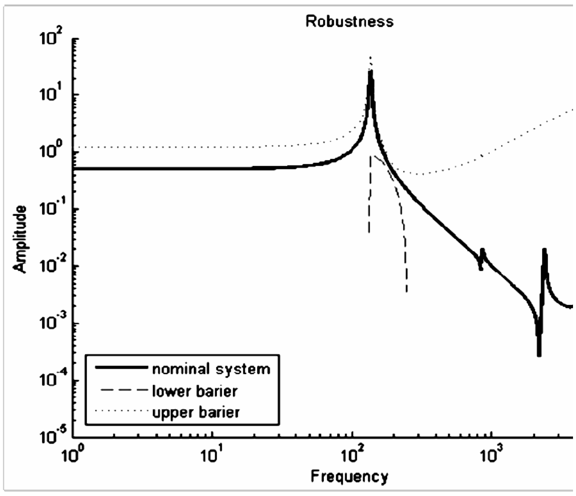

Fig. 4. Bode diagram of feasible family of systems (without controller) (-- goes to zero)

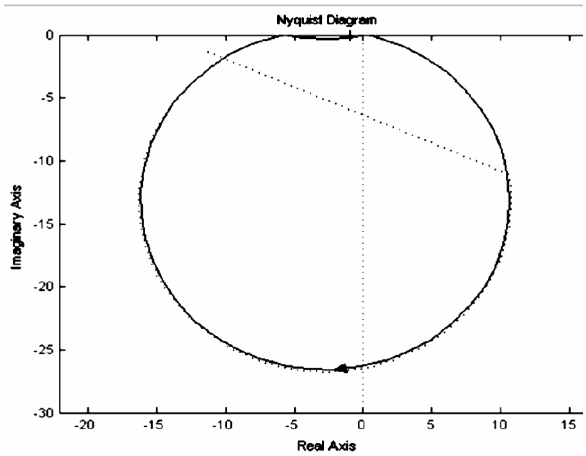

Fig. 3. Identified model of the cantilever beam – mathematical model with time delay '-' and identified model by SC2FA '...'

Cantilever beam experiment

Description of Fig. 1. 1) Industrial PC ADVANTECH, 2) and 3) terminal board, 4) panel of micrometer, 5) sensor – micrometer KEYENCE LS-7001, 6) cantilever beam, 7) voice coil actuator, 8) controlled current source, 9) power supply for 4) 5) 8).

Place of sensor and actuator. The optimal places are chosen for sensor and actuator by the method based on mode shapes – amplitudes and nodal points [3]. The optimal place for sensor is in nodal point of the second mode shape.

Modification of the model. The time delay and scaling factors were added to mathematical model. Fig. 3 shows the Nyquist diagram of the mathematical model comparing to the identified model by SC2FA block (experimental identification).

Results

Because it is very difficult to achieve the Dirac impulse excitation in the experimental way, the rectangular impulse was used instead.

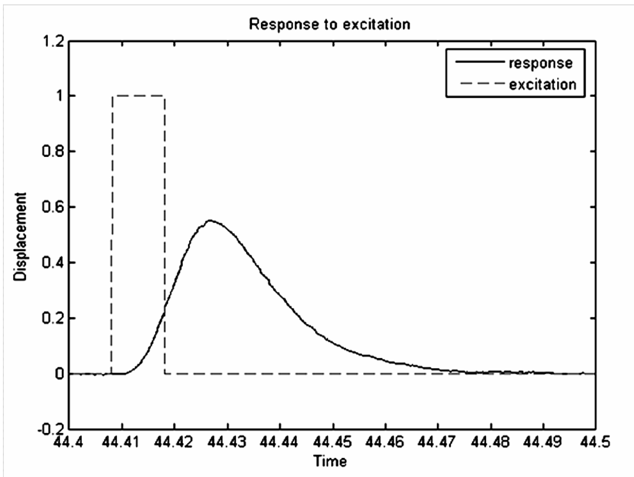

Fig. 5. Response to excitation – controller designed by incomplete pole assignment method – experiment

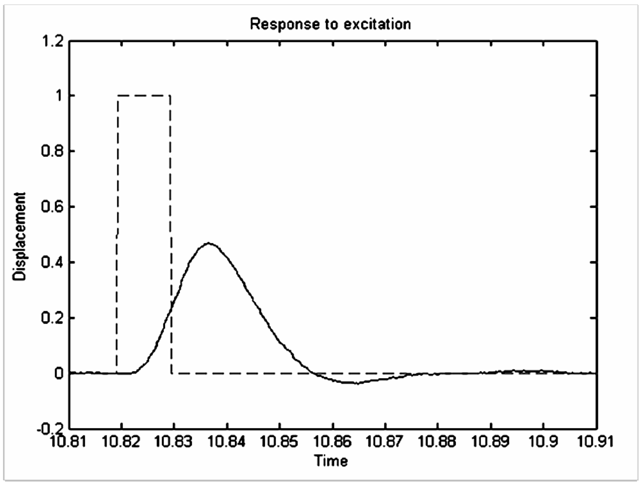

Fig. 6. Response to excitation – self tuning controller – SC2FA – experiment

Conclusion

The purpose of this work is to compare the controller designed by the incomplete pole assignment and more practical self tuning controller experimentally. Both methods give good results, however the controller designed by incomplete pole assignment provides more freedom in the tuning phase in contrast of the self tuning controller, where tuning is simplified.

Acknowledgement

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