265. Validation of Vibration Analysis using Photo-elastic Coatings

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Abstract. The analysis of vibrations is performed using a plate bending element assuming that a coating has no effect to the motion of the plate. The stresses in the coating are calculated by using the procedure of conjugate approximation. Then the relative error norms for the finite elements are determined. They are represented graphically and serve for validation of the results of calculations in hybrid experimental – numerical procedures.

Keywords: plate bending, eigenmode, finite elements, photo-elastic coatings, conjugate approximation, error norms.

Introduction

Calculation of the stress fields in the photo-elastic coating is important in hybrid experimental – numerical procedures [1, 2].

The analysis is performed using a plate bending element of the type described in [3] assuming that a coating has no effect to the motion of the plate. The first eigenmodes are determined and it is assumed that vibrations take place according to the eigenmode.

The stresses in the coating are calculated by using the procedure of conjugate approximation [4, 5, 6]. Then the relative error norms for the finite elements are determined [7, 8]. They are represented graphically in the form of histograms for each of the eigenmodes.

The obtained results serve for validation of the results of calculations in hybrid experimental – numerical procedures developed in [9].

Numerical procedure

Further x, y and z denote the axes of the orthogonal Cartesian system of coordinates. The plate bending element has three nodal degrees of freedom: the displacement w in the direction of the z axis and the rotations Θ_x and Θ_y about the axes x and y. The displacements u and v in the directions of the axes x and y are expressed as $u=z\Theta_y$ and $v=-z\Theta_x$.

The stiffness matrix has the form:

$$[K]=\int\left([B]^T[D][B]+[\overline{B}]^T[\overline{D}][\overline{B}]\right)dxdy, \qquad (1)$$

where:

$$[B]=\begin{bmatrix} 0 & 0 & \frac{\partial N_1}{\partial x} & \dots \\ 0 & -\frac{\partial N_1}{\partial y} & 0 & \dots \\ 0 & -\frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \dots \end{bmatrix},$$

$$[D]=\frac{h^3}{12}\begin{bmatrix} \frac{E}{1-\nu^2} & \frac{E\nu}{1-\nu^2} & 0 \\ \frac{E\nu}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix},$$

$$[\overline{B}]=\begin{bmatrix} \frac{\partial N_1}{\partial y} & -N_1 & 0 & \dots \\ \frac{\partial N_1}{\partial x} & 0 & N_1 & \dots \end{bmatrix},$$

$$[\overline{D}]=\frac{Eh}{2(1+\nu)1.2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad (2)$$

where N_i are the shape functions of the finite element, h is the thickness of the plate, E is the modulus of elasticity of the plate and ν is the Poisson's ratio of the plate.

The mass matrix has the form:

$$[M] = \int [N]^T \begin{bmatrix} \rho h & 0 & 0 \\ 0 & \frac{\rho h^3}{12} & 0 \\ 0 & 0 & \frac{\rho h^3}{12} \end{bmatrix} [N] dxdy, \quad (3)$$

where ρ is the density of the material of the plate and:

$$[N] = \begin{bmatrix} N_1 & 0 & 0 & \dots \\ 0 & N_1 & 0 & \dots \\ 0 & 0 & N_1 & \dots \end{bmatrix}. \quad (4)$$

The nodal values of stresses are determined by using the procedure of conjugate approximation:

$$\int [\hat{N}]^T [\hat{N}] dxdy [\{\delta_x\} \quad \{\delta_y\} \quad \{\delta_{xy}\}] = \\ = \int [\hat{N}]^T [\sigma_x^c \quad \sigma_y^c \quad \tau_{xy}^c] dxdy, \quad (5)$$

where $\{\delta_x\}$, $\{\delta_y\}$ and $\{\delta_{xy}\}$ are the vectors of nodal values of stresses σ_x^c, σ_y^c and τ_{xy}^c in the coating and:

$$[\hat{N}] = [N_1 \quad \dots]. \quad (6)$$

The stresses in the coating are determined as:

$$\begin{Bmatrix} \sigma_x^c \\ \sigma_y^c \\ \tau_{xy}^c \end{Bmatrix} = [D_c] \frac{h}{2} [B] \{\delta\}, \quad (7)$$

where the vector $\{\delta\}$ is the analyzed eigenmode and:

$$[D_c] = \begin{bmatrix} \frac{E_c}{1-\nu_c^2} & \frac{E_c \nu_c}{1-\nu_c^2} & 0 \\ \frac{E_c \nu_c}{1-\nu_c^2} & \frac{E_c}{1-\nu_c^2} & 0 \\ 0 & 0 & \frac{E_c}{2(1+\nu_c)} \end{bmatrix}, \quad (8)$$

where E_c is the modulus of elasticity of the coating and ν_c is the Poisson's ratio of the coating.

The relative error norm for the finite element is calculated as:

$$\psi = \frac{\begin{pmatrix} \int_e \left(\{\varepsilon^c\} - \frac{h}{2} [B] \{\delta\} \right)^T [D_c] \cdot \\ \cdot \left(\{\varepsilon^c\} - \frac{h}{2} [B] \{\delta\} \right) dxdy \end{pmatrix}}{\int_e \{\varepsilon^c\}^T [D_c] \{\varepsilon^c\} dxdy}, \quad (9)$$

where the integrations are performed over the analyzed finite element and the values of $\{\varepsilon^c\}$ are defined from:

$$\{\varepsilon^c\} = [D_c]^{-1} \begin{Bmatrix} \sigma_x^c \\ \sigma_y^c \\ \tau_{xy}^c \end{Bmatrix}, \quad (10)$$

and in this formula the stresses in the coating are obtained from their nodal values by using the shape functions of the analyzed finite element.

Results of analysis

The vibrations of a circular disk are analyzed. On the internal radius of the disk all the generalized displacements are assumed equal to zero. It is proposed to represent the relative error norms as black angles in the circles located at the centers of finite elements.

The histogram of relative error norms for the first eigenmode is presented in Fig. 1, for the second eigenmode in Fig. 2, …, for the sixth eigenmode in Fig. 6.

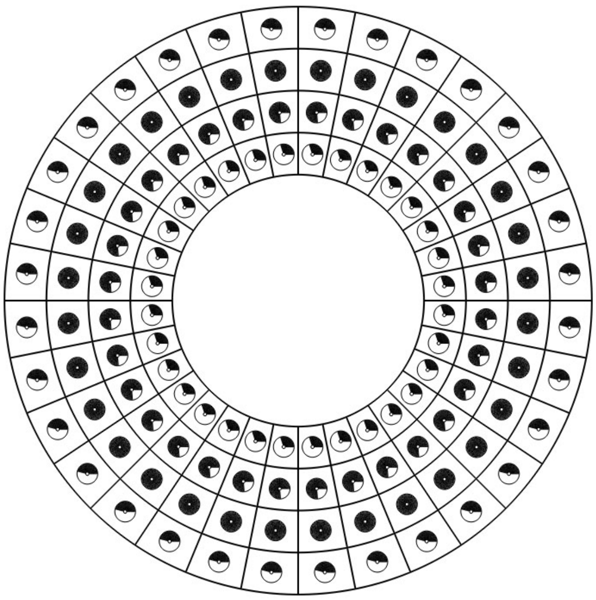

Fig. 1. Histogram of relative error norms for the first eigenmode

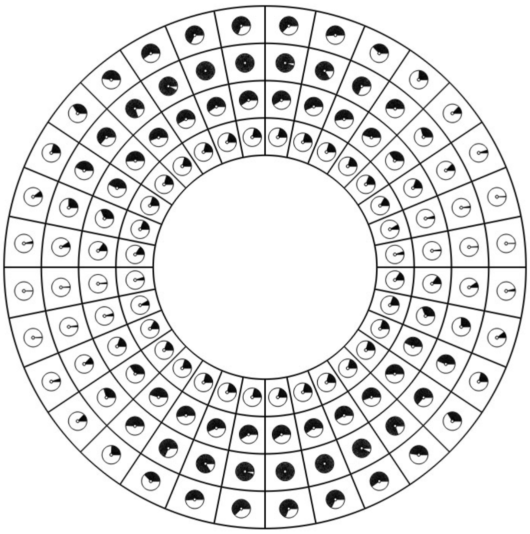

Fig. 2. Histogram of relative error norms for the second eigenmode

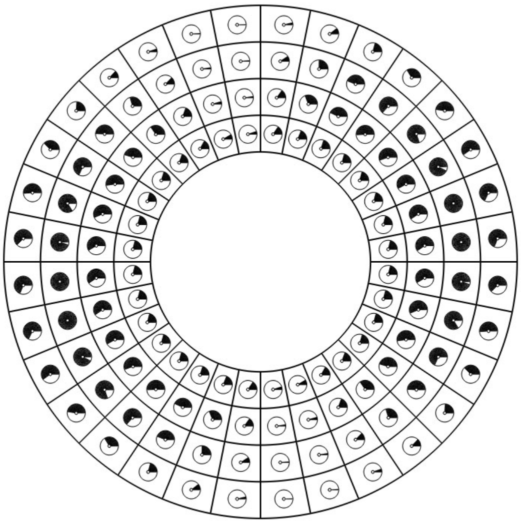

Fig. 3. Histogram of relative error norms for the third eigenmode

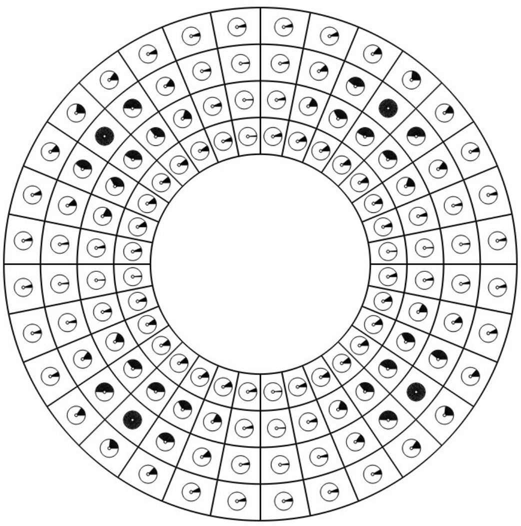

Fig. 4. Histogram of relative error norms for the fourth eigenmode

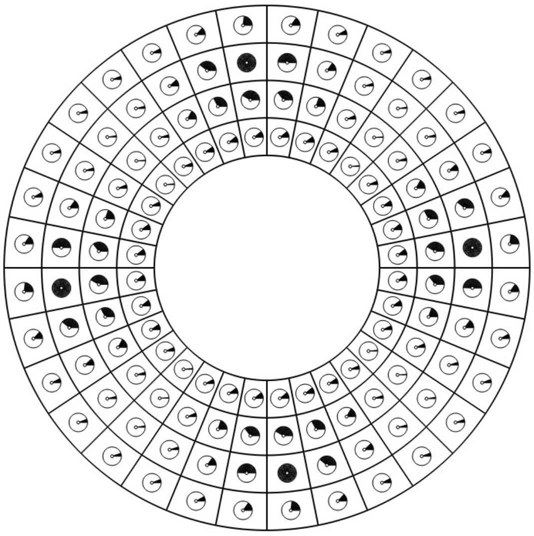

Fig. 5. Histogram of relative error norms for the fifth eigenmode

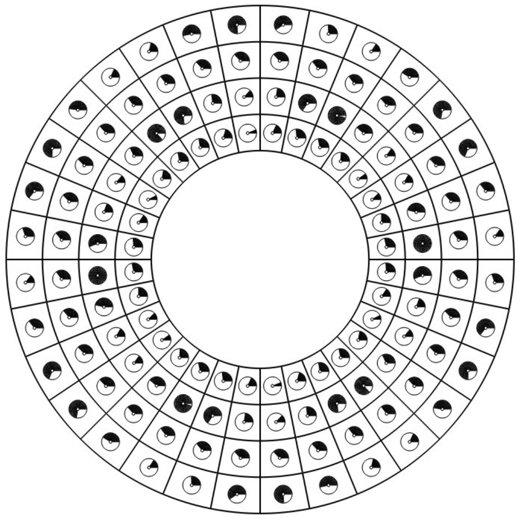

Fig. 6. Histogram of relative error norms for the sixth eigenmode

Photoelastic image of the second eigenmode is presented in Fig. 7.

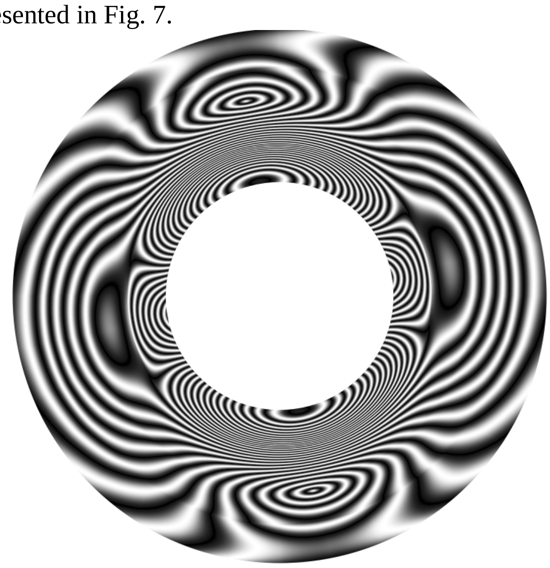

Fig. 7. Photoelastic image of the second eigenmode

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The presented results serve for validation of the calculated stress fields in the coating of a vibrating plate.

On the basis of the relative error norms methods of adaptive control of the smoothing parameter in hybrid experimental – numerical procedures can be proposed.

Conclusions

The first eigenmodes of bending vibrations of a circular plate with fixed internal radius are determined and it is assumed that vibrations take place according to the eigenmode.

The stresses in the coating are calculated by using the procedure of conjugate approximation. Then the relative error norms for the finite elements are determined. Their graphical representation in the form of histograms for each of the eigenmodes is proposed.

The presented results serve for validation of the calculated stress fields in the coating of a vibrating plate. On the basis of the relative error norms methods of adaptive control of the smoothing parameter in hybrid experimental – numerical procedures can be developed.

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