263. Damage Quantification using Modal Parameters

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Abstract

Natural frequencies and mode shapes of a structure change whenever a structure has any kind of damage. However, detection of location and magnitude of damage has always been difficult. This paper presents a technique to locate and quantify the damage when the natural frequencies and mode shapes of undamaged and damaged structure are known. First, the structure is modeled using finite element formulation. Each element is assigned a damage index that is initially zero for undamaged condition. Modal analyses of undamaged and damaged structures are performed. The damage indices are computed using non-negative least squares method. Results based on aluminum beams are presented to establish the effectiveness of this method.

Keywords: Damage detection; Modal parameters; Finite Element Analysis; Experimental Modal Analysis, Non-negative Least Squares.

NOTATION

K Global stiffness matrix of undamaged structure
M Global mass matrix of undamaged structure
$\overline{K}$ Global stiffness matrix of damaged structure
$\overline{M}$ Global mass matrix of damaged structure
k_e Element stiffness matrix of undamaged structure
$\bar{k}_e$ Element stiffness matrix of damaged structure
m_e Element mass matrix of structure
Φ Eigenvector matrix of undamaged structure
$\overline{\Phi}$ Eigenvector matrix of damaged structure
α_e Damage index of the *e*th element
λ Eigenvalue of undamaged structure
$\bar{\lambda}$ Eigenvalue of damaged structure

1. INTRODUCTION

Identification of damage in a structure is always of paramount importance because an early detection may help prevention of catastrophic failure when the damage reaches certain critical value. Non-destructive damage identify-cation methods using vibration modal parameter (i.e. natural frequencies, mode shapes, transfer functions, change in frequency response function curvatures) have attracted extensive interest in the past. Some researchers explored change in natural frequencies due to damage as reviewed by Salawu [1]. However, detection of damage by just variation of natural frequencies would require large damage before it could be detected. Friswell [2] proposed a damage identification method based on catalog of likely damage scenarios. He introduced a power law relation between frequency shifts of first several modes of undamaged and all possible damage scenario. However, this approach requires prior knowledge and cataloging of all possible damage scenarios, which may not be possible always. Stubbs and Oseguenda [3] proposed a method in which an error function for each structural member is computed assuming only one member is damaged. However, in case more than one member is damaged, this approach may not be used.

West [4] used change in modal assurance criteria to locate the damage. Pandey et al [5] used the absolute change in mode shape curvature as indicator of damage. Pandey and Biswas [6] proposed damage detection based on changes in measured flexibility of the structure. However, for structures with rigid body modes, flexibility matrices are not available. Maia et al [7], Silva et al [8], and Davis and Wicks [9, 10] proposed usage of curvature of frequency response functions. Moslehy [11] showed how modal parameters change for damaged beams but did not present any theoretical basis as to how the damage will be located i.e. how to solve the inverse problem. Zhou [12] studied for his MS thesis how modal parameters of damaged and undamaged beams can be used to locate and quantify the damage. This paper is based on that thesis work. Damage detection based on frequency measurements continues to be of interest to the researchers as evident by the recent paper by Kannappan et al [13] where they studied damage in cantilever beams.

Here in this paper a method is proposed which utilizes only the modal parameters (natural frequencies and mode shapes) of damaged structures obtained from experiment and modal parameters and stiffness and mass matrices of undamaged structures from finite element analysis of original or undamaged structures. It may be noted that even for structures where modal analysis is difficult to perform due to ambient excitation; operational modal analysis can be performed to obtain the modal properties. However, operational modal analysis should not be performed unless the excitation force Gaussian white as shown by Rudroju et al [14].

Application of the damage quantification method presented in our paper is demonstrated using aluminum beams though the concept is applicable for any general structure. First in finite element analysis the structure is discretized into n elements and a damage index α_e (which represents the magnitude of damage) is assigned to the *e*th element, $e \in [0, n]$. The magnitude of α varies form 0 to 1 where 0 indicates no damage and 1 indicates complete loss or removal of the element. The subscript e denotes the element number. Thus, knowledge of α_e for all the elements in a structure provides complete information about the magnitude and location of the damage.

Typically whether by measurement or by computation, either way only first few natural frequencies and mode shapes are obtained (instead of all natural frequencies or mode shapes equaling the degrees of freedom). Thus, computation of damage indices involves solving linear algebraic equations simultaneously with more unknowns than equations (a case of underdetermined problem) and to overcome this difficulty non-negative least squares method (Lawson and Hanson [15]) is used. Theoretical details of how natural frequencies and mode shapes of damaged and undamaged structures and stiffness and mass matrices of undamaged structures are used to apply non-negative squares to solve for damage location and quantification are presented next.

THEORY:

In finite element analysis, a structure is discretized into a finite number of elements of small size. In this particular case, beam elements with two degrees of freedom (transverse displacement and slope) are chosen. The structure is discretized into n elements and the stiffness matrix k_e and mass matrix m_e of the *e*th element (Gupta and Foster [16]} are:

$$k_e = \frac{2EI}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix} \tag{1}$$

$$m_e = \frac{\rho Ah}{420}\begin{bmatrix} 156 & -22h & 54 & 13h \\ -22h & 4h^2 & -13h & -3h^2 \\ 54 & -13h & 156 & 22h \\ 13h & -3h^2 & 22h & 4h^2 \end{bmatrix} \tag{2}$$

where E = the Young's modulus, I = the moment of inertia of the cross section, h = length of the element, ρ = the density of the aluminum beam, and A = the cross section area of the aluminum beam.

If there is a damage located in the *e*th element, it is modeled as an effective decrease of the bending stiffness EI to $\bar{E}\bar{I}$. Consequently, the stiffness matrix of the *e*th element of the damaged structure $\bar{k}_e$ is given by:

$$\bar{k}_e = \frac{2\bar{E}\bar{I}}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix} \tag{3}$$

Global stiffness matrices K, $\bar{K}$ and global mass matrices M, $\bar{M}$ are obtained by assembling all discretized elements where K and M are stiffness and mass matrices of the undamaged structure and $\bar{K}$, $\bar{M}$ are stiffness and mass matrices of the damaged structure. For an undamaged structure, the eigenvalue and the associated eigenvector matrices satisfy the equation:

$$(K - \lambda M)\Phi = 0 \tag{4}$$

For the damaged structure, the resulting eigenvalue problem becomes:

$$(\bar{K} - \bar{\lambda}\bar{M})\bar{\Phi} = 0 \tag{5}$$

The change in the stiffness of the *e*th element due to the damage is defined as:

$$\Delta k_e = k_e - \bar{k}_e = \alpha_e k_e \tag{6}$$

where α_e is the damage index of the *e*th element (Araujo dos Santos et.al, [17]), $\alpha_e \in [0, 1]$. When $\alpha_e = 0$, there is no loss of stiffness on the *e*th element (i.e. there is no damage). When $\alpha_e = 1$, the eth element completely fails.

The global stiffness matrices for both undamaged and damaged structure are available by assembling elemental stiffness matrices:

$$K = \sum_{e=1}^{n} k_e \quad \text{and} \quad \bar{K} = \sum_{e=1}^{n} \bar{k}_e = \sum_{e=1}^{n} (1-\alpha_e)k_e$$

The global mass matrix is assumed unchanged before and after damage, which is a reasonable assumption in most real applications leading to $M = \overline{M}$. Premultiplying equation (5) by Φ^T and taking transpose of consequent equation on both sides will result in:

$$\overline{\Phi}^T \overline{K}^T \Phi = \overline{\lambda}\overline{\Phi}^T \overline{M}^T \Phi = \overline{\lambda}\overline{\Phi}^T M^T \Phi \tag{7}$$

Stiffness and mass matrices being symmetric matrices (Ewins, [18]), the equation above becomes:

$$\overline{\Phi}^T \overline{K}\Phi = \overline{\lambda}\,\overline{\Phi}^T M\Phi$$

Also,
$$\begin{aligned}\overline{\Phi}^T \overline{K}\Phi &= \overline{\Phi}^T \sum_{e=1}^{n} (1-\alpha_e) k_e \Phi \\ &= \overline{\Phi}^T \left(\sum_{e=1}^{n} k_e - \sum_{e=1}^{n} \alpha_e k_e \right)\Phi \\ &= \overline{\Phi}^T K\Phi - \overline{\Phi}^T \sum_{e=1}^{n} \alpha_e k_e \Phi \\ &= \overline{\lambda}\,\overline{\Phi}^T M\Phi\end{aligned}$$

Then,
$$\begin{aligned}\overline{\Phi}^T \sum_{e=1}^{n} \alpha_e k_e \Phi &= \overline{\Phi}^T K\Phi - \overline{\lambda}\,\overline{\Phi}^T M\Phi \\ &= \overline{\Phi}^T (\lambda M\Phi) - \overline{\lambda}\,\overline{\Phi}^T M\Phi \\ &= (\lambda - \overline{\lambda})\overline{\Phi}^T M\Phi \end{aligned} \tag{8}$$

The equation (8) also can be written as:

$$\begin{aligned}\overline{\Phi}^T \sum_{e=1}^{n} \alpha_e k_e \Phi &= (\lambda - \overline{\lambda})\overline{\Phi}^T M\Phi \\ &= (\lambda - \overline{\lambda})\overline{\Phi}^T \left(\frac{1}{\lambda} K\Phi \right) \\ &= \left(1 - \frac{\overline{\lambda}}{\lambda} \right)\overline{\Phi}^T K\Phi \end{aligned} \tag{9}$$

Let us assume that i and j are the number of eigenvalues (and eigenvectors) obtained for the undamaged and damaged structure. Since the total number of degrees of freedom is 2n+2, it may be noted that $i \le$ 2n+2 and $j \le$ 2n+2. Equation (9) involves n damage indices as variables. Next the coefficients of these damage indices are collected and put into a matrix A that has the dimension of $m \times n$, where $m = (i \times j)$. Solution of equation (9) implies solution of a set of linear algebraic equations of the form:

$$A_{m\times n}\{\alpha\}_{n\times 1} = \{b\}_{m\times 1} \tag{10}$$

where $\{b\}$ comes from the right hand side of the equation (9). If $m = n$ and $rank(A) = m$, the equation (10) is consistent. If $m > n$ and $rank(A) \le n$, the equation (10) is overdetermined. Also if $n >> m$ and $rank(A) \le m$, which is the case with most applications, the equation (10) is underdetermined. Non-negative least squares method (Lawson and Hanson, [15]) is used to solve these algebraic equations. The solution vector is based on minimization of $\|A\{\alpha\} - \{b\}\|$ subject to the constraint that $\alpha_e \ge 0$

3. RESULTS

To verify the usefulness and accuracy of this method to quantify damage, three aluminum 6061-T6 beams (undamaged or case 1 as shown in Figure.1, beam with a short through crack or case 2 as shown in Figure 2 and beam with a larger through crack at the same location or case 3 as shown in Figure 3) were used. Impact modal testing on these beams was conducted. All beams had the same dimensions and same material properties. There was no damage in case 1 and this beam was used as the baseline. All beams were discretized into 5 elements, each with width of 80 mm. It may be noted that since the damage (crack) was located at 298 mm from left (4 inches from right), it was located in the 4th element (for both case 2 and case 3) i.e. between 240 mm and 320 mm.

All dimensions are in mm

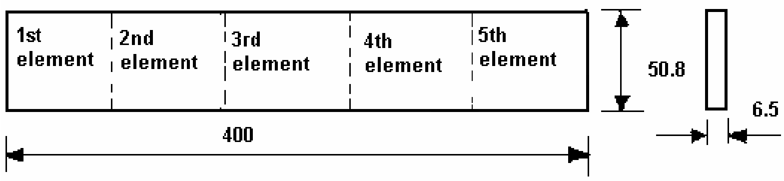

Fig. 1. Case 1 (Undamaged beam)

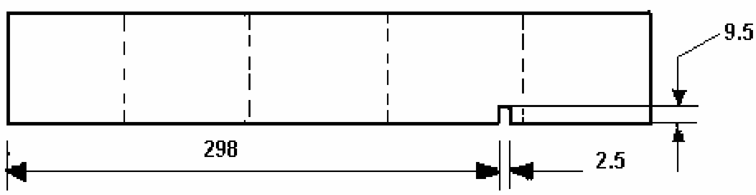

Fig. 2. Case 2 (Beam with short crack)

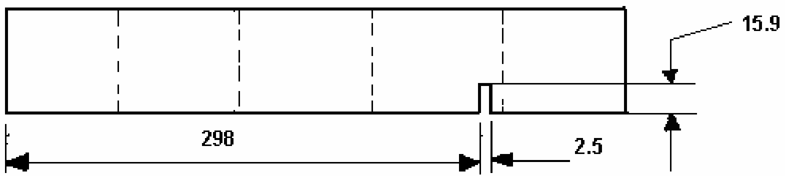

Fig. 3. Case 3 (Beam with long crack)

Since closed form theoretical solution is available for the undamaged beam, natural frequencies obtained from experiment (impact hammer modal testing) were compared with the theoretical values and the results are presented in table 1. Errors were less than 1% indicating satisfactory experimental procedure.

Table 1. The experimental and theoretical natural frequencies of undamaged beam

Method	First Four Bending Natural Frequencies (Hz)			
	1	2	3	4
Experiment	210,9	582,5	1143,2	1875,2
Theoretical	211,3	582,6	1142,1	1888,0
Error in %	0.23	0.02	0.09	0.67

It may be noted that in experiment by impact hammer testing, since response transducer (accelerometer) provides only the transverse displacement and not the slope whereas in beam finite element formulation slope is also needed; cubic spline interpolation was used to obtain the slopes. Next, these natural frequencies and mode shapes (displacements and interpolated slopes) from the experiment along with original stiffness matrices of undamaged beams from the finite element were utilized in the proposed method to compute the damage indices.

Table 2 presents the experimental natural frequencies for the three cases. As expected natural frequencies decreased with damages (cuts) because reduction in stiffness is more pronounced than reduction in mass.

Table 2. The experimental natural frequencies of three aluminum beams

	First Four Bending Natural Frequencies (Hz)			
	1	2	3	4
Undamaged (Case 1)	210,9	582,5	1143,2	1875,2
Beam with short Crack (Case 2)	209,0	573,7	1130,3	1856,1
Beam with Long Crack (Case 3)	207,8	565,5	1116,1	1829,3

Table 3 presents the damage indices obtained for case 2 and table 4 presents damage indices obtained for case 3. Damage indices of all elements other than element 4 (where the crack was located) were found to be zero indicating that the damage was located only within 4th element and thus validated the proposed method. The magnitude of damage index for case 3 was higher than that obtained for obtained for element 4 of case 2 indicating (as expected) larger damage for case 3.

Table 3. Damage indices for damaged beam case 2

Element number	Actual Presence of Crack	Damage Index Computed	Crack Detection by Proposed Method
1	No Crack	$\alpha_1 = 0$	No crack
2	No Crack	$\alpha_2 = 0$	No crack
3	No Crack	$\alpha_3 = 0$	No crack
4	Crack Present	$\alpha_4 = 0{,}1$	Crack present
5	No Crack	$\alpha_5 = 0$	No crack

Table 4. Damage indices for damaged beam case 3

Element number	Actual Presence of Crack	Damage Index	Crack Detection by Proposed Method
1	No crack	$\alpha_1 = 0$	No crack
2	No crack	$\alpha_2 = 0$	No crack
3	No crack	$\alpha_3 = 0$	No crack
4	Longer crack present	$\alpha_4 = 0{,}17$	Longer Crack present
5	No crack	$\alpha_5 = 0$	No crack

4. CONCLUSIONS

A non negative least squares method using modal parameters of damaged beams obtained through experiment and modal parameters as well as stiffness matrices of undamaged beam from finite element analysis has been developed. The proposed method was used successfully to identify the location and the severity of the damage in the aluminum beams. Since finite element results are easily available for undamaged structures and experimental modal results are available for damaged structures, this method has vast potential for damage detection.

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