262. Analysis of vibrations of paper in a printing device

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(Received 1 March 2007; accepted 23 March 2007)

Abstract. The model for the analysis of vibrations of paper in a printing device is presented. It is assumed that paper performs transverse vibrations as a membrane having stiffness due to static tension in its plane. The first eigenmodes are determined. It is shown that the eigenfrequencies of transverse vibrations of the analyzed paper can be controlled by the static tension in the plane of the paper.

The model for the analysis of stability of paper in a printing device is presented. It is assumed that paper moves with constant velocity in a given direction. The analysis is performed using a plate-bending element with the contribution of the forces of inertia from the convective accelerations. This results in the eigenproblem determining the critical velocities and stability eigenmodes.

The stand was made for obtaining the experimental investigations. Using it the nodal lines of the standing waves were determined and using projection moiré the eigenmodes were determined.

The obtained results are used in the process of design of the elements of the printing device.

Keywords: paper, eigenmode, membrane, finite elements, stability, critical velocity, stability eigenmode.

Introduction

The model for the analysis of vibrations of paper in a printing device is proposed on the basis of the models described in [1, 2]. It is assumed that paper performs transverse vibrations as a membrane having stiffness due to static tension in its plane. Thus the analysis consists of two stages:

1) the static problem of plane stress by assuming the displacements at the boundary of the analyzed paper to be given is solved;

2) the vibrations of the investigated paper as of a membrane with stiffness due to the static tension determined in the previous stage of analysis are analyzed (the first eigenmodes are determined).

The model for the analysis of stability of paper in a printing device is proposed on the basis of the model for the analysis of plate bending described in [2] and the principles of analysis of vibrations of elastic structures described in [3, 4].

It is assumed that paper moves with constant velocity in a given direction. The analysis is performed using a plate-bending element with the contribution of the forces of inertia from the convective accelerations. This results in the eigenproblem determining the critical velocities and stability eigenmodes.

Besides, in recent years for the investigation of vibrations moire methods are used rather widely: the method of shadow moire is often used for the analysis of vibrations of a plate [5], geometric moire is used for the analysis of vibrating elastic structures [6], it is also often used for the visualisation of periodic dynamic processes in circular structures [7]; for the identification of plane vibrations stochastic moire is used [8]. In this paper for the analysis of vibrations of a sheet of paper the method of projection moire is used [9].

The purpose of this investigation is to experimentally determine the nodal lines of the standing waves of the paper excited by vibrations and to determine the eigenmodes.

The obtained results are used in the process of design of the elements of the printing device.

Model for the analysis of vibrations of the paper

First, the static problem of plane stress is analyzed. The element has two nodal degrees of freedom: displacements u and v in the directions of axes x and y of the orthogonal Cartesian system of coordinates. The stiffness matrix has the form:

$$[K]=\int [B]^T [D][B] hdxdy, \tag{1}$$

where h are the thickness of the paper and:

$$[B]=\begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \ldots \\ 0 & \frac{\partial N_1}{\partial y} & \ldots \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \ldots \end{bmatrix}, \tag{2}$$

where N_i is the shape functions of the finite element and:

$$[D]=\begin{bmatrix} \frac{E}{1-\nu^2} & \frac{E\nu}{1-\nu^2} & 0 \\ \frac{E\nu}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix}, \tag{3}$$

where E is the modulus of elasticity and ν is the Poisson's ratio.

It is assumed that the displacements at the boundary of the analyzed paper are given and they produce the loading vector $\{F\}$. Thus the vector of displacements $\{\delta\}$ is determined by solving the system of linear algebraic equations:

$$[K]\{\delta\}=\{F\}. \tag{4}$$

In the second stage of the analysis the eigenproblem of the membrane is solved. The element has one nodal degree of freedom: the transverse displacement of paper w. The mass matrix has the form:

$$[\overline{M}]=\int [N]^T \rho [N] hdxdy, \tag{5}$$

where ρ is the density of the material of the paper and:

$$[N]=[N_1 \quad \ldots]. \tag{6}$$

The stiffness matrix has the form:

$$[\overline{K}]=\int [G]^T [M][G] hdxdy, \tag{7}$$

where:

$$[G]=\begin{bmatrix} \frac{\partial N_1}{\partial x} & \ldots \\ \frac{\partial N_1}{\partial y} & \ldots \end{bmatrix}, \tag{8}$$

and:

$$[M]=\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}, \tag{9}$$

where stresses σ_x, σ_y, τ_{xy} are determined from:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}=[D][B]\{\delta\}. \tag{10}$$

The i-th eigenfrequency ω_i and the corresponding eigenmode $\{\delta_i\}$ are determined from:

$$\left([\overline{K}]-\omega_i^2[\overline{M}]\right)\{\delta_i\}=0. \tag{11}$$

If the static displacements of the boundary are changed λ times, the loading vector $\{F\}$ and thus the vector of displacements $\{\delta\}$ and further the stresses σ_x, σ_y, τ_{xy} and thus the stiffness matrix of the membrane are changed the same number of times. From the equation:

$$\left(\lambda[\overline{K}]-\lambda\omega_i^2[\overline{M}]\right)\{\delta_i\}=0, \tag{12}$$

it is evident that the eigenmodes of the membrane remain the same, while the eigenfrequencies are related with the eigenfrequencies of the previous problem as:

$$\tilde{\omega}_i=\sqrt{\lambda}\omega_i. \tag{13}$$

Thus the eigenfrequencies of transverse vibrations of the analyzed paper can be controlled by the static tension in the plane of the paper.

Results of analysis of vibrations of the paper

A square piece of paper is analyzed. For the static problem of plane stress the following boundary conditions are assumed: on the lower boundary it is assumed that u=v=0; on the upper boundary it is assumed that u=0 and v=1; on the left and the right boundaries it is assumed that u=0. For the problem of transverse vibrations on all the boundaries it is assumed that w=0.

The contour plot of the transverse displacement for the first eigenmode is presented in Fig. 1, for the second eigenmode in Fig. 2,…, for the tenth eigenmode in Fig. 10.

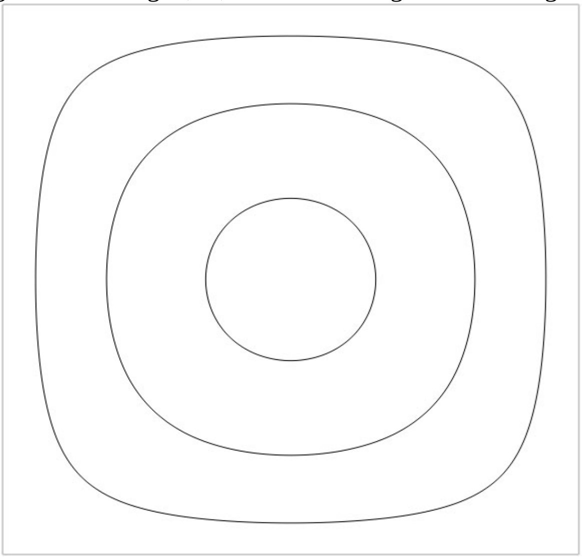


Fig. 1. The first eigenmode

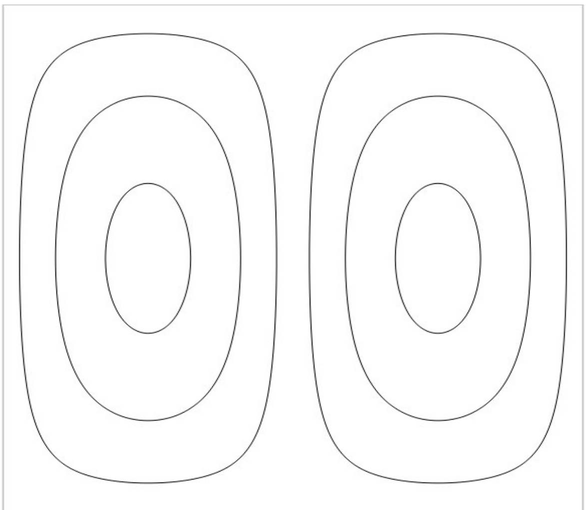


Fig. 2. The second eigenmode

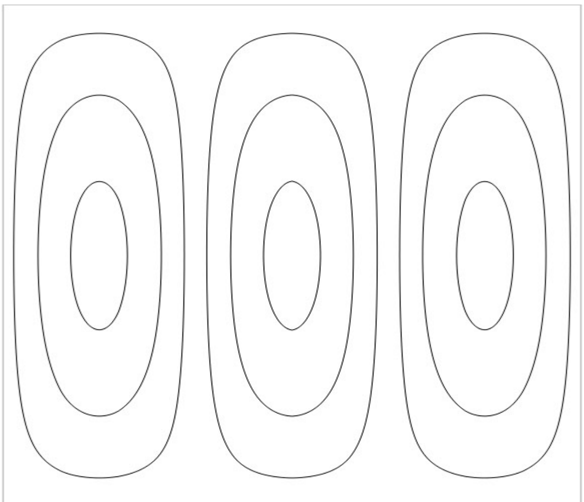


Fig. 3. The third eigenmode

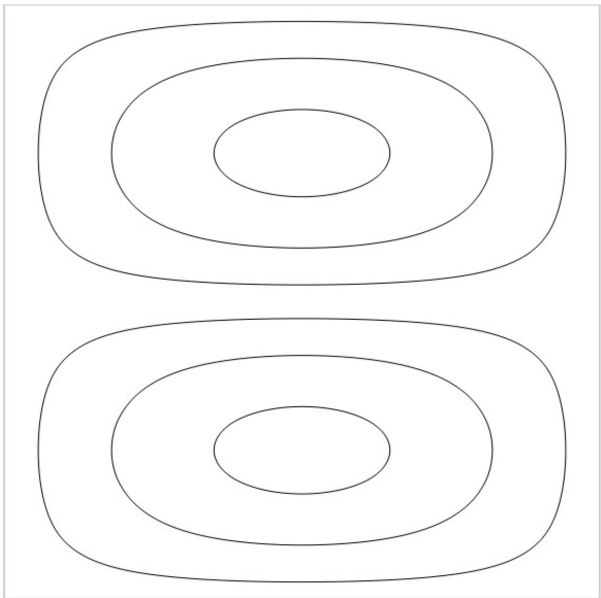


Fig. 4. The fourth eigenmode

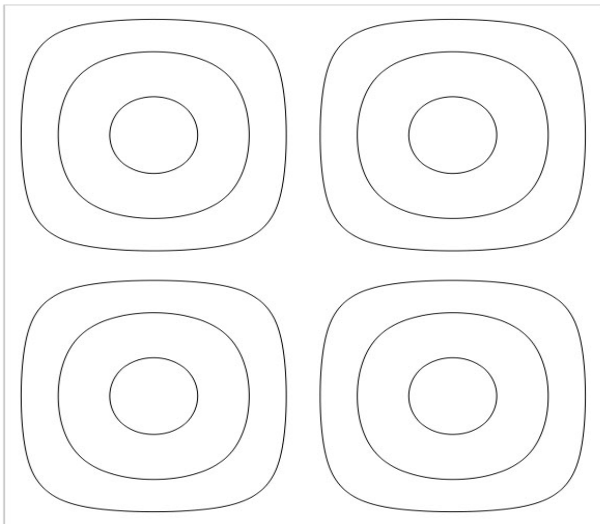


Fig. 5. The fifth eigenmode

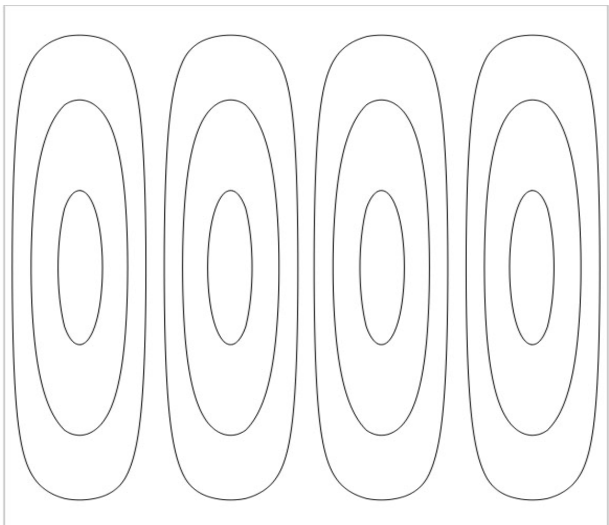


Fig. 6. The sixth eigenmode

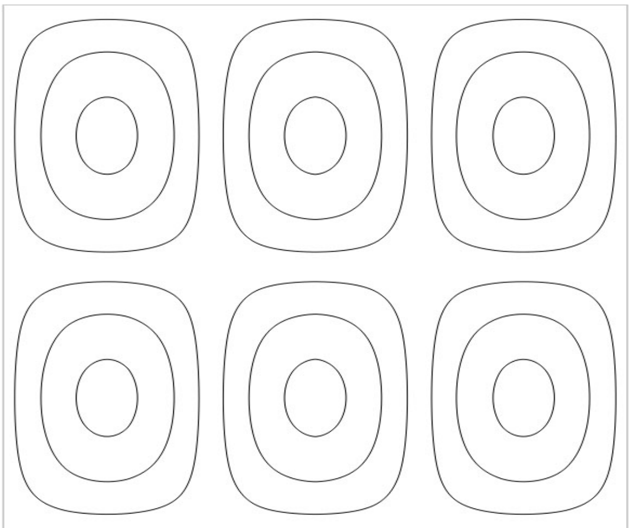


Fig. 7. The seventh eigenmode

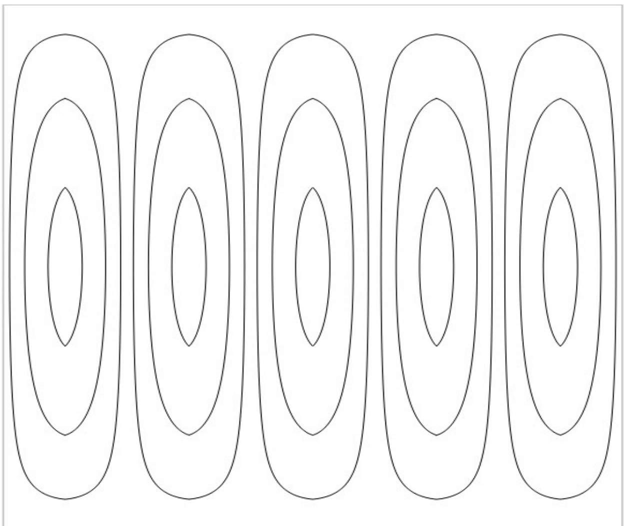


Fig. 8. The eighth eigenmode

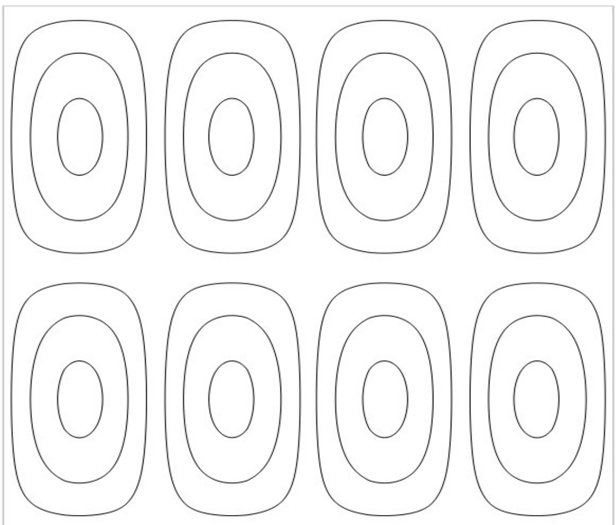


Fig. 9. The ninth eigenmode

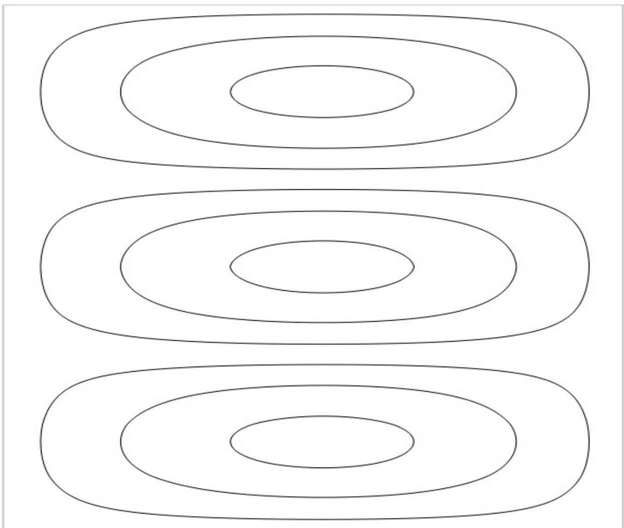


Fig. 10. The tenth eigenmode

Model for the analysis of stability of the paper

Further x, y and z denote the axes of the orthogonal Cartesian system of coordinates. The plate bending element has three nodal degrees of freedom: displacement w in the direction of z axis and rotations Θ_x and Θ_y about axes x and y. Displacements u and v in the directions of axes x and y are expressed as $u=z\Theta_y$ and $v=-z\Theta_x$.

The stiffness matrix has the form:

$$[K]=\int\left([B]^T[D][B]+\left[\overline{B}\right]^T\left[\overline{D}\right]\left[\overline{B}\right]\right)dxdy\,, \qquad (14)$$

where:

$$[B]=\begin{bmatrix} 0 & 0 & \frac{\partial N_1}{\partial x} & \ldots \\ 0 & -\frac{\partial N_1}{\partial y} & 0 & \ldots \\ 0 & -\frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \ldots \end{bmatrix},$$

$$[D]=\frac{h^3}{12}\begin{bmatrix} \frac{E}{1-\nu^2} & \frac{E\nu}{1-\nu^2} & 0 \\ \frac{E\nu}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix},$$

$$\left[\overline{B}\right]=\begin{bmatrix} \frac{\partial N_1}{\partial y} & -N_1 & 0 & \ldots \\ \frac{\partial N_1}{\partial x} & 0 & N_1 & \ldots \end{bmatrix},$$

$$[\overline{D}] = \frac{Eh}{2(1+\nu)1.2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{15}$$

where N_i are the shape functions of the finite element, h is the thickness of the paper, E is the modulus of elasticity and ν is the Poisson's ratio.

It is assumed that the paper is moving in the direction of y axis with constant velocity V. Thus the full acceleration coincides with the convective acceleration:

$$\frac{d^2}{dt^2} = V^2 \frac{\partial^2}{\partial y^2}, \tag{16}$$

where t is the time variable. So the stiffness matrix has the form $[K]$-$V^2[K_V]$, where:

$$[K_V] = \int [B_V]^T \begin{bmatrix} \rho h & 0 & 0 \\ 0 & \frac{\rho h^3}{12} & 0 \\ 0 & 0 & \frac{\rho h^3}{12} \end{bmatrix} [B_V] dxdy, \tag{17}$$

where ρ is the density of the material of the paper and:

$$[B_V] = \begin{bmatrix} \frac{\partial N_1}{\partial y} & 0 & 0 & \ldots \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \ldots \\ 0 & 0 & \frac{\partial N_1}{\partial y} & \ldots \end{bmatrix}. \tag{18}$$

This produces the eigenproblem $([K]$-$V_i^2[K_V])\{\delta_i\}=0$, where V_i are the critical velocities and $\{\delta_i\}$ are the stability eigenmodes.

Results of analysis of stability of the paper

A square piece of paper is analyzed. On the lower and the upper boundaries all the generalized displacements are assumed equal to zero.

The contour plot of the displacement for the first stability eigenmode is presented in Fig. 11, for the second stability eigenmode in Fig. 12, …, for the sixth stability eigenmode in Fig. 16.

The operation of the printing device should take place in the velocity intervals where the full stiffness matrix is positive definite. In order to operate at higher velocities (exceeding the lowest critical velocity), the regions of unstable operation are to be passed. For this purpose special pressing devices to stabilize the motion of the paper are to be used. The locations of the stabilizing elements are determined from the stability eigenmodes (they are to be located at the places of maximum deflections of the eigenmodes that are to be passed when increasing the velocity of the paper motion).

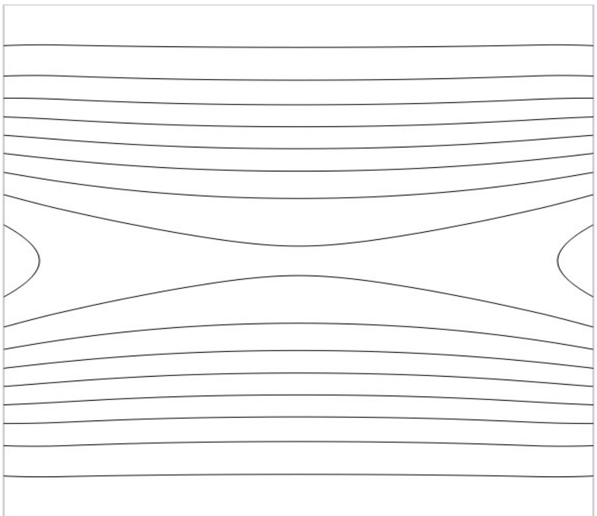


Fig. 11. The first stability eigenmode

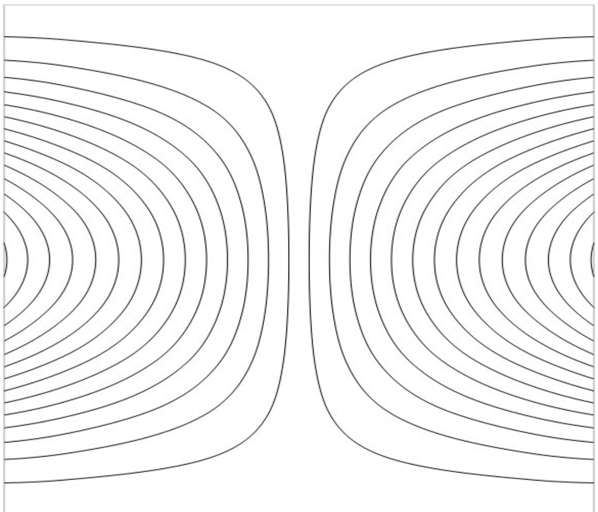


Fig. 12. The second stability eigenmode

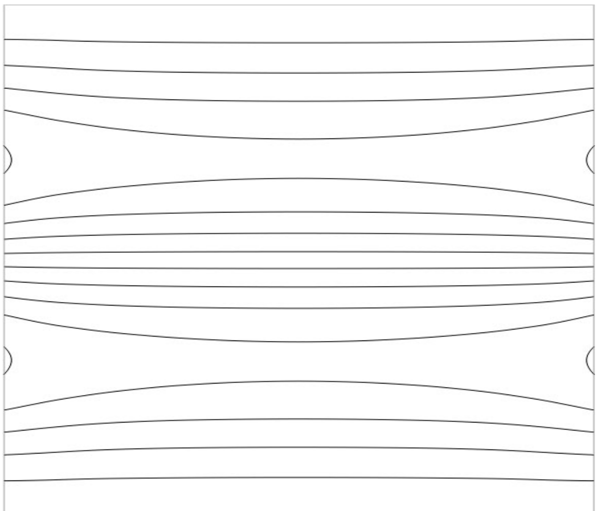


Fig. 13. The third stability eigenmode

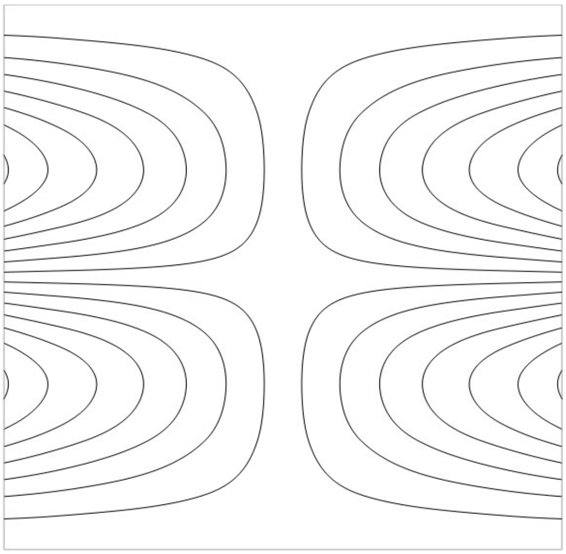


Fig. 14. The fourth stability eigenmode

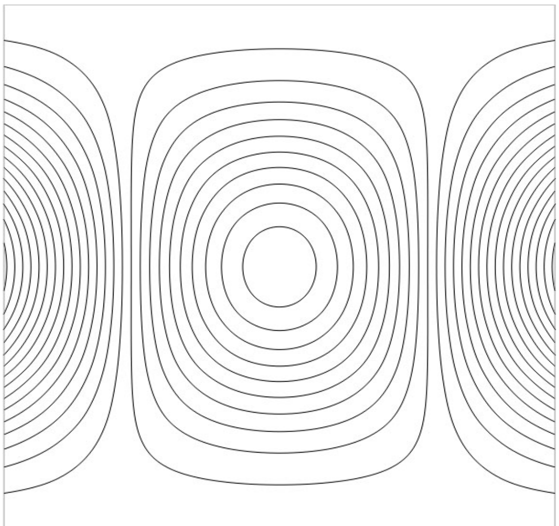


Fig. 15. The fifth stability eigenmode

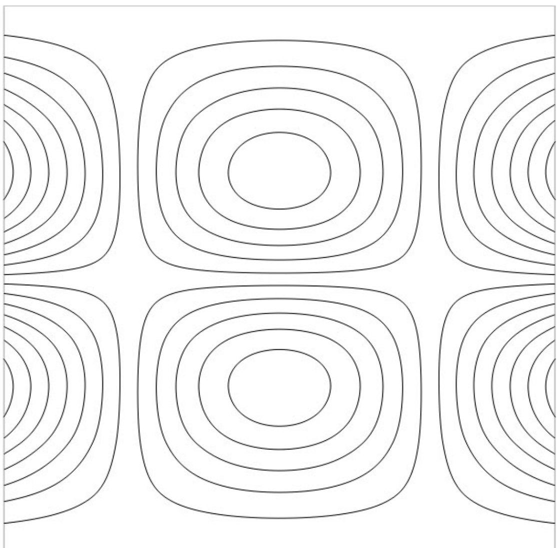
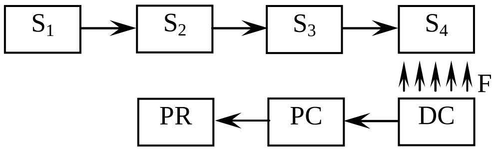


Fig. 16. The sixth stability eigenmode

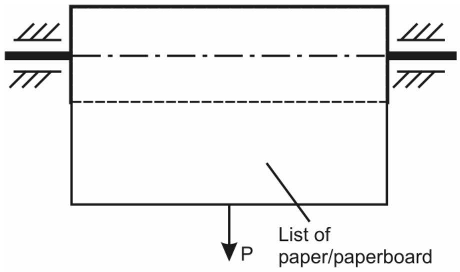
Experimental setup and the method of investigation

In order to determine the dynamical characteristics of the paper under symmetric loading of the sheet of paper, a special experimental setup shown in Fig. 17 was designed and produced. For the visualization of standing waves two methods were used:

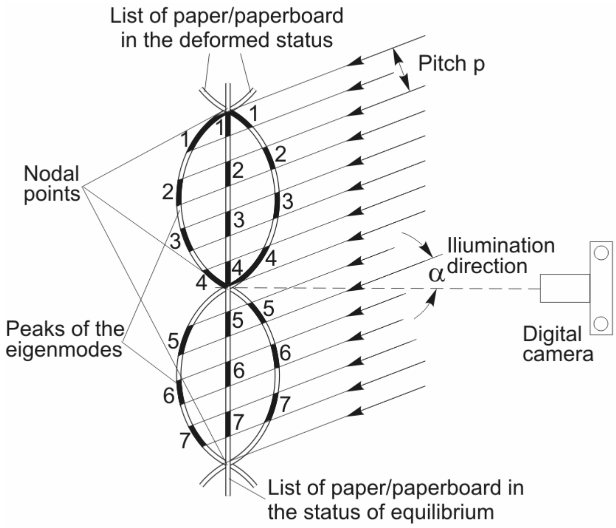
1) a grain of carborundum (150-180 μm);
2) projection moire.



a)



b)



c)

Fig. 17. a) Structural diagram of the experimental setup: S_1 – signal generator; S_2 – amplifier; S_3 – vibroexciter; S_4 – the investigated material; DC – digital camera; PC – personal computer; PR – printer; F – light flux of the digital camera; b) diagram of symmetric loading of the investigated object; P – tension force of the list of paper; c) diagram of the projection moire

The created experimental setup consists of: the generator of vibrations S_1, amplifier S_2, vibroexciter S_3, investigated material S_4, digital camera Canon A700 (*DC*), personal computer *PC*, ink printer HP Deskjet 920c (*PR*) and the optical setup of the projection moire (not indicated in the figure).

The ends of the investigated sheet of paper were fastened between pressing tapes. One of the pressing tapes was fastened to the exciter of vibrations, while the other pressing tape was loaded symmetrically by force *P* of 25.5N. Vibroexciter S_3 was generating longitudinal vibrations of sinusoidal shape of chosen frequency, which excited standing waves in the analyzed material. In the first method the grains of carborundum were put on the surface of the analyzed material for the visualization of nodal lines of standing waves. In the second method for the visualisation of eigenmodes the grid of step *p* was projected by the light rays emitted by a monochromatic light source at a definite angle to the surface of the investigated material [9]. In the first method of experimental investigation the shapes of the nodal lines of the standing waves were obtained. In the second method the eigenmodes were photographed by a digital camera and then processed in the monitor of the personal computer.

In the experiment projection moire was implemented by projecting thin parallel lines of high contrast with the light rays to the vibrating list of paper.

The created experimental setup enables to change:

- the loading force *P* of the investigated material;
- to generate longitudinal vibrations of the investigated material in the frequency interval (8 Hz to 2.2 MHz).

The investigations were performed for Plano Plus paper 80 g/m^2 and Mirabell paperboard 320g/m^2. Technical characteristics of the investigated paper are presented in Table 1.

Table 1

Technical characteristics of the investigated materials

Qualities	Paper Plano Plus	Paper Mirabell
Surface density, g/m^2	80	320
Thickness, μm	98-107	435
Porosity, g/m^3	1	1,36
Longitudinal stiffness, mNm	-	16,2
Perpendicular stiffness, mNm	-	6,4

The obtained results of the experimental investigations under symmetric tension of the sheet are presented in Fig. 18 – Fig. 33. The image of the nodal lines of the standing waves of the first eigenmode for Mirabell paperboard obtained by using the grains of carborundum is presented in Fig. 18, of the second eigenmode – in Fig. 19, of the third eigenmode – in Fig. 20. The image of the nodal lines of the first eigenmode for Plano Plus paper obtained by using the grains of carborundum is presented in Fig. 21, of the second eigenmode – in Fig. 22, ..., of the tenth – in Fig. 30. By using projection moiré, the image of the first eigenmode is presented in Fig. 31, of the second eigenmode – in Fig. 32, of the fifth eigenmode – in Fig. 33.

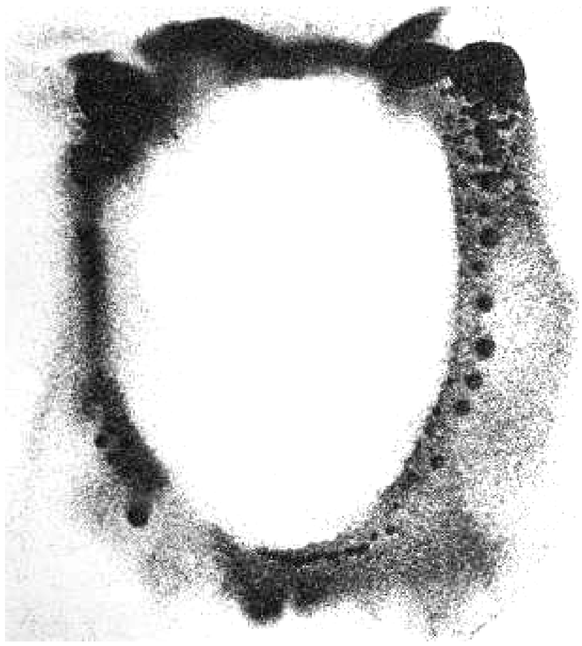


Fig. 18. Nodal line of the standing wave of the first eigenmode of Mirabell paperboard 320 g/m^2: frequency of vibrations 9 Hz, amplitude 2×10^{-6}m, loading force 25.5 N

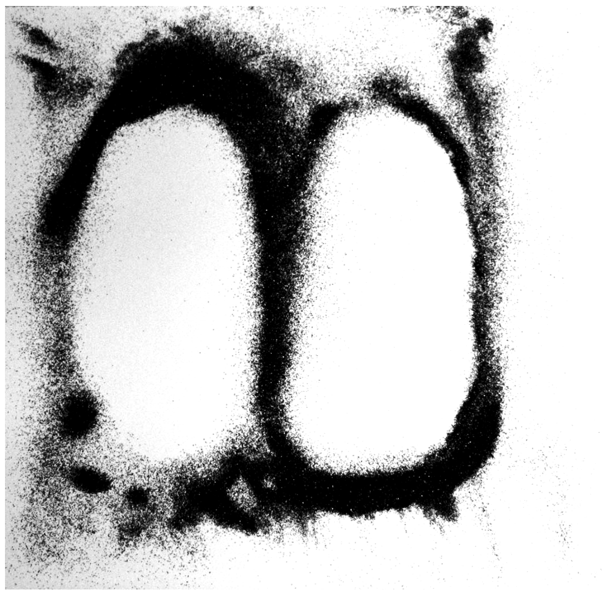


Fig. 19. Nodal lines of the standing waves of the second eigenmode of Mirabell paperboard 320 g/m^2: frequency of vibrations 16 Hz, amplitude 2×10^{-6}m, loading force 25.5 N

Fig. 20. Nodal lines of the standing waves of the third eigenmode of Mirabell paperboard 320 g/m^2: frequency of vibrations 21 Hz, amplitude 2×10^{-6}m, loading force 25.5 N

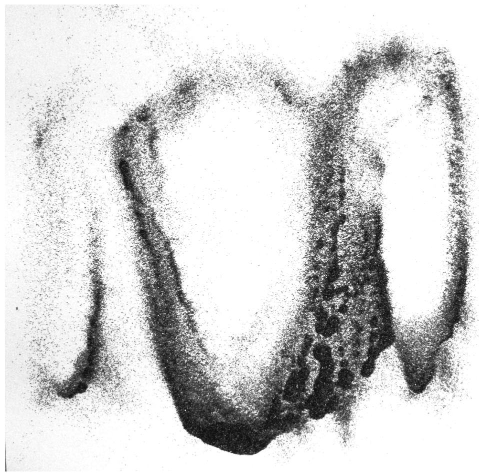


Fig. 23. Nodal lines of the standing waves of the third eigenmode of Plano Plus paper 80 g/m^2: frequency of vibrations 33 Hz, amplitude 2×10^{-6}m, loading force 25.5 N

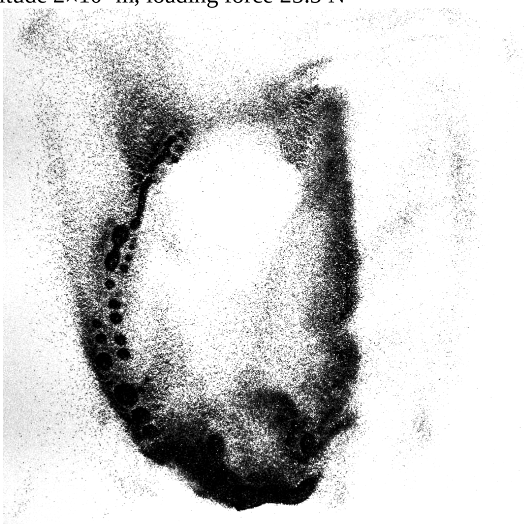


Fig. 21. Nodal lines of the standing waves of the first eigenmode of Plano Plus paper 80 g/m^2: frequency of vibrations 16.8 Hz, amplitude 2×10^{-6}m, loading force 25.5 N

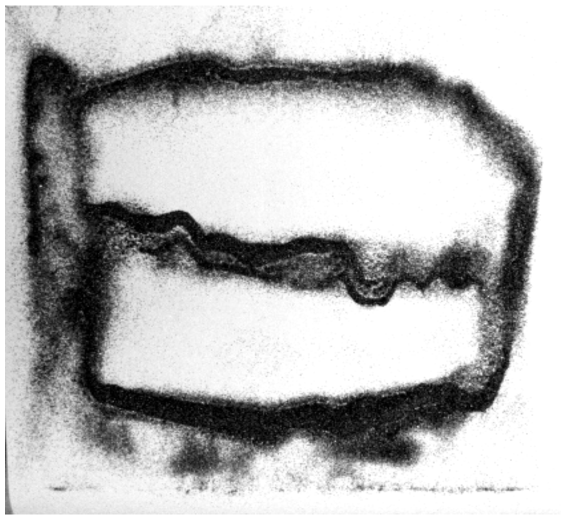


Fig. 24. Nodal lines of the standing waves of the fourth eigenmode of Plano Plus paper 80 g/m^2: frequency of vibrations 46 Hz, amplitude 2×10^{-6}m, loading force 25.5 N

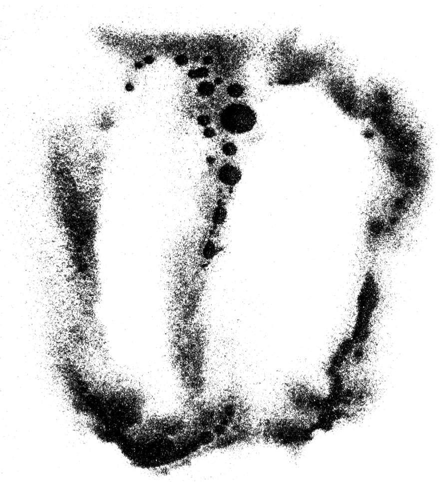


Fig. 22. Nodal lines of the standing waves of the second eigenmode of Plano Plus paper 80 g/m^2: frequency of vibrations 26.5 Hz, amplitude 2×10^{-6}m, loading force 25.5 N

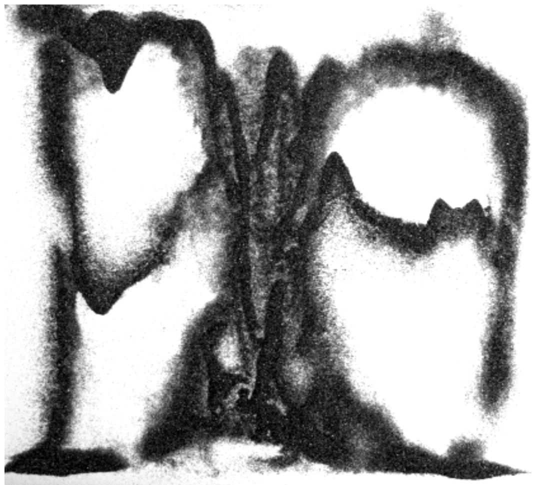


Fig. 25. Nodal lines of the standing waves of the fifth eigenmode of Plano Plus paper 80 g/m^2: frequency of vibrations 65 Hz, amplitude 2×10^{-6}m, loading force 25.5 N

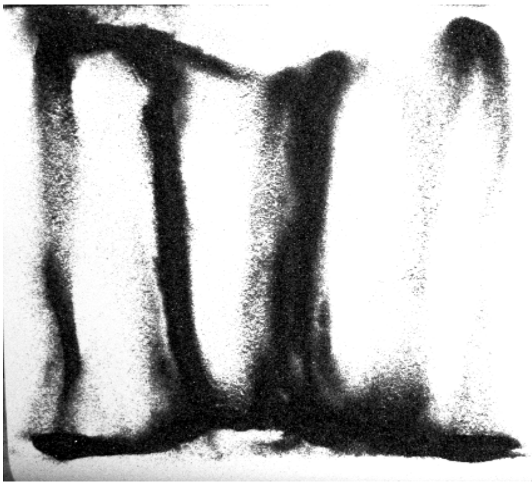


Fig. 26. Nodal lines of the standing waves of the sixth eigenmode of Plano Plus paper 80 g/m^2: frequency of vibrations 83 Hz, amplitude 2×10^{-6}m, loading force 25.5 N

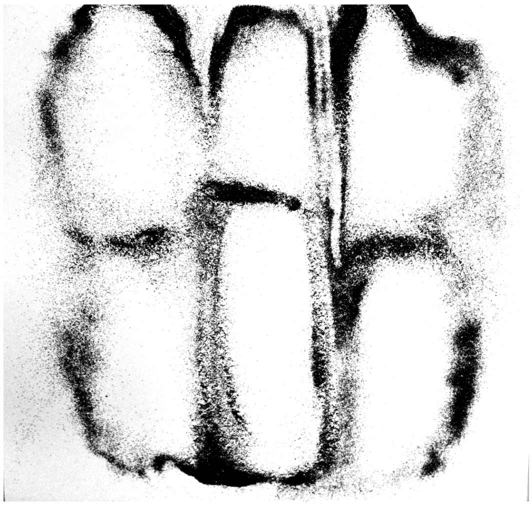


Fig. 27. Nodal lines of the standing waves of the seventh eigenmode of Plano Plus paper 80 g/m^2: frequency of vibrations 102 Hz, amplitude 2×10^{-6}m, loading force 25.5 N

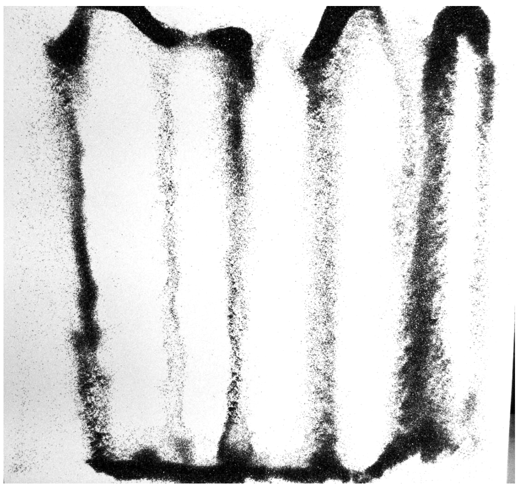


Fig. 28. Nodal lines of the standing waves of the eighth eigenmode of Plano Plus paper 80 g/m^2: frequency of vibrations 118 Hz, amplitude 2×10^{-6}m, loading force 25.5 N

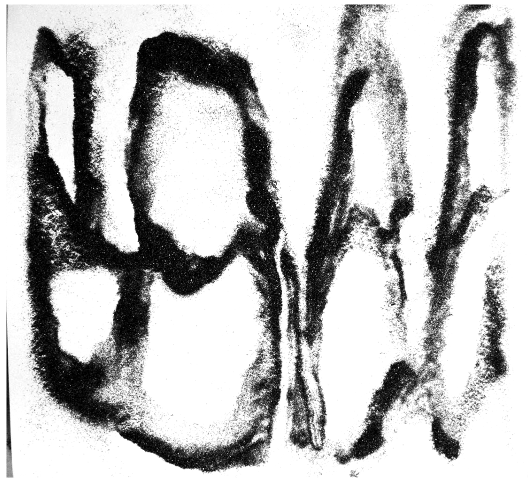


Fig. 29. Nodal lines of the standing waves of the ninth eigenmode of Plano Plus paper 80 g/m^2: frequency of vibrations 139 Hz, amplitude 2×10^{-6}m, loading force 25.5 N

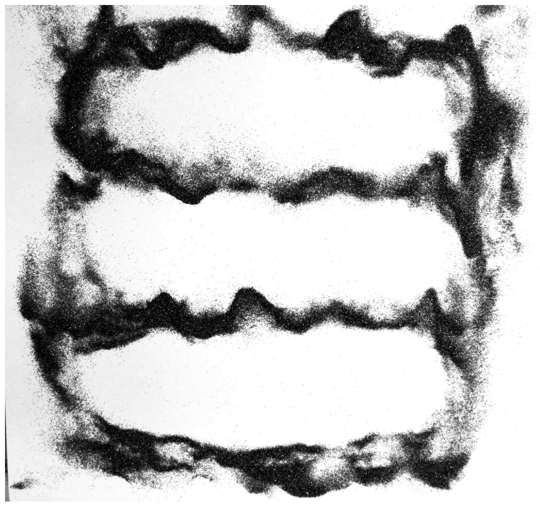


Fig. 30. Nodal lines of the standing waves of the tenth eigenmode of Plano Plus paper 80 g/m^2: frequency of vibrations 225 Hz, amplitude 2×10^{-6}m, loading force 25.5 N

On the basis of experimental investigations by using the grains of carborundum, the nodal lines of standing waves for the first, second, ..., tenth eigenmodes of the sheet of Plano Plus paper and the first, second, third eigenmodes of the sheet of Mirabell paperboard were determined. This rather simple method may be used for the analysis of modal shapes and their distribution in the plane. The precision of this method is not very high. It is used in preparatory investigations, which do not require very high precision.

By using the second method, that is projection moire, the deformations of the sheet of paper were investigated. The shapes of those deformations are similar to the nodal lines of standing waves obtained by the first method. The method of projection moire is more precise and thus it is widely applied for the investigation of the dynamics of paper. In the investigations the first, second and fifth eigenmodes of Plano Plus paper were obtained.

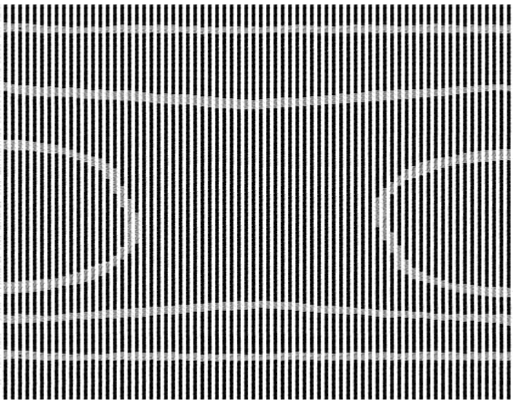


Fig. 31. The first eigenmode of Plano Plus paper 80 g/m^2: frequency of vibrations 128 Hz, amplitude 2×10^{-5}m, loading force 25.5 N

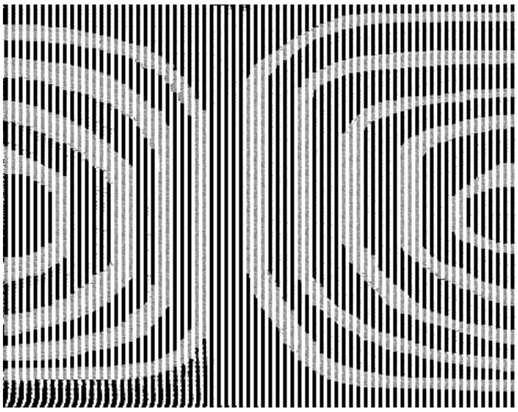


Fig. 32. The second eigenmode of Plano Plus paper 80 g/m^2: frequency of vibrations 157 Hz, amplitude 1.5×10^{-5}m, loading force 25.5 N

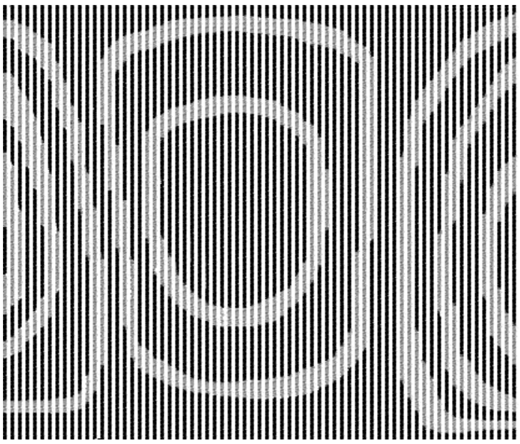


Fig. 33. The fifth eigenmode of Plano Plus paper 80 g/m^2: frequency of vibrations 179 Hz, amplitude 1×10^{-5}m, loading force 25.5 N

Conclusions

The proposed model for the analysis of vibrations of paper in a printing device is based on the assumption that paper performs transverse vibrations as a membrane having stiffness due to static tension in its plane. The static problem of plane stress is solved by assuming that the displacements at the boundary of the analyzed paper are given. Then the vibrations of the investigated paper as of a membrane with stiffness due to the static tension determined previously are analyzed.

It is shown that the eigenfrequencies of transverse vibrations of the analyzed paper can be controlled by the static tension in the plane of the paper.

The model for the analysis of stability of paper in a printing device is proposed.

It is assumed that paper moves with constant velocity in a given direction. It is analyzed using a plate-bending element with the contribution of the forces of inertia from the convective accelerations.

The critical velocities and stability eigenmodes are determined. The operation of the printing device should take place in the velocity intervals where the full stiffness matrix is positive definite. In order to operate at velocities exceeding the lowest critical velocity, the regions of unstable operation are to be passed. For this purpose the locations of the stabilizing elements are determined from the stability eigenmodes.

The experimental setup is created for the investigations on the basis of which the nodal lines of the standing waves of Mirabell paperboard (320 g/m^2) and Plano Plus paper (80 g/m^2) were determined by using grain material. By the method of projection moire the first, second and fifth eigenmodes of Plano Plus paper (80 g/m^2) were determined.

The obtained experimental and theoretical findings are used in the process of design of the elements of the printing device.

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